Fluctuational conductivity in superconductors with nonuniform pairing

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(Submitted 23 February 1993)
Pis'ma Zh. Eksp. Teor. Fiz. **57**, No. 6, 363–365 (24 March 1993)

The critical exponent for the fluctuational conductivity is higher for superconductors with a nonuniform pairing than for those with a uniform pairing.

A nonuniform superconducting state arises in a self-consistent way in a superconducting ferromagnet.[1,2] This state has recently attracted interest because of the possible realization of a spatially nonuniform component of the superconducting order parameter in high-T_c superconductors. This component would be induced by spatially nonuniform insulating correlations (e.g., by antiferromagnetic correlations in the copper-oxide perovskites[3]).[4,5]

It was recently shown[6] that in the region in which a fluctuational average order exists the Larkin–Ovchinnikov–Fulde–Ferrell nonuniform superconducting state is characterized by fluctuational effects which are much stronger than in the uniform case.[7,8] In particular, the critical region, the correlation length ξ, and the pseudogap in the density of states are all larger. The reason for these differences is that the phase volume of the correlated states is larger by a factor of $(\xi q_0)^2$ (q_0 is the modulus of the nonuniform-pairing vectors) than that of the uniform case.

In this letter we calculate the fluctuational conductivity above the transition point for the case of a nonuniform superconducting state. Actual measurements of this conductivity, as well as other kinetic coefficients, may prove a reliable method for observing a nonuniform superconductivity.

The static conductivity σ at low frequencies ω is found from the relation

$$\mathbf{j}_\omega = i\omega\sigma\mathbf{A}_\omega.$$

Because of strong pair-breaking effects (in particular, because of the internal magnetic field), the Aslamazov-Larkin diagram is predominant[8] (Fig. 1). Corresponding to this diagram is the expression

$$\mathbf{j}_\omega = \frac{e^2}{m^2} T \sum_{\omega_n} \int \frac{d^3q}{(2\pi)^3} [\mathbf{C}(\mathbf{q},\omega,\omega_n)\mathbf{A}_\omega]\mathbf{C}(q,\omega,\omega_n)\Gamma(\omega_n,\mathbf{q})(\Gamma(\omega_n-\omega,\mathbf{q}), \quad (1)$$

where the vertex $C(\mathbf{q},\omega,\omega_n)$ (Fig. 2) is

$$\mathbf{C}(\mathbf{q},\omega,\omega_n) = T \sum_{\epsilon_n} \int \frac{d^3p}{(2\pi)^3} \mathbf{p}[G^+(\mathbf{p},\epsilon_n)G^-(\mathbf{q}-\mathbf{p},\omega_n-\omega-\epsilon_n)G^+(\mathbf{p},\epsilon_n+\omega)$$
$$+G^-(\mathbf{p},\epsilon_n)\Gamma^+(\mathbf{q}-\mathbf{p},\ \omega_n-\omega-\epsilon_n)G^-(\mathbf{p},\epsilon_n+\omega)]. \quad (2)$$

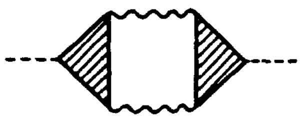

FIG. 1.

We will go through the calculations for a superconductor with an internal magnetic field h.[1,2] In this case the Green's functions of the pairing electrons are

$$G^{\pm}(\mathbf{p},\epsilon_n)=\frac{1}{i\epsilon_n-\xi(\mathbf{p})\pm h}. \tag{3}$$

The fluctuation propagator $\Gamma(\omega_n,\mathbf{q})$ for this system was calculated in Ref. 6:

$$\Gamma(\omega_n,\mathbf{q})=-\frac{1}{N(0)}\frac{1}{\eta+D(q-q_0)^2+\beta|\omega_n|}. \tag{4}$$

Here $N(0)$ is the density of states at the Fermi level, $\eta=(h-h_c)/h_c$, h_c is the critical field, $D=v_F^2[2(v_F^2q_0^2-4h_c^2)]^{-1}$, v_F is the Fermi velocity, and $\beta=\pi/(2v_Fq_0)$. For the Larkin–Ovchinnikov–Fulde–Ferrell nonuniform superconducting state, the quantity q_0 is related in a self-consistent way to the internal field h by

$$q_0=\frac{\alpha h}{v_F}\left(1+\frac{8\pi^2}{3(\alpha^2-4)}\frac{T^2}{h^2}\right),$$

where T is the temperature, and $\alpha=2.4$. Since all the singularity near the transition point in expression (1) is in the propagators $\Gamma(\omega_n,\mathbf{q})$, we can treat $C(\mathbf{q},\omega,\omega_n)$ as independent of the frequency and take it to be

$$\mathbf{C}(\mathbf{q},\omega,\omega_n)=\mathbf{q}\,\frac{p_F^3}{4\pi(v_Fq_0)^2}[C(h)+C(-h)],$$

where

$$C(h)=\mathrm{Re}\left[\psi\left(\frac{1}{2}+\frac{ih}{2\pi T}+\frac{iq_0v_F}{4\pi T}\right)+\psi\left(\frac{1}{2}+\frac{ih}{2\pi T}-\frac{iq_0v_F}{4\pi T}\right)\right]$$

$$\mathbf{j}_\omega=i\omega\sigma\mathbf{A}_\omega.$$

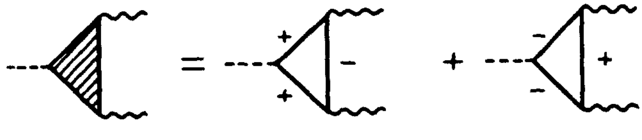

FIG. 2.

$$-4\mathrm{Re}\left[\frac{\pi T}{iq_0v_F}\ln\frac{\Gamma\left(\frac{1}{2}+\frac{ih}{2\pi T}+\frac{iq_0v_F}{4\pi T}\right)}{\Gamma\left(\frac{1}{2}+\frac{ih}{2\pi T}-\frac{iq_0v_F}{4\pi T}\right)}\right], \tag{5}$$

p_F is the Fermi momentum.

After an analytic continuation of expression (1) to real ω, we find the fluctuational correction to the conductivity, σ', in the approximation linear in ω:

$$\sigma'=\frac{e^2}{m^2}\frac{q_0^2}{3}\left[\frac{p_F^3}{4\pi(v_Fq_0)^2}[C(h_c)+C(-h_c)]\right]^2\frac{N^{-2}(0)}{2\pi\beta^2}$$

$$\times\int\frac{d^3q}{(2\pi)^3}\int dyc\tanh\frac{y}{2T}\frac{y(z^2-y^2)}{(z^2+y^2)^2}, \tag{6}$$

where $z=\beta^{-1}[\eta+D(q-q_0)]^2$.

Analysis of expression (6) reveals two regions which differ in the characteristic behavior of σ':

$$1)\quad \eta\ll\beta T;\quad \sigma'=T\frac{\eta^{-5/2}}{D^{1/2}}f\left(\frac{h_c}{T},\frac{v_Fq_0}{T}\right); \tag{7a}$$

$$2)\quad \eta\gg\beta T;\sigma'=\frac{20}{9}\beta\eta^{-7/2}\frac{T^2}{D^{1/2}}f\left(\frac{h_c}{T},\frac{v_Fq_0}{T}\right); \tag{7b}$$

where

$$f\left(\frac{h_c}{T},\frac{v_Fq_0}{T}\right)=\frac{\beta}{\pi 2^7}\frac{[C(h_c)+C(-h_c)]^2}{m(p_Fv_F)}\frac{e^2n}{m},$$

and n is the density of electrons.

This correction is valid at those temperatures and those fields h_c for which it does not exceed the conductivity of a normal metal.

If the nonuniform superconducting state arises because of spatially nonuniform insulating correlations (in high-T_c superconductors), it exists against the background of ordinary uniform superconductivity. Here we are no longer talking about low temperatures, since correction (7a) corresponds to the nonuniform component. This correction differs from the Aslamazov-Larkin correction for the uniform component in that its divergence is sharper (the critical exponent is 5/2 in comparison with the Aslamazov-Larkin value of 1/2). However, the overall correction also depends on the relative weights of the uniform and nonuniform components, as is seen in the appearance of slope changes on the experimental curves.

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Translated by D. Parsons