Pumping of a static magnetic field by an alternating field in a type-II superconductor

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When an alternating magnetic field is applied to a soft type-II superconductor, a static magnetic induction often arises in the interior of the sample. An external field $H_0 + H_1 \sin(\omega t)$ creates a static induction $B_\infty \gg H_0$ in the interior under the conditions $H_{c2} > H_1 \gg H_0 > H_{c1}$.

If a static magnetic field H stronger than the lower critical field H_{c1} is applied to a soft type-II superconductor, a local magnetic induction $B_\infty \approx H$ is set up in the interior of the superconductor. What value does B_∞ take on when an alternating magnetic field is also applied? More generally, what state is established in the interior of a soft superconductor in the presence of an external alternating magnetic field? Below we attempt to answer these questions.

An experimental study of this situation was undertaken just recently.[1] The diffusion of magnetic vortices in a superconductor is usually studied in cases which allow the use of linear equations.[2] In the case at hand, this linearization is not possible.

To construct equations describing the evolution of the magnetic induction $\mathbf{B}(\mathbf{r}, t)$, we work from the model of a two-component gas of vortices. The first component, with a density $n_1(\mathbf{r}, t)$, consists of vortices with a magnetic moment $\mathbf{\Phi}$, while the second, with a density $n_2(\mathbf{r}, t)$, consists of vortices with a magnetic moment $-\mathbf{\Phi}$. The vector $\mathbf{\Phi}$ is directed along the z axis and is equal in magnitude to cosh/2e, the flux quantum. The system is assumed to be spatially uniform along the z axis.

The continuity equations are

$$\frac{\partial n_1}{\partial t}+\mathrm{div}(n_1\mathbf{v}_1)+\frac{n_1 n_2}{\vartheta}=0,$$

$$\frac{\partial n_2}{\partial t}+\mathrm{div}(n_2\mathbf{v}_2)+\frac{n_1 n_2}{\vartheta}=0. \tag{1}$$

Here $\mathbf{v}_i$ is the velocity of component i, and the term $n_1 n_2/\vartheta$ describes the recombination of the components. Obviously, $\mathbf{v}_1 = -\mathbf{v}_2 \equiv \mathbf{v}$.

We turn now to equations for $n = n_1 + n_2$ and $\nu = n_1 - n_2$:

$$\frac{\partial \nu}{\partial t}+\mathrm{div}(n\mathbf{v})=0,$$

$$\frac{\partial n}{\partial t}+\mathrm{div}(\nu\mathbf{v})+\frac{n^2-\nu^2}{2\vartheta}=0. \tag{2}$$

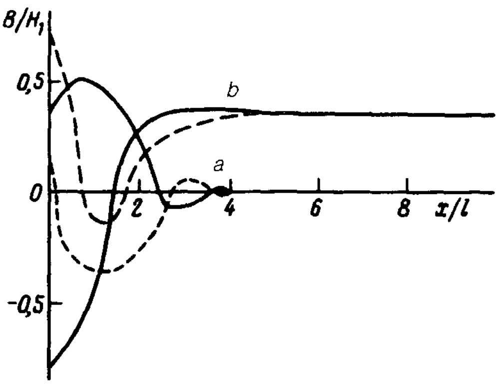

FIG. 1. Instantaneous distributions of the magnetic induction inside a sample to which a periodic external magnetic field $H(t)$ has been applied, after the initial stage of relaxation. These are the results of a numerical solution of Eq. (3). a—$H(t)=H_1 \sin \omega t$; b—$H(t)=H_0+H_1 \sin \omega t$, $H_0/H_1=0.1$. The solid and dashed lines show the $B(x)$ distributions at different times.

In the approximation of viscous motion we would have $\eta \mathbf{v}=-\nabla U$, where η is the viscosity coefficient, and $U=\Phi \mathrm{B}/4\pi$ is the constant energy of the first component. Using $\mathbf{B}=\Phi \nu$, we find $v=-\Phi^2 \nabla \nu / 4\pi\eta$. This relation and Eqs. (2) describe the dynamics of the gas of vortices and antivortices.

Equations (2) simplify significantly in the limit of intense recombination, $\vartheta \to 0$. In this limit we have $n^2 \simeq \nu^2$, i.e., $n \simeq |\nu|$. From (2) we find a closed equation for the induction, which takes the following form in the 1D case:

$$\frac{\partial B}{\partial t}=\frac{\Phi_0}{2\eta}\frac{\partial}{\partial x}\left(\operatorname{sign} B \frac{\partial B^2}{\partial x}\right), \tag{3}$$

where Φ_0 is the flux quantum, and $B \equiv B_z(x, t)$. In contrast with the nonlinear diffusion equations for classical problems, discussed in Refs. 3 and 4, Eq. (3) contains a factor of sign B. This factor arises because of a possible change in the sign of B. At $B=0$, vortices annihilate with antivortices. The function sign B on the right side of (3) ensures continuity of the electric field $E_y=-(\Phi_0/2\eta c)\operatorname{sign} \mathrm{B}(\partial B^2/\partial x)$. The nonlinear equations for the case of creep in hard superconductors are given in Ref. 5.

We consider a semi-infinite sample, at $x>0$. In the $x<0$ half-plane, there is a periodic alternating external magnetic field

$$H(t)=H_0+\sum_{m=1}^{\infty} H_m \sin(\omega m t+\varphi_m). \tag{4}$$

Expression (4) imposes a boundary condition on (3): $B(x=0, t)=H(t)$. For an individual harmonic, say the mth, Eq. (3) is put in dimensionless form through the use of the quantities B/H_m, $t\omega m$, and x/l, where $l=(\Phi_0 H_m/\eta \mathrm{m}\omega)^{1/2}=c(H_m \rho_n/H_{c2}\omega m)^{1/2}$ is a characteristic length (ρ_n is the resistivity in the normal phase, and H_{c2} is the upper critical field). With $H_m/H_{c2} \approx 10^{-3}$ and $\omega m \approx 1$ Hz, the value of l is in the interval $10^{-2}-10^{0}$ cm. It follows from dimensionality considerations that the amplitude of the induction oscillations for one individual harmonic falls off over a distance on the order of l with depth in the sample. This $B(x, t)$ behavior is demonstrated by Fig. 1. This result means, in particular, that a sample with a thickness much greater than l can be regarded as semi-infinite. If there are several harmonics, the induction $B(x=\infty)\equiv B_\infty$ is independent of the time after some transients, and it by no means has to vanish (curve b in Fig. 1).

To determine B_∞ we multiply (3) by x and integrate over x from zero to infinity. Integrating by parts, we find

$$\frac{\partial}{\partial t}\int_0^\infty dxxB(x,t)=\frac{\Phi_0}{2\eta}\,[H(t)\,|H(t)|-B_\infty\,|B_\infty|\,]. \tag{5}$$

By virtue of our assumption that B is independent of x at large x, we find from Eq. (3) that B_∞ is also independent of the time. After some transients, $B(x,\,t)$ becomes a periodic function of the time at any x, and the integral of the left side of (5) over the period vanishes. This condition is equivalent to the vanishing of the zeroth Fourier component of the electric potential induced by moving vortices. Integration (3) over the period, we find an expression for B_∞:

$$B_\infty=|f|^{1/2}\,\text{sign}\,f,\quad f=\frac{\omega}{2\pi}\int_0^{2\pi/\omega}dtH(t)\,|H(t)|. \tag{6}$$

This expression can be generalized to the case of a nonzero current through the sample. An important aspect of Eq. (6) is its simplicity: The induction B_∞ is determined by only the external field. It does not depend on even the frequency of this field (the duration of the transients does depend on the frequency, as does the thickness of the surface layer within which the induction depends on the coordinate and oscillates in time). As expected, we find from (6) that if $H(t)$ consists of only a single harmonic, the induction is zero: $B_\infty=0$. An exceptional case is the zeroth harmonic (*i.e.*, a static external field), for which we have $B_\infty=H_0$.

The most interesting effects arise from an interaction of different harmonics. The result for the interaction of the zeroth and first harmonics can be derived most simply: $H(t)=H_0+H_1 sin\omega t$. In this case the integration in (6) can be carried out easily:

$$B_\infty=\text{sign}\,H_0\begin{cases}(H_0^2+H_1^2/2)^{1/2},\\ \pi^{-1/2}[\,(2H_0^2+H_1^2)\arcsin|H_0/H_1|+3|H_0|(H_1^2-H_0^2)\,]\,.\end{cases} \tag{7}$$

The upper and lower versions of Eq. (7) are valid for $|H_0|\geqslant|H_1|$ and $|H_0|\leqslant|H_1|$, respectively. We also note that at the point $|H_0|=|H_1|$ the first derivatives of B_∞ with respect to H_0 and H_1 are continuous. In the region $|H_0|\ll|H_1|$ we have the asymptotic behavior $B_\infty=2(|H_0H_1|/\pi)^{1/2}\,\text{sign}\,H_0$, which can be interpreted as a paramagnetic response with a nonanalytic $B(H_0)$ dependence such that we find $\partial B_\infty/\partial H_0\to\infty$ in the limit $H_0\to 0$ (Fig. 2).

Equation (6) also predicts a "rectification", *i.e.*, the appearance of a constant induction B_∞ in the interior of the sample when there is an alternating external magnetic field as in (4) without a static component ($H_0=0$). A comparatively simple analytic expression for the integral in (6) can be found if the amplitude of the first harmonic is the largest: $|H_1|\gg|H_m|$, $m\neq 1$. Without loss of generality, we can set $\varphi_1=0$ in (4). We then have

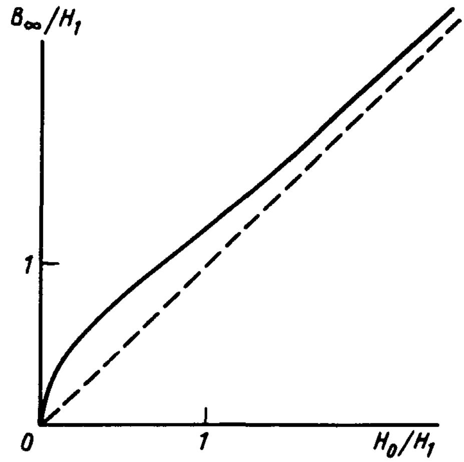

FIG. 2. The static induction B_∞ in the interior of a soft type II superconductor as a function of the static component of the external magnetic field $H(t) = H_0 + H_1 sin\omega t$.

$$B_\infty = \text{sign}\left[H_1\left(H_0 - \sum_{m=1}^{\infty} \frac{H_{2m}\sin\varphi_{2m}}{4m^2-1}\right)\right] 2\left[\left|H_1\left(H_0 - \sum_{m=1}^{\infty} \frac{H_{2m}\sin\varphi_{2m}}{4m^2-1}\right)\right|/\pi\right]^{1/2}. \quad (8)$$

According to (8), if only odd harmonics are present we have $B_\infty = 0$, and there is no rectification. In addition, it is a straightforward matter to show that no combination of odd harmonics of arbitrary amplitudes will lead to a rectification. If, on the other hand, only the first and second harmonics (in particular) are present, and the condition $|H_1| \gg |H_2|$ holds, then

$$B_\infty = -2\text{sign}(H_1H_2\sin\varphi_2)(|H_1H_2\sin\varphi_2|/3\pi)^{1/2}.$$

As a result, the sign of B_∞ is determined by the phase of the second harmonic with respect to the first. In this limit B_∞ is a square-root function of the product H_1H_2, just as we had $B_\infty \propto (H_0H_1)^{1/2}$ in the case of the combination of the zeroth and first harmonics, under the condition $|H_0| \ll |H_1|$. At $|H_2| \gg |H_1|$ we easily find

$$B_\infty = -\text{sign}(H_1H_2\sin\varphi_2)|H_1|(|\sin\varphi_2|/\pi)^{1/2};$$

i.e., the induction reaches saturation with increasing amplitude of the second harmonic.

A soft type-II superconductor thus acts as a sort of rectifying nonlinear element and damper in this case. On the other hand, it can be assumed that a periodic external magnetic field pumps a static magnetic induction inside the sample.

We carried out a numerical simulation of an equation which generalizes Eq. (3) to the case of a nonzero critical current. We found that effects similar to those discussed above are also realized in the case of hard superconductors in several situations.

The effects discussed in this letter may prove useful in the development of superconducting pickups for alternating magnetic fields.[6]

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