Phase diagram of a superconductor with a twin plane

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A new model explains the behavior of the absolute-instability field of the supercooled normal state observed experimentally in crystalline superconductors with a twin plane. The *H-T* phase diagram of tin with a twin plane is discussed.

A superconductivity localized near a twin plane in a crystal was discovered by Khlyustikov and Buzdin about a decade ago (see the reviews).[1,2] It was established experimentally, in particular, that the superconducting transition temperature near a twin plane, T_d, may be either higher or lower than the bulk superconducting transition temperature T_c (below we assume $T_d > T_c$). Studies have also been made of the behavior of a superconductivity localized near a twin plane in a magnetic field and the effect of this superconductivity on the onset of a bulk superconductivity.[3]

The experimental picture is shown in Fig. 1 (for tin with an angle $\alpha = 1.4\times10^{-3}$ at the vertex of the twin wedge).[4] Here $H_d(T)$ is the critical magnetic field of a thermodynamic equilibrium with normal and superconducting states of the twin plane, $H_m(T)$ is the critical supercooling field (the absolute-instability field) of the normal state of the twin plane, $H_b(T)$ is the field below which the volume of the sample containing the twin plane cannot be supercooled, and $H^*(T)$ is the critical supercooling field of a sample with a twin-plane superconductivity. Also shown in this figure are the bulk critical field $H_c(T)$ and the surface-superconductivity field $H_{c3}(T)$ for tin (a type-I superconductor). The temperature T_b is found by extrapolating $H_b(T)$ to the temperature axis. The orientation of the field with respect to the twin plane cannot be controlled experimentally.

An important feature of this phase diagram is that the absolute-instability field of the normal state, H_m, is parallel to the field H_{c3}, in contrast with (for example) experiments[1,2] on niobium, in which H_m was parallel to H_{c2}.

To find the absolute-instability field of the normal state of the twin plane (lines H_m and H_b in Fig. 1), we write a Ginzburg–Landau functional, taking into account the changes in the conditions for the onset of superconductivity near the twin plane and the finite reflection coefficient of the twin plane for electrons [5] (the z axis runs perpendicular to the twin plane):

$$F = F_v + F_s,$$

$$F_v = \int dV \left\{ a_0\tau|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{1}{4m}|D_i\psi|^2 + \frac{1}{8\pi}B^2 \right\}, \qquad (1)$$

$$F_s = \int dV \left\{ \gamma(\mathbf{r})(|\psi_+|^2 + |\psi_-|^2) + \frac{1}{4m\alpha}\left|\psi_+ - \psi_-\right|^2 \delta(z) \right\},$$

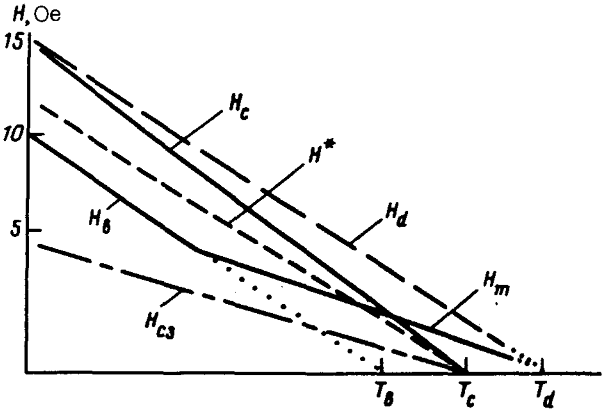

FIG. 1.

where $D_i = -i\nabla_i - (2e/c)A_i$, $\mathbf{B} = \text{curl}\ \mathbf{A}$ is the magnetic field, $\psi_\pm = \psi(z = \pm 0)$, $\gamma(\mathbf{r})$ is a function, localized near the twin plane, which characterizes the change in the superconducting coupling constant [below we assume $\gamma(\mathbf{r}) = -\gamma\delta(z)$], and α is determined by the transmission of the twin plane.

The assumption $\alpha = 0$ (that the twin plane is transparent to electrons) was used in Refs. 1 and 2. In this case the linearized Ginzburg–Landau equation in a magnetic field which follows from (1) has the form of a one-dimensional Schrödinger equation with a potential which is the sum of the harmonic-oscillator potential and an attractive δ-function. Analysis of the resulting transcendental equation for H_m (see below) leads to the relation $H_m \| H_{c2}$ mentioned above.

Let us examine the opposite limiting case, in which the twin plane has a low transmission. In the limit $\alpha \to \infty$, the twin plane is essentially an insulating interlayer. The twins on the right and left can then be treated as independent, and we can take up the problem of finding H_m for a half-space.

It is convenient to work below in terms of the dimensionless variables[5]

$$F_v = \int dV \left\{ t|\tilde{\psi}|^2 + \frac{1}{2}|\tilde{\psi}|^4 + |(-i\tilde{\nabla}_i - a_i)\tilde{\psi}|^2 + k^2h^2 \right\}, \tag{2}$$

where

$$t = \frac{T - T_c}{T_d - T_c}, \quad \tilde{\psi} = \frac{\psi}{\psi_d}, \quad \tilde{x}_i = x_i\frac{\sqrt{\tau_d}}{\xi}, \quad a_i = A_i\frac{2\pi\xi\sqrt{\tau_d}}{\Phi_0},$$

$$h_i = B_i\frac{2\pi\xi^2}{\Phi_0\tau_d}, \quad \psi_d = \sqrt{\frac{a_0\tau_d}{b}}, \quad \tau_d = \frac{T_d - T_c}{T_c},$$

$\Phi_0 = hc/2e$ is the flux quantum, and k is the Ginzburg–Landau parameter (we will be omitting the tildes below).

We assume that the magnetic field is directed along the y axis, $a = (hz, 0, 0)$, and $\psi=\psi(x, z)$. From (2) we find the equation $(z>0)$

$$\left[-\frac{\partial^2}{\partial z^2}+\left(-i\frac{\partial}{\partial x}-hz\right)^2\right]\psi(x,z)=-t\psi(x,z). \tag{3}$$

A boundary condition on ψ is found from the δ-function term in (1):

$$\frac{\partial\psi_+}{\partial z}=-\psi_+ .$$

We seek the order parameter in the form $\psi(x, z) = \exp(ihz_0x)f(z)$. The equation for $f(z)$ then becomes

$$-\frac{d^2f}{dz^2}+h^2(z-z_0)^2f=-tf, \quad z>0.$$

Proceeding as in the known problem [6] of finding the surface-superconductivity field, we find a Schrödinger equation with a potential

$$V(z)=\begin{cases} h^2(z-z_0)^2, & z>0,\\ h^2(z+z_0)^2, & z<0,\end{cases} \tag{4}$$

with the boundary conditions $f(+0)=f(-0)$ and $df(+0)/dz=-f(0)$. A solution of this equation is

$$f(z)=\begin{cases} C_+\exp\left(-\frac{h}{2}(z-z_0)^2\right)H_\nu(\sqrt{h}(z-z_0)), & z>0,\\ C_-\exp\left(-\frac{h}{2}(z+z_0)^2\right)H_\nu(-\sqrt{h}(z+z_0)), & z<0,\end{cases} \tag{5}$$

where $H_\nu(x)$ is the Hermite function,[7] and $\nu=-\frac{1}{2}(1+t/h)$.

Joining the solutions with the help of the boundary conditions, we find the following equation, which implicitly determines the function $h(t, z_0)$:

$$\frac{H_{\nu-1}(-\sqrt{h}z_0)}{H_\nu(-\sqrt{h}z_0)}=\frac{1+hz_0}{(1+t/h)\sqrt{h}}. \tag{6}$$

The $H_m(T)$ dependence is determined by the maximum value of the function $h(t, z_0)$ over z_0. Let us analyze some limiting cases of (6). In the case $\gamma = 0$ (in which there is no enhancement of the superconductivity near the twin plane), the one drops out of the numerator, and we return to the known equation [6] for H_{c3}. In the case $z_0 = 0$, by using the property $H_\nu(0)=\Gamma(-\nu/2)/2\Gamma(-\nu)$ of the Hermite functions [$\Gamma(x)$ is the gamma function], we find the equation

$$B\left[\frac{1}{2},\frac{1}{4}\left(1+\frac{t}{h}\right)\right]=2\sqrt{\pi h} \tag{7}$$

[$B(x, y)$ is the beta function], which was found in Refs. 1 and 2 for H_m in the case $\alpha = 0$. In the limit $h \to 0 (t \to 1)$, H_m has a square-root behavior, while in strong fields

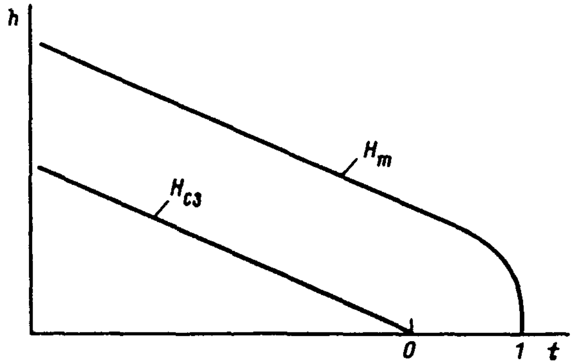

FIG. 2.

we find $t/h \to -1$; i.e., H_m becomes parallel to H_{c2}. (A behavior of this sort has been seen experimentally[1,2] in niobium; this result apparently means that the twin plane in niobium is transparent to electrons.)

It is difficult to find from (6) the explicit temperature dependence of the maximum of $h(t, z_0)$, i.e., $H_m(T)$. We therefore content ourselves with a simple qualitative analysis, which yields the following results. In strong fields, $h \gg 1$, we have $H_m(T) \| H_{c3}(T)$. In weak fields, ignoring the second term in the numerator, we find Eq. (7). In the limit $h \to 0$, $t \to 1$, this equation yields $h \sim \sqrt{1-t}$. Figure 2 shows the qualitative behavior of $H_m(T)$.

The assumption that the twin boundary in tin has only a low transmission for electrons thus leads to an explanation of the experimental behavior of the absolute-instability field of the normal state.

The apparent reason for the $H_m - H_b$ slope change is that, when twinning occurs in the crystal, regions characterized by a distinct transition temperature T_b and a distinct coherence length ξ_0 form in the crystal. As a result, the absolute-instability field has a different slope.

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