Is the frequency of one-electron oscillations under Coulomb blockade conditions equal to *I*/*e*?

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Under certain conditions, the frequency of one-electron oscillations is determined by the energy of the Coulomb repulsion of electrons.

When electrons tunnel through a junction with a small capacitance, the Coulomb repulsion of electrons is important. Under Coulomb blockade conditions, the change in the charge at the junction due to the tunneling of a single electron is so large that the simultaneous tunneling of yet another electrons is prevented by energy relations. As a result, the tunneling of individual electrons is correlated in time. At a fixed value of the direct current flowing across the junction, the potential across the junction thus oscillates in time, at a frequency[1,2]

$$\omega_e = 2\pi I/e. \tag{1}$$

Another interesting entity in whose behavior Coulomb blockade effects are important is a system consisting of a quantum dot (or quantum well) which is coupled by tunnel junctions with two metal electrodes. In the present letter we wish to demonstrate that the statistical properties of the fluctuations (including fluctuations of the voltage) in such a system as a direct current flows through it are characterized by another frequency. This other frequency is determined directly by the energy of the Coulomb repulsion of two electrons in the well; it is not directly related to the magnitude of the current.

The oscillations at frequency (1) in the voltage across the junction are associated with fluctuations in the charge at the banks of the junction. Here, in contrast, we are interested in fluctuations in the number of electrons (in the amount of charge) in the well.

The use of a simple model of a junction enables us to pursue the calculation of the frequency of one-electron oscillations to completion. The system consists of two ideal banks which are coupled by a "bottleneck" (a quantum dot, a quantum well, etc.), in which there is one quantum-well level. The Coulomb repulsion of the electrons at the level is described by the Hubbard model.[3] This model refers to the quantum limit and contains all features characteristic of the Coulomb blockade. The Hamiltonian of the system is

$$\hat{H} = \sum_{\alpha=L,R} \epsilon_{k\alpha} c^+_{k\alpha} c_{k\alpha} + \sum_{\sigma} \epsilon_0 c_\sigma{}^+ c_\sigma + U n_\downarrow n_\uparrow + \sum_{\substack{\alpha=L,R \\ \sigma}} (I_{k\alpha} c^+_{k\alpha} c_\sigma + \text{H.a.}). \tag{2}$$

The index α describes the states in the left (L) and right (R) electrodes, $\epsilon_{k\alpha}$ is the electron spectrum in the banks, ϵ_0 is the energy of the one-particle level in the quantum well, U is the Hubbard energy of the electron repulsion in the well, and $T_{k\alpha}$ are the matrix elements for tunneling into the banks. The static voltage V_0 is incorporated in the usual way, through shifts of the chemical potentials in the banks: $\mu_L - \mu_R = eV_0$.

The expectation value of the number of electrons in the well is given by

$$\langle n_\sigma \rangle = \int d\omega G_\sigma^{<}(\omega)/2\pi, \tag{3}$$

where $G_\sigma^{<}(\omega)$ is the Keldysh Green's function[4] for electrons in the well; here the Coulomb repulsion and the tunneling into the banks have been taken into account. The Green's function for an isolated center can be found exactly, while tunneling into the banks can be dealt with by a perturbation theory in the barrier transmission levels $\gamma_{L,R}$:

$$\gamma_{L,R} = \pi \sum_k |T_{k\alpha}|^2 \delta(\omega - \epsilon_{kL,R}), \quad \gamma = \gamma_L + \gamma_R. \tag{4}$$

This is the approach which was taken in Refs. 5 and 6, among other places (below we assume that the rate of the tunneling into the banks is independent of the energy). For the Keldysh Green's function we have

$$G_\sigma^{<}(\omega) = F(\omega)\rho_\sigma(\omega), \quad F(\omega) = [\gamma_L f_L(\omega) + \gamma_R f_R(\omega)]/\gamma,$$

$$\rho_\sigma(\omega) = \frac{1}{\pi}\mathrm{Im}[G_\sigma^A(\omega)], \tag{5}$$

$$\rho_\sigma(\omega) = \frac{\gamma[\omega - \epsilon_0 - U(1 - \langle n_{-\sigma}\rangle)]}{(\omega - \epsilon_0)^2(\omega - \epsilon_0 - U)^2 + [\omega - \epsilon_0 - U(1 - \langle n_{-\sigma}\rangle]^2\gamma^2},$$

where $f_{L,R}(\omega)$ are the distribution functions in the banks, and the density of states $\rho_\sigma(\omega)$ in the well consist of two peaks, at energies $\simeq \epsilon_0$ and $\simeq (\epsilon_0 + U)$.

In the limit $\gamma \to 0$ the expression for the density of electron states at a center is given by the standard expression[3]

$$\rho_\sigma(\omega) = (1 - \langle n_{-\sigma}\rangle)\delta(\omega - \epsilon_0) + \langle n_{-\sigma}\rangle\delta(\omega - \epsilon_0 - U). \tag{6}$$

A point which we wish to emphasize (and one which is important to the discussion below) is that the heights of these peaks depend on the occupation numbers $\langle n_\sigma \rangle$.

Fluctuations of the electron density in a junction are determined by the charge-charge correlation function

$$K(\tau) = \langle [n_\sigma(t) + n_{-\sigma}(t)][n_\sigma(t+\tau) + n_{-\sigma}(t+\tau)]\rangle \tag{7}$$

(we are omitting the charge of the electron). Approximating the Fourier transform of the correlation function $K(\tau)$ by a product of Keldysh Green's functions, we find

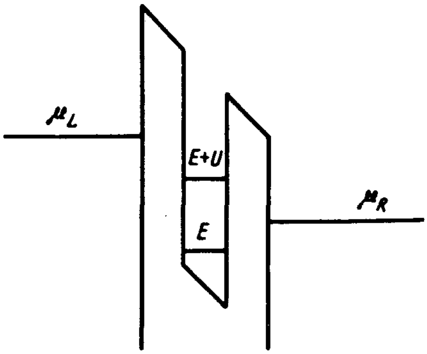

FIG. 1.

$$K(\Omega)=\int d\tau e^{i\Omega\tau}K(\tau)$$

$$=\sum_\sigma \int d\omega[G_\sigma^{<}(\omega)G_\sigma^{>}(\omega+\Omega)+G_\sigma^{<}(\omega+\Omega)G_\sigma^{>}(\omega)]. \quad (8)$$

We first consider the limit $\gamma \to 0$, which is a very clear case. In this limit we find

$$K(\Omega)=\sum_\sigma \{[(1-\langle n_\sigma\rangle)^2+\langle n_\sigma\rangle^2]\delta(\Omega)+(1-\langle n_\sigma\rangle)\langle n_\sigma\rangle\delta(\Omega-U)\}$$
$$\times\{[F(\epsilon_0+F(\epsilon_0+\Omega)-2F(\epsilon_0)F(\epsilon_0+\Omega)]+[F(\epsilon_0+U)$$
$$+F(\epsilon_0+U+\Omega)-2F(\epsilon_0+U)F(\epsilon_0+U+\Omega)]\}. \quad (9)$$

The first term corresponds to an uncorrelated tunneling of electrons (in the case $\gamma\neq 0$; more on this below). The second term is zero except at the voltage $\mu_L - \mu_R = eV_0 > U$, which is above the Coulomb repulsion energy. This term arises from an overlap of the peaks in the densities of states $\rho_\sigma(\omega)$ and $\rho_\sigma(\omega+\Omega)$ corresponding to the energies ϵ_0 and ϵ_0+U. The coefficient is proportional to $\langle n_\sigma\rangle(1-\langle n_\sigma\rangle)$ and is zero except when $\langle n_\sigma\rangle$ is not an integer [for an isolated level, the value of $\langle n_\sigma\rangle$ might be 0.1, and the coefficient at $\delta(\Omega-U)$ would always be zero]. This case is realized if the condition $\mu_L-\mu_R=eV_0U$ holds ($\epsilon_0<\mu_L<\epsilon_0+U, \mu_R>\epsilon_0, \epsilon_0+U$; Fig. 1). From (5) and (6) we find $\langle n_\sigma\rangle=\gamma_L/(2\gamma_L+\gamma_R)$.

From statistical radiophysics[7] we know that the second term in $K(\Omega)$ describes a random process of the $\cos(\omega_e t+\phi)$ type with a constant frequency $\omega_e=U$ and with a random phase ϕ, distributed uniformly over an interval of 2π. This process is quite different from Gaussian.[7] Consequently, fluctuations of the charge (and associated fluctuations of the potential across a junction) are harmonic oscillations with a constant frequency and a random phase.

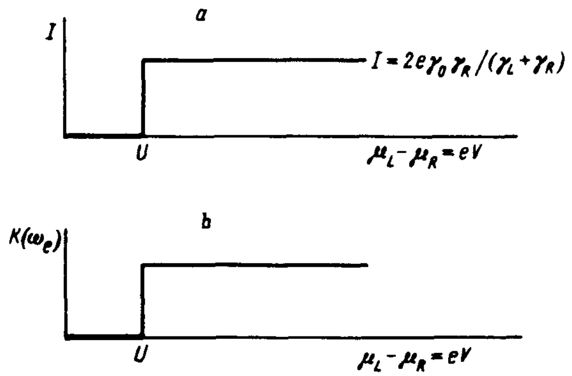

FIG. 2. Tunneling current and correlation function at the Coulomb frequency as a function of the voltage across the junction.

The part of the correlation function which is responsible for the single-electron oscillations thus comes into play abruptly as the voltage is varied, since the heights of the peaks in the spectral density $\rho_\sigma(\omega)$ also vary abruptly as the voltage is varied, at $eV=U$.

How is the spectrum of charge fluctuations related to the magnitude of the current? The tunneling current for a system of this sort can be found from the expression[6]

$$I=e\frac{\gamma_L\gamma_R}{(\gamma_L+\gamma_R)}\sum_\sigma\int d\omega\rho_\sigma(\omega)[f_L(\omega)-f_R(\omega)]. \tag{10}$$

It is not difficult to verify that the voltage dependence of the current in a step function (Fig. 2a): At $\mu_L-\mu_R<U$ we have $I=0$, while at $\mu_L-\mu_R>U$ we have $I = 2e\gamma_L\gamma_R/(2\gamma_L + \gamma_R)$.[6] Figure 2b shows the behavior of the peak height in the $K(\Omega)$ spectrum near $\omega = U$. We wish to stress that what changes abruptly is not the frequency of the peak in $K(\Omega)$ (this peak is always at $\omega_e = U$) but the height of this peak.

If the voltage is lower than the Coulomb repulsion energy in the well, $eV<U$, there is thus no current, and the coefficient of $\delta(\Omega-U)$ in Eq. (9) is simultaneously zero. There is accordingly no contribution of the type $\cos(\omega_e t+\phi)$ to the correlation function, since under the condition $\epsilon_0<\mu_L, \mu_R<\epsilon_0+U$, the lower level, with an energy ϵ_0, is filled, while the upper one is empty. A second electron cannot tunnel through the well, since this event is prevented by the Coulomb repulsion energy U. As the external voltage is raised above the threshold repulsion energy ($\epsilon_0<\mu_L<\epsilon_0+U$, $\mu_R>\epsilon_0$, ϵ_0+U), a current appears abruptly (Fig. 2a), because of a possible tunneling of a second electron through the level with the energy ϵ_0+U. However, only a single electron can pass through this level. The tunneling of electrons turns out to be correlated in time in this case. The frequency of the charge oscillations in the well (and thus the frequency of the oscillations in the potential across the junction) is the frequency of the one-electron oscillations, which appears as a voltage jump (Fig. 2b). More precisely, what appears abruptly are an oscillatory component of the time dependence $n(t)$ and a δ-function component of the correlation function. The tunneling of individual electrons might be thought of as a motion of wave packets for individual electrons which follow at a constant frequency ω_e and which have a random phase.

If γ is not zero, the δ-functions in (9) spread out, and in the case $U=0$ the density of electron states in the well becomes

$$\rho_\sigma(\omega)=\gamma/[(\omega-\epsilon_0)^2+\gamma^2]. \tag{11}$$

In this case the correlation function $K(\Omega)$ has the behavior

$$K(\Omega)\propto\frac{1}{\Omega^2+4\gamma^2},\quad K(\tau)\propto e^{-2\gamma\tau}, \tag{12}$$

as follows from (9). It can be seen from the form of correlation function (12) (Ref. 7, for example) that this correlation function corresponds to a random Poisson process. The jumps in the voltage are alternating exponential pulses of the $e^{-\gamma t}$ type, with a time scale γ, whose alternation in time is a Poisson distribution. Such a process corresponds to the response of an *RC* circuit to a train of random square pulses.

In the case of a small but finite $\gamma\neq0$, with $U\neq0$, the δ-functions in correlation function $K(\Omega)$ spread out into peaks with a width on the order of γ, which lie near $\Omega \propto U$. The correlation function for such a peak corresponds to a random process with a characteristic frequency $\omega \simeq U$ and a random phase.[7]

In the case of a quantum well, the fluctuation spectrum of the system is thus characterized by a frequency which is governed by the energy scale of the Coulomb repulsion of electrons in the well. It does not depend on the current level. The oscillatory component of the correlation function changes abruptly as a function of the applied voltage, tracking the excursions of the system into the Coulomb blockade regime. Our model also yields a natural transition to a Poisson statistics for noninteracting electrons.

If there are many levels in the system (this would be the macroscopic limit), we would expect abrupt changes in correlation function $K(\omega)$ at a large number of voltages, corresponding to the repulsion energies upon the addition of successive electrons to the well.

One might observe oscillations of the charge density in a well experimentally by connecting an auxiliary electrode in the immediate vicinity of the well and measuring the frequency spectrum of the fluctuations in the potential of this electrode.

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