Massless gauge superfields of higher half-integer superspins

S. M. Kuzenko, V. V. Postnikov,* and A. G. Sibiryakov
Tomsk State University, 634050 Tomsk, Russia
**L. D. Landau Institute of Theoretical Physics, Russian Academy of Sciences, 117334 Moscow, Russia*

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Two superfield formulations (dual with respect to one another) are proposed for free massless $N=1$, $D=4$ supersymmetric theories of superspin $(s+1/2)$, where $s=2,3,\ldots$. For $s=1$ the first version reduces to linearized $n=-1$ nonminimal supergravity and the second version reduces to linearized minimal supergravity.

It is considered to be generally accepted that all $N=1$, $D=4$ supersymmetric field theories admit an explicitly supersymmetric formulation in terms of superspace and superfields. Thus far, however, such realizations have been obtained only for theories describing multiplets of superspin $s\leqslant 3/2$; the case of massless superspin $s=3/2$ corresponds to supergravity [the massless multiplet of superspin s is often designated as $(s,\ s+1/2)$]. Free models of massless multiplets of higher superspins $s>3/2$ were constructed comparatively a long time ago[1] and were based on the results of Refs. 2 and 3 for the Lagrangian formulation for fields of arbitrary spin in terms of symmetric tensors and spin-tensors. More precisely, Curtright showed that the combined effect for massless fields of spin s and $(s+1/2)$, where $s=2,\ 5/2,\ 3,\ \ldots$, is invariant under supersymmetry transformations and the corresponding wave equations describe the massless representation of the Poincaré superalgebra of superspin s. Supersymmetry transformations have also been found on the basis of a formulation for higher-spin fields in terms of nonsymmetric tensors and spin tensors.[4] In the approaches of Refs. 1 and 4, however, supersymmetry was realized implicitly in the sense that the corresponding transformations form a closed algebra only on the mass shell. In this letter we propose two explicitly supersymmetric formulations, which are dual with respect one another, for free massless multiplets of arbitrary half-integer superspins $s>3/2$. The extension to the case of integer superspins as well as massless theories in anti-de Sitter superspace will be presented in separate publications.

The proposed superfield theories, which realize the massless representation of the Poincaré superalgebra of superspin $(s+1/2)$, where $s=2,\ 3,\ \ldots$, are described by the following sets of boson superfields:[1)]

$$1.\ H_{\alpha(s)\dot\alpha(s)},\quad \Gamma_{\alpha(s-1)\dot\alpha(s-1)},\quad \bar\Gamma_{\alpha(s-1)\dot\alpha(s-1)};\tag{1}$$

$$2.\ H_{\alpha(s)\dot\alpha(s)},\quad G_{\alpha(s-1)\dot\alpha(s-1)},\quad \bar G_{\alpha(s-1)\dot\alpha(s-1)}.\tag{2}$$

In each case $H_{\alpha(s)\dot\alpha(s)}\equiv H_{\alpha_1\ldots\alpha\ \dot\alpha 1\ldots\dot\alpha^s}\ (z)$ is a real superfield of the Lorentz type $(s/2,\ s/2)$, i.e., it is symmetric separately with respect to undotted and dotted indices.

The complex superfields $\Gamma_{\alpha(s-1)\dot\alpha(s-1)}$ and $G_{\alpha(s-1)\dot\alpha(s-1)}$ transform according to the same representation $[(s-1)/2,(s-1)/2]$ of the Lorentz group, but they are subject to significantly different constraints:

$$\bar D^{\dot\beta}\Gamma_{\alpha_1\ldots\alpha_{s-1}\dot\beta\dot\alpha_1\ldots\dot\alpha_{s-2}}=0; \quad (3)$$

$$\bar D_{\dot\alpha}G_{\alpha_1\ldots\alpha_{s-1}\dot\alpha_2\ldots\dot\alpha_s}=0. \quad (4)$$

Here symmetrization extends only to the dotted indices. Equations (3) and (4) mean that Γ and G are linear superfields,

$$\bar D^2\Gamma_{\alpha(s-1)\dot\alpha(s-1)}=0; \quad (5)$$

$$\bar D^2G_{\alpha(s-1)\dot\alpha(s-1)}=0. \quad (6)$$

where $\bar D^2=\bar D_{\dot\beta}\bar D^{\dot\beta}$ $(D^2=D^\beta D_\beta)$. The superfield $\Gamma_{\alpha(s-1)\dot\alpha(s-1)}$ can be called transversely linear field and the superfield $G_{\alpha(s-1)\dot\alpha(s-1)}$ can be called longitudinally linear field.

The general solution of Eq. (3) can be written in the form

$$\Gamma_{\alpha_1\ldots\alpha_{s-1}\dot\alpha_1\ldots\dot\alpha_{s-1}}=\bar D^{\dot\alpha_s}\xi_{\alpha_1\ldots\alpha_{s-1}(\dot\alpha_1\ldots\dot\alpha_s)}, \quad (7)$$

where $\xi_{\alpha(s-1)\dot\alpha(s)}$ is an arbitrary fermion superfield of the Lorentz type $[(s-1)/2, s/2]$. Here $\xi_{\alpha(s-1)\dot\alpha(s)}$ is determined from the modulus of the transformations:

$$\delta\xi_{\alpha_1\ldots\alpha_{s-1}\dot\alpha_1\ldots\dot\alpha_s}=\bar D^{\dot\alpha_{s+1}}\Lambda_{\alpha_1\ldots\alpha_{s-1}(\dot\alpha_1\ldots\dot\alpha_{s+1})}, \quad (8)$$

with an arbitrary boson superparameter $\Lambda_{\alpha(s-1)\dot\alpha(s+1)}$ of the Lorentz type $[(s-1)/2, (s+1)/2]$. Similarly, the general solution of Eq. (4) is

$$G_{\alpha_1\ldots\alpha_{s-1}\dot\alpha_1\ldots\dot\alpha_{s-1}}=\bar D_{(\dot\alpha_1}\zeta_{\alpha_1\ldots\alpha_{s-1}\dot\alpha_2\ldots\dot\alpha_{s-1})}, \quad (9)$$

where $\zeta_{\alpha(s-1)\dot\alpha(s-2)}$ is an arbitrary fermion superfield of the Lorentz type $[(s-1)/2, (s-2)/2]$, which is determined from the shear modulus:

$$\delta\zeta_{\alpha_1\ldots\alpha_{s-1}\dot\alpha_1\ldots\dot\alpha_{s-2}}=\bar D_{(\dot\alpha_1}\kappa_{\alpha_1\ldots\alpha_{s-1}\dot\alpha_2\ldots\alpha_{s-2})}, \quad (10)$$

with an arbitrary boson superparameter $\kappa_{\alpha(s-1)\dot\alpha(s-3)}$ of the Lorentz type $[(s-1)/2, (s-3)/2]$ The superfield $\Gamma_{\alpha(s-1)\dot\alpha(s-1)}$, which satisfies the condition (3), can therefore be interpreted as a gauge-invariant strength for the superpotential $\xi_{\alpha(s-1)\dot\alpha(s)}$ under the gauge transformations (8) (infinite-order reducibility). The superfield $G_{\alpha(s-1)\dot\alpha(s-1)}$,, which satisfies the condition (3), is a gauge-invariant strength for the superpotential $\xi_{\alpha(s-1)\dot\alpha(s-2)}$ under the gauge transformations (10) (finite-order reducibility).

We now define the gauge transformations for the superfields $H_{\alpha(s)\dot\alpha(s)}$, $\Gamma_{\alpha(s-1)\dot\alpha(s-1)}$, and $\Gamma_{\alpha(s-1)\dot\alpha(s-1)}$ according to the law

$$\delta H_{\alpha_1\ldots\alpha_s\dot\alpha_1\ldots\dot\alpha_s}=\bar D_{(\dot\alpha_1}L_{\alpha_1\ldots\alpha_s\dot\alpha_2\ldots\dot\alpha_s}-D_{(\alpha_1}\bar L_{\alpha_2\ldots\alpha_s)\dot\alpha_1\dot\alpha_s}; \quad (11)$$

$$\delta\Gamma_{\alpha_1\ldots\alpha_{s-1}\dot\alpha_1\dot\alpha_{s-1}}=-\tfrac{1}{4}\bar D^{\dot\beta}D^2\bar L_{\alpha_1\ldots\alpha_{s-1}\dot\beta\dot\alpha_1\ldots\dot\alpha_{s-1}}; \quad (12)$$

$$\delta G_{\alpha_1\ldots\alpha_{s-1}\dot\alpha_1\ldots\dot\alpha_{s-1}}=-\tfrac{1}{4}\bar D^2 D^\beta L_{\beta\alpha_1\ldots\alpha_{s-1}\dot\alpha_1\ldots\dot\alpha_{s-1}}+i(s-1)\partial^{\beta\dot\beta}\bar D_{(\alpha_1}L_{\beta\alpha_1\ldots\alpha_{s-1}\dot\alpha_2\ldots\dot\alpha_{s-1})\dot\beta}. \tag{13}$$

Here $L_{\alpha(s)\dot\alpha(s-1)}$ is an arbitrary fermion superfield of the Lorentz type $[s/2,\ (s-1)/2]$

The action functional that is (1) quadratic in the superfields H, Γ, and $\bar\Gamma$, (2) invariant under the transformations (11) and (12), and (3) leads in the components to field equations of order no higher than second in the derivatives, is determined, within a constant, as follows:

$$\begin{aligned}S^{\perp}_{(s+1/2,s+1)}=(-\tfrac{1}{2})^2\int d^8z\Big\{&\tfrac{1}{8}H^{\alpha(s)\dot\alpha(s)}D^\beta\bar D^2 D_\beta H_{\alpha(s)\dot\alpha(s)}\\&+H^{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)}(D_\beta\bar D_{\dot\beta}\Gamma_{\alpha(s-1)\dot\alpha(s-1)}-\bar D_{\dot\beta}D_\beta\bar\Gamma_{\alpha(s-1)\dot\alpha(s-1)})\\&+\Big(\bar\Gamma^{\alpha(s-1)\dot\alpha(s-1)}\Gamma_{\alpha(s-1)\dot\alpha(s-1)}\\&+\frac{s+1}{s}\Gamma^{\alpha(s-1)\dot\alpha(s-1)}\Gamma_{\alpha(s-1)\dot\alpha(s-1)}+\text{H.c.}\Big)\Big\}.\end{aligned} \tag{14}$$

The gauge arbitrariness of the transformations (11) and (12) enables us to choose the Wess–Zumino gauge

$$\begin{aligned}H_{\alpha_1\ldots\alpha_s\dot\alpha_1\ldots\dot\alpha_s}=&\theta^\beta\bar\theta^{\dot\beta}h_{(\beta\alpha_1\ldots\alpha_s)(\dot\beta\dot\alpha_1\ldots\dot\alpha_s)}+\bar\theta^2\theta^\beta\Psi_{(\beta\alpha_1\ldots\alpha_s)\dot\alpha_1\ldots\dot\alpha_s}-\theta^2\bar\theta^{\dot\beta}\bar\Psi_{\alpha_1\ldots\alpha_s(\dot\beta\dot\alpha_1\ldots\dot\alpha_s)}\\&+\theta^2\bar\theta^2A_{\alpha_1\ldots\alpha_s\dot\alpha_1\ldots\dot\alpha_s},\end{aligned}$$

$$\begin{aligned}\Gamma_{\alpha_1\ldots\alpha_{s-1}\dot\alpha_1\ldots\dot\alpha_{s-1}}=\exp(i\theta\sigma^a\bar\theta\partial_a)\,[&h_{\alpha_1\ldots\alpha_{s-1}\dot\alpha_1\ldots\dot\alpha_{s-1}}+\theta^\beta\bar\Psi_{(\beta\alpha_1\ldots\alpha_{s-1})\dot\alpha_1\ldots\dot\alpha_{s-1}}\\&+\theta_{(\alpha_1}\bar\psi_{\alpha_2\ldots\alpha_{s-1})\dot\alpha_1\ldots\dot\alpha_{s-1}}+\bar\theta^{\dot\beta}\bar\lambda_{\alpha_1\ldots\alpha_{s-1}(\dot\beta\dot\alpha_1\ldots\dot\alpha_{s-1})}\\&+\theta^2B_{\alpha_1\ldots\alpha_{s-1}\dot\alpha_1\ldots\dot\alpha_{s-1}}+\theta^\beta\bar\theta^{\dot\beta}U_{(\beta\alpha_1\ldots\alpha_{s-1})(\dot\beta\dot\alpha_1\ldots\dot\alpha_{s-1})}\\&+\bar\theta^{\dot\beta}\theta_{(\alpha_1}F_{\alpha_2\ldots\alpha_{s-1})}(\dot\beta\dot\alpha_1\ldots\dot\alpha_{s-1})+\theta^2\bar\theta^{\dot\beta}\rho_{\alpha_1\ldots\alpha_{s-1}(\dot\beta\dot\alpha_1\ldots\dot\alpha_{s-1})}.\end{aligned} \tag{15}$$

All field components written above are completely symmetric with respect to undotted and dotted indices; the boson fields $h_{\alpha(s+1)\dot\alpha(s+1)}$, $h_{\alpha(s-1)\dot\alpha(s-1)}$, and $A_{\alpha(s)\dot\alpha(s)}$ are real and the fields $B_{\alpha(s-1)\dot\alpha(s-1)}$ and $U_{\alpha(s)\dot\alpha(s)}$ are complex. It is easy to see that the boson fields $A_{\alpha(s)\dot\alpha(s)}$, $B_{\alpha(s-1)\dot\alpha(s-1)}$, $U_{\alpha(s)\dot\alpha(s)}$, and $F_{\alpha(s-2)\dot\alpha(s)}$ and the fermion fields $\lambda_{\alpha(s)\dot\alpha(s-1)}, \rho_{\alpha(s)\dot\alpha(s-1)}$ are auxiliary fields. After integrating in Eq. (14) over θ and $\bar\theta$ and eliminating the auxiliary fields, we obtain a unified theory of free boson fields

$$h_{\alpha(s+1)\dot\alpha(s)},\quad h_{\alpha(s-1)\dot\alpha(s-1)}$$

and fermion fields

$$\Psi_{\alpha(s+1)\dot\alpha(s)},\quad \Psi_{\alpha(s-1)\dot\alpha(s)},\quad \Psi_{\alpha(s-1)\dot\alpha(s-2)}+\text{H.c.}$$

The boson part of the action is identical to Fronsdal's action[2] for a massless field of spin $(s+1)$ and the fermion part is identical to the Fang–Fronsdal action[3] for a massless field of spin $(s+1/2)$. The theory (14) thus describes a massless multiplet of superspin $(s+1/2)$.

We note that a somewhat different choice of the Wess–Zumino gauge, which differs from the gauge (15), leads to a description of the boson part of the action (14) in terms of the generalized tetrad introduced by Vasil'ev.[4]

We now write the action functional of the superfields H, G, and $\bar{G}$, which is invariant under the gauge transformations (11) and (13):

$$S^{\|}_{(s+1/2,s+1)}=\left(-\tfrac{1}{2}\right)^s\int d^8z\Big\{\tfrac{1}{8}H^{\alpha(s)\dot{\alpha}(s)}D^\beta\bar{D}^2D_\beta H_{\alpha(s)\dot{\alpha}(s)}$$
$$-\tfrac{1}{8}\frac{s}{2s+1}[D_\beta,\bar{D}_{\dot{\beta}}]H^{\beta\alpha(s-1)\dot{\beta}\dot{\alpha}(s-1)}[D^\gamma,\bar{D}^{\dot{\gamma}}]H_{\gamma\alpha(s-1)\dot{\gamma}\dot{\alpha}(s-1)}$$
$$+\frac{s}{2}\partial_{\beta\dot{\beta}}H^{\beta\alpha(s-1)\dot{\beta}\dot{\alpha}(s-1)}\partial^{\gamma\dot{\gamma}}H_{\gamma\alpha(s-1)\dot{\gamma}\dot{\alpha}(s-1)}+\frac{2is}{2s+1}\partial_{\beta\dot{\beta}}H^{\beta\alpha(s-1)\dot{\beta}\dot{\alpha}(s-1)}$$
$$\times(G_{\alpha(s-1)\dot{\alpha}(s-1)}-\bar{G}_{\alpha(s-1)\dot{\alpha}(s-1)})+\frac{1}{2s+1}\Big(\bar{G}^{\alpha(s-1)\dot{\alpha}(s-1)}$$
$$\times G_{\alpha(s-1)\dot{\alpha}(s-1)}-\frac{s+1}{s}G^{\alpha(s-1)\dot{\alpha}(s-1)}G_{\alpha(s-1)\dot{\alpha}(s-1)}+\text{H.c.}\Big)\Big\}. \tag{16}$$

This action also describes a massless multiplet of superspin $(s+1/2)$. Actually, the theories (14) and (16) are equivalent, since they are related through the following auxiliary action by a duality transformation:

$$S[H,G,V]=\left(-\tfrac{1}{2}\right)^s\int d^8z\Big\{\tfrac{1}{8}H^{\alpha(s)\dot{\alpha}(s)}D^\beta\bar{D}^2D_\beta H_{\alpha(s)\dot{\alpha}(s)}$$
$$+\Big(H^{\beta\alpha(s-1)\dot{\beta}\dot{\alpha}(s-1)}D_\beta\bar{D}_{\dot{\beta}}V_{\alpha(s-1)\dot{\alpha}(s-1)}$$
$$-\frac{2}{s}G^{\alpha(s-1)\dot{\alpha}(s-1)}V_{\alpha(s-1)\dot{\alpha}(s-1)}+\bar{V}^{\alpha(s-1)\dot{\alpha}(s-1)}V_{\alpha(s-1)\dot{\alpha}(s-1)}$$
$$+\frac{s+1}{s}V^{\alpha(s-1)\dot{\alpha}(s-1)}V_{\alpha(s-1)\dot{\alpha}(s-1)}+\text{H.c.}\Big)\Big\}. \tag{17}$$

Here the variable $V_{\alpha(s-1)\dot{\alpha}(s-1)}$ is an arbitrary complex superfield of the Lorentz type $[(s-1)/2,\ (s-1)/2]$

There exists a purely superfield proof of the fact that theories (14) and (16) describe a massless superspin $(s+1/2)$. Specifically, the only gauge-invariant strength which survives on the mass shell is the chiral superfield of the Lorentz type $(s+1/2,\ 0)$

$$W_{\alpha_1\ldots\alpha_{2s+1}}=\bar{D}^2\partial_{(\alpha_1}{}^{\dot\beta_1}\ldots\partial_{\alpha_s}{}^{\dot\beta_s}D_{\alpha_{s+1}}H_{\alpha_{s+2}\ldots\alpha_{2s+1})\dot\beta_1\ldots\dot\beta_s} \tag{18}$$

and its conjugate field. In the equations of motion the strength $W_{\alpha(2s+1)}$ satisfies the equation

$$D^\beta W_{\beta\alpha(2s)}=0, \tag{19}$$

and is therefore an on-shell massless superfield of superhelicity $(s+1/2)$.[6]

In conclusion, let us consider the case which occurs at $s=1$. In terms of the scheme described above this is a special choice, since Eq. (3) is meaningful only if $s>1$. For $s=1$, however, we can employ, as before, the corollary of this equation (5), assuming Γ to be an arbitrary complex superfield. The corresponding functional (14) will then be identical to the action of linearized $n=-1$ nonminimal supergravity.[7] Next, for $s=1$ (and only in this case) Eq. (4) represents chirality, $\bar{D}_{\dot\alpha}G=0$; the expression on the right-hand side of the gauge law (13) also becomes purely chiral. The corresponding functional (16) is identical to the action of the linearized minimal $(n=-1/3)$ supergravity.[7] The new minimal formulation $(n=0)$ is probably specific only for supergravity (superspin 3/2) and cannot be used as a basis for a generalization to higher superspins.

[1)]We employ the notation adopted in the book by Wess and Bagger.[5] In particular, $z^A=(x^a,\theta^\alpha,\bar\theta_{\dot\alpha})$ are coordinates in $N=1$, $D=4$ superspace; $D_A=(\partial_a,D_\alpha,\bar{D}^{\dot\alpha})$ are flat covarient derivatives; and $\partial_{\alpha\dot\alpha}=(\sigma^a)_{\alpha\dot\alpha}\partial_a$. Parentheses indicate symmetrization of spinor indices. For example, $\Phi_{(\alpha_1\ldots\alpha_s)}=(1/s!)\Phi_{\alpha_1\ldots\alpha_s}+(s!-1)$ permutations of the indices $\alpha_1\ldots\alpha_s$. The undotted and dotted indices are symmetrized independently.

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