Aharonov-Bohm effect in lattice field theory

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The generating functional of an Abelian Higgs model is expressed as a sum over world surfaces of Nielsen–Olsen strings. It is shown that there exists a nontrivial topological interaction which corresponds to the Aharonov–Bohm effect in field theory.

We consider a 4D Abelian Higgs model in which the scalar field $\Phi = |\Phi| e^{i\varphi}$, with a charge Ne, is condensed. Although the gauge field has a mass, there are long-range forces[1,2] between (on the one hand) charges Me which are not multiples of Ne, and (on the other) Nielsen–Olsen strings. We will show that this interaction is related to the number of couplings of the world surfaces of the strings and the particle trajectories, i.e., that it is topological. We use a lattice formulation of the theory. The potential of the scalar field, $V(|\Phi|)$, is assumed for simplicity to be so deep that the radial component of the scalar field is frozen: $|\Phi| =$const. Selecting the Villeĭn action as the remaining dynamic variable, i.e., the phase (φ), we find the following partition function for the theory:

$$Z=\int_{-\infty}^{+\infty} \mathscr{D}A \int_{-\pi}^{+\pi} \mathscr{D}\varphi \sum_{l(c_1)\in \mathbb{Z}} \exp\left\{-\frac{1}{2e^2}\, \|dA\|^2-\frac{\kappa}{2}\, \|d\varphi+2\pi l-NA\|^2\right\}. \quad (1)$$

The notation here is that which is used in lattice differential geometry.[3] It allows us to write the results of the calculations in compact form. The fields are defined on the lattice cells c_k, of dimensionality k. The gauge field is defined on the edges, $A = A(c_1)$, and the scalar field at the nodes, $\varphi = \varphi(c_0)$. The external differentiation operator d raises the dimensionality by one. For example, the field intensity, defined on placquettes, is constructed from the edge variables in the standard way: $F(c_2) = dA(c_1)$. Here $d\varphi$ corresponds to an edge if $\varphi = \varphi(c_0)$. The scalar product is defined as $(\phi, \psi) = \Sigma_{c_k}\phi(c_k)\psi(c_k)$; the sum is over all the cells c_k. The norm is defined in the standard way, $\|\phi\|^2=(\phi, \phi)$, so a sum over all placquettes is implied in the first term in the summation in (1), while a sum over all edges is implied in the second term. Corresponding to each field $\phi(c_k)$ is a field defined on the dual lattice: $*\phi(*c_k)$, where $*c_k$ is the $(D-k)$-dimensional cell which is the dual of c_k. The codifferentiation operator $\delta=*d*$, which lowers the dimensionality of the fields by one, makes possible an "integration by parts": $(\phi, d\psi)=(\delta\phi, \psi)$.

After fixing the gauge, $d\varphi=0$, and factoring out the terms in the argument of the exponential function in (1) which are quadratic in A, we see that the field A acquires a mass $m = N\kappa^{1/2}e$ (this is the Higgs mechanism). Applying to (1) a transformation analogous to that which was used by Banks *et al.*[4] for a compact electrodynamics and by Polikarpov and Wiese[5] for a 4D XY model, we find a partition function expressed in terms of the world surfaces of Nielsen–Olsen strings:

$$Z^{\mathrm{BKT}}=\sum_{\substack{*\sigma(*c_2)\in Z\\ \delta*\sigma=0}} \exp\{-2\pi^2\kappa\,[*\sigma,(\Delta+m^2)^{-1}*\sigma]\}\cdot \tag{2}$$

Here $\Delta=\delta d+d\delta$ is the lattice Laplacian,[3] and the summation is over all integers $*\sigma(*c_2)$ which belong to the placquettes of the dual lattice. The condition $\delta*\sigma=0$ corresponds to the circumstance that the world surfaces of strings which are described by the numbers $\{*\sigma\}$ are closed.[5] We are using the notation Z^{BKT}, since analogous transformations were first carried out by Beresinskii[6] and Kosterlitz and Thouless[7] for a 2D XY model.

We now consider a Wilson loop for the charge Me: $W_M(C)=\exp\{iM(A,\,j_c)\}$, $j_c=1$, on the edges which belong to contour C and with $j_c=0$ on the other edges. Repeating the transformations which lead from (1) to (2), we find, in the BKT representation,

$$\langle W_M(C)\rangle=\frac{1}{Z^{\mathrm{BKT}}}\sum_{\substack{*\sigma(*c_2)\in Z\\ \delta*\sigma=0}}\exp\left\{-2\pi^2\kappa\,[*\sigma,(\Delta+m^2)^{-1*}\sigma]-\frac{M^2e^2}{2}\,[\,j_c,(\Delta+m^2)^{-1}j_c]\right.$$

$$\left.-2\pi i\frac{M}{N}\,[\,j_c,(\Delta+m^2)^{-1}\delta\sigma]+2\pi i\frac{M}{N}\,\mathbb{L}(\sigma,\,j_c)\right\}. \tag{3}$$

The first three terms in the exponential function describe short-range forces (the Yukawa interaction), $(\Delta+m^2)^{-1}$; the quantity $\mathbb{L}=(*j_C,\,\Delta^{-1}d*\sigma)$ in the last term can be written in the form $\mathbb{L}=(*j_c,\,*n)$, where n is an integer solution of the equation $\delta*n=*\sigma$. It follows that $\mathbb{L}$ is an integer equal to the number of intersections of contour C and the 3D volume bounded by the surface $\{*\sigma\}$. The expression for $\mathbb{L}$ is thus a lattice version of Gauss's law for the number of couplings of contour C and the world surfaces of the strings, $\{*\sigma\}$. The corresponding forces are long-range: Even if any point of the surface $\{*\sigma\}$ is arbitrarily far from any point on contour C, $\mathbb{L}$ can be finite. The long-range nature of the forces here corresponds to the Aharonov–Bohm effect in field theory: Nielsen–Olsen strings play the role of solenoids which are interacting with particles of charge Me. It follows from (3) that if M/N is an integer, then the condensate of the scalar field completely screens the charge Me, and there is no long-range interaction (in quantum mechanics, the Aharonov–Bohm effect disappears when the product of the magnetic flux of the solenoid and the scattered charge is a multiple of 2π). An actual long-range interaction between particles of charge Me can arise in that phase of the theory in which a condensate of strings exists.

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