Massless gauge superfields of higher integer superspins

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(Submitted 31 March 1993)
Pis'ma Zh. Eksp. Teor. Fiz. **57**, No. 9, 526–529 (10 May 1993)

Two (dual) superfield formulations are proposed for free massless $N=1$, $D=4$ supermultiplets $(s, s+1/2)$, where $s=1, 2, \ldots$. For $s=1$ one of these versions is identical to the nonminimal formulation for the supermultiplet $(1, 3/2)$ [E. S. Fradkin and M. Vasiliev, Nuovo Cim. Lett. **25**, 79 (1979); B. de Witt and J. W. van Holten, Nucl. Phys. B **155**, 530 (1979); S. J. Gates and W. Siegel, Nucl. Phys. B **164**, 484 (1980)].

In the preceding letter[1] two (dual) superfield formulations were constructed for free massless $N=1$, $D=4$ supermultiplets of half-integer superspin $(s+1/2)$, where $s=2, 3, \ldots$. In this letter consider multiplets of integer superspin. We will show that as in the case of higher half-integer superspins, for each massless multiplet of superspin s,[1)] where $s=1,2, \ldots$, there exist two dual superfield formulations.

The proposed superfield theories which realize the massless representation of Poincaré superalgebra with nonzero integer superspin s are described by the following sets of boson superfields:

$$1.\ H_{\alpha(s-1)\alpha(s-1)},\quad \Gamma_{\alpha(s)\dot\alpha(s)},\quad \bar\Gamma_{\alpha(s)\dot\alpha(s)}; \tag{1}$$

$$2.\ H_{\alpha(s-1)\dot\alpha(s-1)},\quad G_{\alpha(s)\dot\alpha(s)},\quad \bar G_{\alpha(s)\dot\alpha(s)}. \tag{2}$$

In each case $H_{\alpha(s-1)\dot\alpha(s-1)} \equiv H_{\alpha_1\ldots\alpha_{s-1}\dot\alpha_1\ldots\dot\alpha_{s-1}}(z)$ is a real superfield of the Lorentz type $[(s-1)/2,(s-1)/2]$, i.e., it is symmetric separately with respect to the undotted and dotted spinor indices (we use the same notation as in Ref. 1). The complex superfields $\Gamma_{\alpha(s)\dot\alpha(s)}$ and $G_{\alpha(s)\dot\alpha(s)}$ transform according to the $(s/2, s/2)$ representation of the Lorentz group, but they are subject to significantly different constraints:

transverse-linear

$$\bar D^{\dot\beta}\Gamma_{\alpha_1\ldots\alpha_s\dot\beta\dot\alpha_1\ldots\dot\alpha_{s-1}}=0 \tag{3}$$

and longitudinal-linear

$$\bar D_{(\dot\alpha_1}G_{\beta_1\ldots\beta_s\dot\alpha_2\ldots\dot\alpha_{s+1})}=0, \tag{4}$$

respectively. The properties of the superfields (3) and (4) are discussed in detail in Ref. 1.

We define the gauge transformations for the superfields $H_{\alpha(s-1)\dot\alpha(s-1)}$, $\Gamma_{\alpha(s)\dot\alpha(s)}$, and $G_{\alpha(s)\dot\alpha(s)}$ according to the law:[2)]

$$\delta H_{\alpha_1\ldots\alpha_{s-1}\dot\alpha_1\ldots\dot\alpha_{s-1}}=D^\beta L_{\beta\alpha_1\ldots\alpha_{s-1}\dot\alpha_1\ldots\dot\alpha_{s-1}}-\bar D^{\dot\beta}\bar L_{\alpha_1\ldots\alpha_{s-1}\dot\beta\dot\alpha_1\ldots\dot\alpha_{s-1}}; \tag{5}$$

$$\delta\Gamma_{\alpha_1\ldots\alpha_s\dot\alpha_1\ldots\dot\alpha_s}=-\tfrac{1}{4}\bar D^2 D_{(\alpha_1}\bar L_{\alpha_2\ldots\alpha_s)\dot\alpha_1\ldots\dot\alpha_s)}+i(s+1)\bar D^{\dot\beta}\partial_{(\alpha_1(\dot\beta_1}\bar L_{\alpha_2\ldots\alpha_s)\dot\alpha_1\ldots\dot\alpha_s)}; \tag{6}$$

$$\delta G_{\alpha_1\ldots\alpha_s\dot\alpha_1\ldots\dot\alpha_s}=-\tfrac{1}{4}\bar D_{(\dot\alpha_1}D^2L_{\alpha_1\ldots\alpha_s\dot\alpha_2\ldots\dot\alpha_s)}\,. \tag{7}$$

Here $L_{\alpha(s)\dot\alpha(s-1)}$ is an arbitrary fermion superfield of the Lorentz type $[s/2, (s-1)/2]$.

The action functional that 1) is quadratic in the superfields H, Γ, and $\bar\Gamma$, 2) invariant under the transformations (5) and (6), and 3) leads in the components to field equations of order no higher than second in the derivatives, is determined, within a constant, by the expression

$$\begin{aligned}S^{\perp}_{(s,s+1/2)}=&-\left(-\frac{1}{2}\right)^s\int d^8z\left\{-\frac{1}{8}H^{\alpha(s-1)\dot\alpha(s-1)}D^\beta\bar D^2D_\beta H_{\alpha(s-1)\dot\alpha(s-1)}\right.\\&+\frac{1}{8}\frac{s^2}{(s+1)(2s+1)}[D^\beta,\bar D^{\dot\beta}]H^{\alpha_1\ldots\alpha_{s-1}\dot\alpha_1\ldots\dot\alpha_{s-1}}[D_{(\beta},\bar D_{(\dot\beta}]H_{\alpha_1\ldots\alpha_{s-1})\dot\alpha_1\ldots\dot\alpha_s)}\\&+\frac{1}{2}\frac{s^2}{s+1}\partial^{\beta\dot\beta}H^{\alpha_1\ldots\alpha_{s-1}\dot\alpha_1\dot\alpha_{s-1}}\partial_{(\beta(\dot\beta}H_{\alpha_1\ldots\alpha_{s-1})\dot\alpha_1\ldots\dot\alpha_{s-1})}\\&+\frac{2is}{2s+1}H^{\alpha(s-1)\dot\alpha(s-1)}\partial^{\beta\dot\beta}(\Gamma_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)}-\bar\Gamma_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)})\\&\left.+\frac{1}{2s+1}\left(\bar\Gamma^{\alpha(s)\dot\alpha(s)}\Gamma_{\alpha(s)\dot\alpha(s)}-\frac{s}{s+1}\Gamma^{\alpha(s)\dot\alpha(s)}\Gamma_{\alpha(s)\dot\alpha(s)}+\text{H.c.}\right)\right\}.\end{aligned} \tag{8}$$

The gauge arbitrariness (5) and (6) enables us to choose for $s>1$ the Wess–Zumino gauge:

$$\begin{aligned}H_{\alpha_1\ldots\alpha_{s-1}\dot\alpha_1\ldots\dot\alpha_{s-1}}=&\beta_{\alpha_1}\bar\theta_{(\dot\alpha_1}h_{\alpha_2\ldots\alpha_{s-1})\dot\alpha_2\ldots\dot\alpha_{s-1})}+\theta^2\bar\theta_{(\dot\alpha_1}\Psi_{\alpha_1\ldots\alpha_{s-1}\dot\alpha_2\ldots\dot\alpha_{s-1})}\\&-\bar\theta^2\theta_{(\alpha_1}\bar\Psi_{\alpha_2\ldots\alpha_{s-1})\dot\alpha_1\ldots\dot\alpha_{s-1}}+\theta^2\bar\theta^2A_{\alpha_1\ldots\alpha_{s-1}\dot\alpha_1\ldots\dot\alpha_{s-1}},\\ \Gamma_{\alpha_1\ldots\alpha_s\dot\alpha_1\ldots\dot\alpha_s}=&\exp(i\theta\sigma^a\bar\theta\partial_a)[h_{\alpha_1\ldots\alpha_s\dot\alpha_1\ldots\dot\alpha_s}+\theta^\beta\Psi_{(\beta\alpha_1\ldots\alpha_s)\dot\alpha_1\ldots\dot\alpha_s}+\theta_{(\alpha_1}\Psi_{\alpha_2\ldots\alpha_s)\dot\alpha_1\ldots\dot\alpha_s}\\&+\bar\theta^{\dot\beta}\bar\lambda_{\alpha_1\ldots\alpha_s(\dot\beta\dot\alpha_1\ldots\dot\alpha_s)}+\theta^2B_{\alpha_1\ldots\alpha_s\dot\alpha\alpha\ldots\dot\alpha_s}+\theta^\beta\bar\theta^{\dot\beta}U_{(\beta\alpha_1\ldots\alpha_s)(\dot\beta\dot\alpha_1\ldots\dot\alpha_s)}\\&+\bar\theta^{\dot\beta}\theta_{(\alpha_1}F_{\alpha_2\ldots\alpha_s)(\dot\beta\dot\alpha_1\ldots\dot\alpha_s)}+\theta^2\bar\theta^{\dot\beta}\bar\rho_{\alpha_1\ldots\alpha_s(\dot\beta\dot\alpha_1\ldots\dot\alpha_s)}].\end{aligned} \tag{9}$$

All component fields written out are completely symmetric in the undotted and dotted indices; the boson fields $h_{\alpha(s)\dot\alpha(s)}$, $h_{\alpha(s-2)\dot\alpha(S-2)}$, and $A_{\alpha(s-1)\dot\alpha(s-1)}$ are real and the fields $B_{\alpha(s)\dot\alpha(s)}$ and $U_{\alpha(s+1)\dot\alpha(s+1)}$ are complex. It is clear from dimensional considerations that the component fields $A_{\alpha(s-1)\dot\alpha(s-1)}$, $B_{\alpha(s)\dot\alpha(s)}$, $U_{\alpha(s+1)\dot\alpha(s+1)}$, $F_{\alpha(s-1)\dot\alpha(s+1)}$, $\lambda_{\alpha(s+1)\dot\alpha(s)}$, and $\rho_{\alpha(s+1)\dot\alpha(s)}$ are auxiliary fields. After integrating in Eq. (8) over θ and $\bar\theta$ and eliminating the auxiliary fields, we obtain a unified theory of free boson fields

$$h_{\alpha(s)\dot\alpha(s)},\quad h_{\alpha(s-2)\dot\alpha(s-2)}$$

and fermion fields

$$\Psi_{\alpha(s+1)\dot\alpha(s)},\quad \Psi_{\alpha(s-1)\dot\alpha(s)},\quad \Psi_{\alpha(s-1)\dot\alpha(s-2)}+\text{H.c.}$$

The boson sector of the action is identical to Fronsdal's action[2] for a massless field of spin s and the fermion sector is identical to the Fang–Fronsdal action[3] for a massless spin $(s+1/2)$ field. The theory (8) with $s>1$ thus describes a massless multiplet of superspin s. It is easy to show that in the case $s=1$ the theory (8) describes the supermultiplet (1, 3/2).

The action functional of the superfields H, G, and $\bar{G}$, which is invariant under the gauge transformations (5) and (7), has a simpler structure than that of (8):

$$S^{\|}_{(s,S+1/2}=\left(-\frac{1}{2}\right)^{s}\int d^{8}z\left\{\frac{1}{8}H^{\alpha(s-1)\dot{\alpha}(s-1)}D^{\beta}\bar{D}^{2}D_{\beta}H_{\alpha(s-1)\dot{\alpha}(s-1)}+\frac{s}{s+1}H^{\alpha(s-1)\dot{\alpha}(s-1)}\right.$$
$$\times(D^{\beta},\bar{D}^{\dot{\beta}}G_{\beta\alpha(s-1)\dot{\beta}\dot{\alpha}_{s}(s-1)}-\bar{D}^{\dot{\beta}}D^{\beta}G_{\beta\alpha(s-1)\beta\dot{\alpha}(s-1)})$$
$$\left.+\left(\bar{G}^{\alpha(s)\dot{\alpha}(s)}G_{\alpha(s)\dot{\alpha}(s)}+\frac{s}{s+1}G^{\alpha(s)\dot{\alpha}(s)}G_{\alpha(s)\dot{\alpha}(s)}+\text{H.c.}\right)\right\}. \tag{10}$$

The theories (8) and (10) are equivalent, since they are related through the following auxiliary action by a duality transformation:

$$S[H,\Gamma,V]=\left(-\frac{1}{2}\right)^{s}\int d^{8}z\left\{\frac{1}{8}H^{\alpha(s-1)\dot{\alpha}(s-1)}D^{\beta}\bar{D}^{2}D_{\beta}H_{\alpha(s-1)\dot{\alpha}(s-1)}\right.$$
$$+\frac{1}{s+1}(sH^{\alpha(s-1)\dot{\alpha}(s-1)}D^{\beta}\bar{D}^{\dot{\beta}}V_{\beta\alpha(s-1)\dot{\beta}\dot{\alpha}_{s}(s-1)}+2\Gamma^{\alpha(s)\dot{\alpha}(s)}V_{\alpha(s)\dot{\alpha}(s)}$$
$$\left.+(s+1)\bar{V}^{\alpha(s)\dot{\alpha}(s)}V_{\alpha(s)\dot{\alpha}(s)}+sV^{\alpha(s)\dot{\alpha}(s)}V_{\alpha(s)\dot{\alpha}(s)}+\text{H.c.})\right\}, \tag{11}$$

where $V_{\alpha(s)\dot{\alpha}(s)}$ is an arbitrary complex superfield of the Lorentz type $(s/2,\ s/2)$. Therefore, the functional (10) describes a free massless multiplet of superspin s. This fact can be proved by a different method as follows. The only gauge-invariant strength which survives on the mass shell of the theory (10) is the chiral superfield of Lorentz type $(s,\ 0)$

$$W_{\alpha_1\ldots\alpha\beta_1\ldots\beta_s}=\frac{si}{2}\partial_{(\alpha_1}{}^{\dot{\alpha}_1}\ldots\partial_{\alpha_{s-1}}{}^{\dot{\alpha}_{s-1}}\bar{D}^{\dot{\alpha}_s}D_{\alpha s}G_{\beta_1\ldots\beta_s)\dot{\alpha}_1\ldots\dot{\alpha}_s}+\partial_{(\alpha_1}{}^{\dot{\alpha}_1}\ldots\partial_{\alpha_s}{}^{\dot{\alpha}_s}G_{\beta_1\ldots\beta_s)\dot{\alpha}_1\ldots\dot{\alpha}_s} \tag{12}$$

and its conjugate field. In the equations of motion $W_{\alpha(2s)}$ becomes D-transverse:

$$D^{\beta}W_{\beta\alpha(2s-1)}=0; \tag{13}$$

i.e., it is the on-shell massless superfield of the superhelicity s.

In conclusion let us consider the case $s=1$, which has been studied in the literature. There are two known off-shell formulations for the (1, 3/2) supermultiplet: minimal[4] and nonminimal.[5-7] The functional (10) with $s=1$ is identical to the superfield action for the nonminimal (1, 3/2) multiplet.[7] The theory (8) with $s=1$ realizes a new off-shell formulation for the (1, 3/2) supermultiplet. In this new formulation the dynamics is described by the spin-tensor superfield $\Phi_{(\alpha\beta)\dot{\alpha}}$ which arises according to the rule $\bar{\Gamma}_{\alpha\dot{\alpha}}=D^{\beta}\Phi(\alpha\beta)\dot{\alpha}$.

[1)]We recall that a massless multiplet of superspin s describes two particles with spin s and $(s+1/2)$ and is often designated by $(s, s+1/2)$.
[2)]The undotted and dotted indices are symmetrized independently.

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Translated by M. E. Alferieff