# Mechanism for the breaking of symmetry of the directions of the velocity shear in the Rayleigh–Taylor instability

S. V. Bazdenkov and O. P. Pogutse
*Kurchatov Institute Russian Science Center, 123182, Moscow*

(Submitted 4 March 1993)
Pis'ma Zh. Eksp. Teor. Fiz. **57**, No. 7, 410–413 (10 April 1993)

A few-parameter model (similar to the Lorentz model) is proposed for describing the generation of shear flow in a plasma or liquid as a result of the onset of the Rayleigh-Taylor instability. The sign asymmetry of the velocity shear in the flow which is generated and also the sign asymmetry of the electric field in the plasma are explained on the basis of this model.

So-called *L–H* transitions, in which the plasma confinement time roughly doubles, are frequently observed in tokamaks. They are usually accompanied by an elevated MHD activity at the edge of the plasma, manifested in the excitation of ELMs (edge localized modes) and the generation of a shear flow (or an electric field) at the plasma periphery. Numerous phenomenological models have been proposed to explain these effects, but these models clearly underestimate the role of the shear flow which arises. It was not until the last international conference on plasma physics, in Würzburg, that some more self-consistent models were proposed in a few papers.[1–3] These models are based on a relationship between the flute instability, on the one hand, and *L–H* transitions and the generation of a shear flow in the plasma, on the other. Two of these papers[2,3] describe the plasma inside the separatrix, where the magnetic field lines are closed, where the curvature of a field line varies, and where the problem is fundamentally three-dimensional. The plasma outside the separatrix and near the separatrix was studied in Ref. 1. We believe that this region is the more important one, since the dissipation is greater here, and the curvature of the field lines has a constant sign. The (flute) instability is thus more pronounced here. In this case we can average the equations for the plasma dynamics along a field line, and we can lower the dimensionality of the problem. The corresponding equations, in dimensionless variables, are[1]

$$\frac{\partial n}{\partial t}+[\nabla\phi,\nabla n]+g_T\frac{\partial\phi}{\partial y}=D\cdot\Delta n+\frac{n}{\tau_n}, \tag{1}$$

$$\frac{\partial u}{\partial t}+[\nabla\phi,\nabla u]+g\frac{\partial n}{\partial y}=\mu\cdot\Delta u+\frac{\phi}{\tau_\phi}, \tag{2}$$

$$u=\Delta\phi. \tag{3}$$

The equations were constructed in the approximation of a strong magnetic field. Within terms with $\tau_n$ and $\tau_\phi$, these equations are the same as the equations describing the convection of a heated liquid in the gravitational field in the Boussinesq approximation. The customary boundary conditions for system (1)–(3) correspond to two impenetrable walls which bound the plasma or liquid layer in the "radial" (or vertical) direction, $x$, while there is periodicity along the coordinate $y$ (an analog of the poloidal angle).

A numerical simulation of system (1)–(3) has shown[1] that a shear flow is generated in the poloidal direction for certain values of the parameters of the problem. There are two characteristic regimes here. Slightly above the critical level (with a small average density gradient), the velocity of the flow which arises is also extremely small, while the particle fluxes to the wall are large (this is the $L$ regime). As we move progressively further above the critical level, the velocity of the generated flow also increases, the fluxes to the wall decrease (this is the $H$ regime), and the flow become quasiperiodic in time (ELMs). A point of importance to the discussion below is that only a few harmonics (about five to eight) are excited in these regimes. This circumstance suggests that an attempt should be made to describe these processes by a far simpler system of ordinary differential equations for the amplitudes of the harmonics, similar to the Lorentz system.

To describe the generation of a flow, it is sufficient to consider only five amplitudes, $X$, $Y$, $Z$, $V$ and $W$:

$$n=Y\cdot\sin(2\pi my/y_0)\sin(\pi x/x_0)+Z\cdot\sin(2\pi x/x_0), \tag{4}$$

$$\phi=(V+X\cdot\cos(2\pi my/y_0))\sin(\pi x/x_0)+W\cdot\sin(2\pi my/y_0)\sin(2\pi x/x_0). \tag{5}$$

Here $x_0$ and $y_0$ are the dimensions of the plasma layer, and $m$ is a characteristic index of the poloidal mode. The functions $X$, $Y$, and $Z$ are the same as those in the Lorentz model, while $V$ and $W$ are required to describe a new effect: the generation of an average poloidal flow (the amplitude $V$). The corresponding correction to the "vortex" ($W$) is necessary in order to couple $V$ with the Lorentz system.

Using (4) and (5), we find from Eqs. (1)–(3) the following system of equations:

$$\frac{dX}{dt}=VW+\gamma_g Y-\gamma_X X, \tag{6}$$

$$\frac{dY}{dt}=-ZX+\gamma_g X-\gamma_Y Y, \tag{7}$$

$$\frac{dZ}{dt}=YX-\gamma_Z Z, \tag{8}$$

$$\frac{dV}{dt}=-WX-\gamma_V V, \tag{9}$$

$$\frac{dW}{dt}=-cVX-\gamma_W W. \tag{10}$$

All the $\gamma's$ are positive here and are expressed in terms of characteristic parameters of the problem; here $0<c<3/4$.

The few-parameter model in (6)–(10) reproduces the results of a more detailed two-dimensional simulation of the plasma in a qualitatively correct way {see Ref. 1

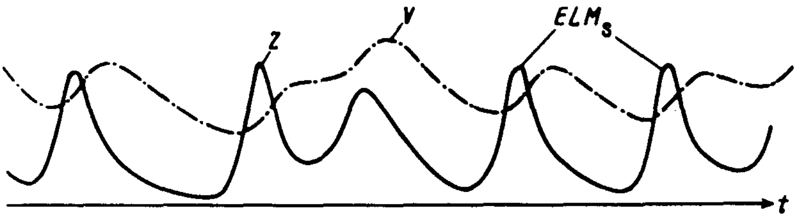


FIG. 1. Time evolution of the amplitude of the shear velocity $V$ and of the average particle flux to the wall, $Q \propto -\partial n/\partial x \propto Z$. This is an example of the numerical integration of Eqs. (6)–(10) with $\gamma_g = 1$, $c=0$, and damping coefficients $\gamma \approx 0.1$.

and Fig. 1, which shows some typical results of a numerical integration of Eqs. (6)–(10)}. This model also yields a description of the experimentally observed[4] sign of the shear in the flow velocity. Here, however, we should take into account an effect which has been passed over previously: the onset of an electric field in the plasma as a result of the difference between the confinement of ions and that of electrons, particularly near the separatrix. As we will show below, even an obviously weak startup electric field in the plasma, due to the difference between the classical mechanisms for the transport of ions and electrons, unambiguously determines the sign of the velocity shear in the flow which is generated.

Figure 2 shows the distribution $P$ of the relative time which the velocity $V$ spends in a given interval. This distribution was found from two numerical solutions of Eqs. (6)–(10), differing in the signs of the initial conditions [long time intervals of the process were considered, up to 1000 (or even more) reciprocal growth rates of the Rayleigh-Taylor instability]. The primary feature of this probability distribution—its essentially symmetric shape—stems from the absence of a special direction for the vorticity in the original equations (1)–(3), and thus in model (6)–(10). A source of asymmetry in $P$ might be the difference in the confinement of electrons and ions which

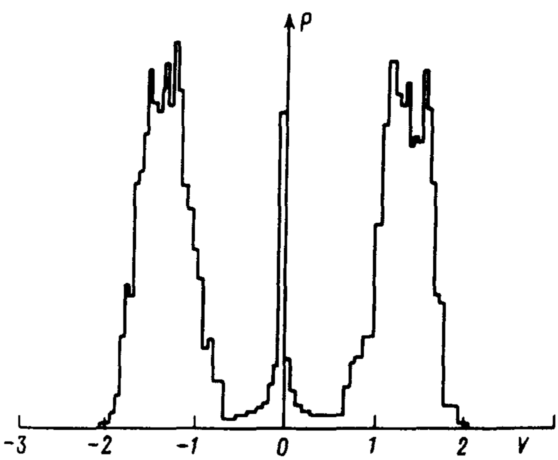


FIG. 2. The probability distribution $P(V)$ for $V_0 = 0$ and $\gamma_g = 1$ found from two solutions differing in the signs of the initial conditions (specifically, in the sign of the variable $X$).

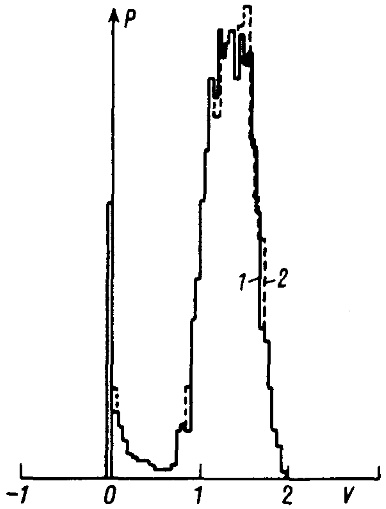


FIG. 3. The same as in Fig. 2, but for $V_0 = 0.01$ (curve 1) and $V_0 = 0.05$ (2).

is characteristic of a plasma, particularly near and outside the separatrix. The loss of electrons along closed field lines is usually greater than the transverse ion transport, so the plasma is charged.[4] In other words, a radial electric field arises, along with a corresponding shear in the poloidal rotation velocity. This factor or mechanism, which is always at work, can formally be incorporated in our few-parameter model by adding a source of a "seed" velocity $V_0$ to the equation for the velocity $V$. While the equations otherwise remain unchanged, Eq. (9) becomes

$$\frac{dV}{dt} = -WX - \gamma_V(V - V_0). \tag{9a}$$

Analysis of numerical solutions of system (6)–(8), (9a), (10) as a function of the seed velocity amplitude $V_0$ shows that the characteristic amplitude of the solution is substantially higher (by two or more orders of magnitude) than $V_0$ and that it is sensitive only to the sign of $V_0$—not its magnitude. The sign of the velocity shear which is generated is the same as the sign of $V_0$. In other words, the electric field which is generated is in the same direction as the startup field. Specifically, values $V_0 \approx 10^{-3}$–$10^{-2}$ in our case "control" the sign of the velocity shear at the characteristic amplitudes of this shear, $V \approx 1-10$, i.e., when the difference between the amplitudes of the controlling and controlled signals is two orders of magnitude. Figure 3 shows distributions of the probability $P$ for the same parameter values (and for the same initial conditions) as in Fig. 2, but with $V_0 = 10^{-2}$ and $V_0 = 5 \times 10^{-2}$. These distributions demonstrate the obvious asymmetry of the sign of the shear velocity. The dynamics and energetics of the flow which is generated are controlled by the Rayleigh–Taylor instability. On the other hand, a weak seed deviation from an ambipolar loss of ions and electrons controls the sign of the velocity shear.

[1] N. N. Kukharkin, M. V. Osipenko, O. P. Pogutse, and V. M. Gribkov, "The studies of convection in the

edge tokamak plasma," D-4-14, *Fourteenth International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Würzburg, Germany, 1992.*
[2] P. H. Diamond, V. Shapiro, and V. V. Schevchenko *et al.*, *ibid*, "Self-regulation of fluctuation levels and transport by generation of sheard electric fields," D-2-4.
[3] J. F. Drake, J. M. Finn, P. Guzdar *et al.*, *ibid*, "Tokamak edge transport, sheared flow and the *L–H* transition," D-2-5(C).
[4] R. J. Taylor, M. L. Brown, B. D. Fried *et al.*, Phys. Rev. Lett. **63**, 2365 (1989); R. J. Groebner, K. H. Burrell, and R. P. Seraydarian, Phys. Rev. Lett. **64**, 3015 (1990).

Translated by D. Parsons