Asymptotic freedom at large distances and the IR renormalon problem

Yu. A. Simonov
Institute of Theoretical and Experimental Physics, 117259 Moscow, Russia

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Perturbation theory is formulated in the confining background field B_μ. All diagrams are gauge invariant and at a large N_c limit depend on B_μ only through the Wilson loops $W(C,B_\mu)$. Assuming the area law for the latter one obtains the charge renormalization at large distances, which yields the behavior $\alpha_s(p) \sim 4\pi[\ln(p^2+m^2)/(\Delta^2)]^{-1}$, where m is a process-dependent mass on the order of 1 GeV. For $m \gg \Lambda$ the asymptotic freedom then persists for all Euclidean monenta, and IR renormalons are shown to disappear.

1. Perturbation expansion (PE) in the free vacuum of QCD is diverging at large distances due to the ghost pole in the running coupling $\alpha_s(p)$. In addition, the sum of PE is not Borel summable due to the IR renormalons.[1] In this letter we instead consider PE in the framework of the background field formalism[2,3] which splits in the Lagrangian L the total field A_μ into the nonperturbative (NP) part B_μ and perturbation a_μ:

$$A_\mu = B_\mu + a_\mu, \quad L(A) = L_0(B) + \sum_{i=1}^{4} L_i(B,a). \tag{1}$$

The usual diagrammatic technique can then be used for the expansion in powers of ga_μ, where the propagators of the quantum field a_μ, $G_{\mu\nu}(x,y;B)$ depend on the background field B_μ. We also use, as in Ref. 3, the gauge fixing (gf) term $\frac{1}{2\xi}[D_\mu(B)A_\mu]^2$ and the corresponding Faddeev-Popov ghosts. Prescribing gauge transformations for a_μ, $B_\mu \to V^+[a_\mu, B_\mu - (i/g)\partial_\mu]V$, we can write all physical amplitudes for large N_c with the help of the Feynman-Schwinger representation (FSR)[4] through the average of the product of the Wilson loops $W(C_i B)$ of the field B_μ. We are interested, in particular, in the large distance behavior of the new PE, and assume for $\langle W(C_i,B)\rangle$ the area law $\exp(-\sigma S_{\min})$, which follows naturally from the cluster expansion[5] or lattice calculations.[6] Thus the large distances are described by only one input parameter: the string tension σ, which is used as the only characteristics of the NP fields $\{B_\mu\}$.

Our specific goal is to calculate the running coupling $\alpha_s(p)$ at all Euclidean momenta p, including the IR region $p \to 0$. Two different *NP* definitions of the charge are used:

i) the energy of static charges at a distance R

$$E(R) = E_{NP}(R;B) - \frac{4}{3}\frac{\alpha_s(R)}{R}, \tag{2}$$

where $E(R)$ is computed as

$$E(R) = -\lim_{T\to\infty}\left\{\frac{1}{T}\ln\langle W(C,B+a)\rangle\right\} \tag{3}$$

and the contour C is a rectangular $R\times T$;

ii) the extended method of Ref. 3, where the two-point functions with external B lines have been calculated. In the lowest order for $B=0$ the charge Z factor, Z_g, is given by a gluon a_μ loop and ghost loop diagrams, which yield standard charge renormalization.

In the full treatment for $B\neq 0$ one considers a gauge-invariant generalization of the same loop diagrams ,where the gluon and ghost Green's functions are not free but contain B_μ to all orders.

In the first definition i) the case without background renormalization of the Wilson loop was considered in Ref. 7 and the result for $\alpha_s(R)$ can be obtained from Ref. 8, taking the limit of heavy quark masses:

$$\alpha_s(R) = \alpha_s(\mu)[1+\alpha_s(\mu)f^{(0)}(R)+\cdots]. \tag{4}$$

The same diagrams as in the free vacuum case can be considered with the confining background which manifests itself as the area law for the Wilson loop. In this specific case, where the side length T of the Wilson loop tends to infinity, and N_c is large, so that gluon lines are replaced by double fundamental lines, we can show that the internal fundamental loops have the asymptotic behavior[9] at large $|x-y|$

$$\langle\Pi(x,y;B)\rangle_B \sim \exp(-m_1|x-y|)\,. \tag{5}$$

The mass m_1 is a mass of two charges which are connected by the string, like the mass of ρ meson, and which can be computed through[10] σ. Typically,[9,10] we have $m_1\approx(3\text{–}4)\sqrt{\sigma}\sim 1$ GeV. Using the asymptotic expression (5) in the computation of $f^{(0)}(R)$, we obtain instead of (4) the expression

$$\frac{1}{R}f^{(B)}(R)\approx I(R)\equiv\int_\delta^\infty\frac{dr}{r}e^{-m_1 r}\left[\frac{1}{R}\Theta(R-r)+\frac{1}{r}\Theta(r-R)\right]. \tag{6}$$

At small R, $I(R)$ has the familiar asymptotic freedom:

$$I(R)\approx\frac{1}{R}\ln\frac{R}{\delta},\quad m_1R\ll 1. \tag{7}$$

This value coincides with $f^{(0)}$ upon renormalization $1/\delta\to\mu$.

For large R, $m_1R\gg 1$, the logarithmic growth of $f^{(B)}(R)$ is "screened" in (6):

$$f^{(B)}(R)\approx\ln\frac{1}{m_1\delta}\sim\ln\frac{\mu}{m_1}+0\left(\frac{1}{Rm_1}\right). \tag{8}$$

From the Fourier transform of (2) we easily obtain

$$\alpha_s(p)\approx\alpha_s(\mu)\left[1+\alpha_s(\mu)\frac{b_0}{4\pi}\ln\frac{p^2+m_1^2}{\mu^2}+\cdots\right]. \tag{9}$$

Similar results can be obtained for the definition ii), where the contribution to Z_g is calculated from the two-point function:

$$H_{\mu\nu}(x,y)=\frac{g^2}{(4\pi)^2}b_0(\partial_\mu\partial_\nu-\partial^2\delta_{\mu\nu})\bar{\Pi}(x,y), \tag{10}$$

and the one-loop function $\bar{\Pi}$ contains the sum of contributions of the gluon a_μ and the ghost against the background B_μ, which can be computed using FSR[4,10] and the proper-time Hamiltonian technique.[9,10]

We are concerned only with the asymptotic behavior which can be found by the aforementioned methods:

$$\bar{\Pi}(x,y)\sim\exp(-m_2|x-y|), \quad |x-y|\to\infty. \tag{11}$$

Here the value of m_2 is close to the two-gluon glueball mass in the 1^{-+} state. As was computed in Ref. 11, m_2 is ≈ 2 GeV. Using (11), we compute Z_g to one-loop approximation, and $\alpha_s(p)$ appears to be the same as in (9) with the replacement $m\to m_2$. We note that m_i depends on the process, since the infrared asymptotic behavior of the renormalization gluon loop depends on the surroundings in the diagrams which it enters: The loop is attached by strings to the overall Wilson contour.

It is important that due to the gauge invariance of PE the renormalizations of g and B_μ are connected:[3] $Z_g=Z_B^{-1/2}$, and the product gB_μ is renormalization invariant, implying the same property for σ and m_1,m_2. Thus, it is not surprising that m^2 in (9) is on the same footing as the momentum p^2. Moreover, the dependence on the normalization mass (the scale parameter) μ is the same as in the free case, which means that Gell-Mann–Low equations do not change (at least to one-loop order)

$$\frac{d\ln g}{d\ln\mu}=-\frac{b_0g^2}{16\pi^2}. \tag{12}$$

Solving (12) with the initial condition (9), we obtain $\alpha_s(p)$ in the form with the parameter $\bar{\Lambda}$

$$\alpha_s(p)=\frac{4\pi}{b_0\ln[(m^2+p^2)/\bar{\Lambda}^2]}. \tag{13}$$

Here $\bar{\Lambda}$ may differ from the usual QCD parameter, and it coincides with it when $m\to 0$.

The form (13) becomes the familiar form when $p^2\gg m_i^2$ and is universal in this limit. However, for $p^2<m_i^2$ the value of $\alpha_s(p)$ is not universal (i.e., it depends on the process where it enters) which is reflected by the nonuniversality of m_i^2 in our two definitions.

In addition, we have systematically dropped in the derivation of (9) and (13) the constant terms and the powers of $[\mu^2/(p^2+m^2)]$, so that the result (9) is a leading term for large $\ln[m^2+p^2)/\mu^2]\gg 1$.

The estimate of $\alpha_s(p=0)$ in (13) for $m\cong 1$ GeV and $\Lambda\simeq 0.2$ GeV, $b_0=9$ yields $\alpha_s(p=0)=0.43$ and $\alpha_s(p\approx m_\rho)\approx 0.38$, in agreement with the sum rule estimates in Ref. 12. Equation (13) thus reflects a phenomenologically reasonable behavior, where the asymptotic freedom persists to small values of the momentum.

Finally, we turn to the renormalon problem.[1] Following Ref. 13, we write the generalization of the contribution of the set of IR renormalon diagrams to the Euclidean correlator $\Pi(Q^2)$ of e.m. currents as follows:

$$\Delta\Pi = \frac{\alpha_s(Q^2)}{8\pi^3} \sum_n \left(\frac{b_0\alpha_s(Q^2)}{4\pi}\right)^n \int_0^{Q^2} \frac{k^2dk^2}{Q^4} \ln^n\left(\frac{Q^2+m^2}{k^2+m^2}\right), \tag{14}$$

where we have used (13) and have taken the normalization point at Q^2+m^2, $Q^2 \gg m^2$. When $m=0$, Eq. (14) becomes the familiar expression [see, e.g., Eq. (9) in Ref. 13].

The integration in (14) yields

$$\Delta\Pi(Q^2) = \frac{1}{2\pi^2 b_0} \sum_{n\gg 1}^{\infty} \left(\frac{\alpha_s(Q^2)b_0}{8\pi}\right)^n q_n, \tag{15}$$

where

$$q_n = \begin{cases} (n-1)!, & n<2n_0 \\ \dfrac{(2n_0)^n}{n}, & n>2n_0 \end{cases}, \tag{16}$$

and $n_0=\ln[Q^2+m^2)/m^2] \gg 1$.

In the limit $m\to 0$, $n_0 \to \infty$ we obtain in (15) a factorially diverging series, where the Borel transform $\Delta\Pi(t)$ has a pole in the Borel parameter t at $8\pi/b_0$—the so-called IR renormalon[1] which precludes the Borel summation of the series (15). For finite values of m, however, the factorial growth of q_n stops at $n \approx 2n_0$ and the series (15) is summable by the usual methods which yield asymptotically, $n_0 \gg 1$, the simple expression

$$\Delta\Pi(Q^2) \cong -\frac{1}{2b_0\pi^2} \ln\frac{\alpha_s(Q^2)}{\alpha_s(0)}. \tag{17}$$

Thus the confining background drastically improves the properties of PE and raises hope that the perturbation theory of QCD can be managed at all distances. Accordingly, the NP background can be described by the lowest correlators $\langle FF\rangle$, $\langle FFF\rangle$..., or just by the string tension (at large distances), while from the PE one can keep only a few lower-order terms.

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