Electron g-factor anisotropy in asymmetric GaAs/AlGaAs quantum well

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It is shown that in an asymmetric quantum well, grown in the direction $z \parallel [001]$ from semiconductors with a zinc blende structure, the conduction-band spin splitting attributable to the absence of an inversion center results in conduction-electron g-factor anisotropy in the plane of the well. It is demonstrated for the example of a GaAs/AlGaAs quantum well that the off-diagonal components $g_{xy}=g_{yx}$ ($x \parallel [100]$), which characterize the electron g-factor anisotropy in the plane of the well, can be comparable in magnitude to the diagonal components $g_{xx}=g_{yy}$.

A fundamental characteristic of a conduction electron in a semiconductor is its g-factor. In our case the g-factor is a tensor of rank two ($\hat{g}$), whose form is determined by the symmetry of the crystal. For example, in bulk GaAs (point symmetry group T_d) the tensor for an electron at the bottom of the conduction band degenerates into a scalar and the g-factor is isotropic. In the heterostructure GaAs/AlGaAs with a symmetric quantum well (QW), grown in the direction $z \parallel [001]$, the components of the g-factor in the direction of, and perpendicular to, the plane of the QW become different due to the lowering of the symmetry ($T_d \rightarrow D_{2d}$) at the bottom of the quantum-size subbands: $g_{xx}=g_{yy}\neq g_{zz}$ ($x \parallel [100]$). Such g-factor anisotropy was investigated theoretically in Ref. 1 and observed experimentally in Ref. 2. In an asymmetric quantum well the further lowering of symmetry ($D_{2d}\rightarrow C_{2v}$) causes the off-diagonal elements $g_{xy}=g_{yx}$ of the tensor $\hat{g}$, in addition to the diagonal elements, to be different from zero. This is why the g-factor anisotropy appears in the plane of the well.

In this letter we consider a specific mechanism for the appearance of off-diagonal components of the g-factor conduction electrons in an asymmetric QW, grown from semiconductors with zinc blende structure. Because of the spin-momentum coupling, in this mechanism, the external magnetic field **H** influences the spin of a conduction electron by changing the momentum p of the electron. This coupling is present in crystals without an inversion center and in the case of bulk GaAs it leads to cubic (in the momentum) spin splitting of the conduction band.

First, we consider qualitatively the nature of the appearance of g_{yx}. Spin splitting of the conduction band in crystals without an inversion center can be represented as resulting from the action on the electron spin of an effective magnetic field $\mathbf{H}_{\text{eff}}$, whose magnitude and direction are determined by the magnitude and direction of the electron momentum $\mathbf{p}$.[3] For example, in a GaAs/AlGaAs QW, grown in the direction $z \parallel [001]$, the spin of an electron with momentum $\mathbf{p}=(0, p_y, p_z)$ precesses around the field $\mathbf{H}_{\text{eff}}=(0, \beta p_z^2 p_y, 0)$.[1)] Here $y \parallel [010]$ and β is a parameter characterizing the magnitude of the spin splitting of the conduction band. Let us assume that an external magnetic field **H** is applied to the sample in the direction $x \parallel [100]$. An electron moving in the z direction with a velocity $V_z = p_z/m$ for a time δt without colliding with the walls of the well will then acquire, under the action of the Lorentz force, in the y direction the additional momentum

$$\delta p_y' = -\frac{e}{c} H_x v_z \delta t = -\frac{e}{c} H_x (z - z_0).$$

Here z_0 is the initial position of the electron in the well. Averaging δp_y over the initial position of the electron, we obtain

$$\delta p_y(z) = -\frac{e}{c} H_z (z - \langle z \rangle),$$

where $\langle z \rangle$ is the average coordinate of the electron in the well. Averaging $\delta p_y(z)$ over the fast motion in the direction of size quantization gives $\langle \delta p_y(z) \rangle = 0$. However, the correction to the effective magnetic field $(\delta H_{\rm eff})_y = \beta p_z^2 \delta p_y$ generally does not vanish under such averaging. Indeed,

$$\langle (\delta H_{\rm eff})_y \rangle = \frac{e}{c} \beta (\langle p_z^2 \rangle \langle z \rangle - \langle p_z^2 \rangle) H_x,$$

and in an asymmetric well $\langle (\delta H_{\rm eff})_y \rangle \neq 0$. Application of an external magnetic field along the x axis therefore causes a precession of the electron spins around the y axis, which corresponds to a nonzero off-diagonal element $g_{yx} = (e/c)\beta(\langle p_z^2 \rangle \langle z \rangle - \langle p_z^2 z \rangle)$.

We now give a more rigorous proof. We consider a nondegenerate electron gas and assume that motion in the z direction is determined entirely by size quantization and that the energy of this motion is significantly higher than that of motion in the plane of the QW. We orient the external magnetic field parallel to the plane of the well ($\mathbf{H} = (H_x, H_y, 0)$) and choose the gauge for the vector potential in the form $\mathbf{A} = (H_y z, -H_x z, 0)$. The Hamiltonian[4] describing the spin splitting (which is attributable to the fact that the crystal has no inversion center) of the conduction band of a bulk GaAs semiconductor, in the structure with a QW grown in the [001] direction, can then be written in the form

$$\hat{V} = \frac{\gamma_c}{\hbar^3} \left\{ \hat{p}_z^2, \left[-\hat{\sigma}_x \left(p_x + \frac{e}{c} H_y z \right) + \hat{\sigma}_y \left(p_y - \frac{e}{c} H_x z \right) \right] \right\}_{\rm symm}, \tag{1}$$

where p_x and p_y are the components of the electron quasimomentum in the plane of the QW in the absence of a magnetic field, $\hat{p}_z$ is the momentum operator, $\hat{\sigma}_x$ and $\hat{\sigma}_y$ are Pauli matrices, γ_c is a constant characterizing the spin splitting (cubic in $\mathbf{p}$) of the conduction band in the bulk semiconductor (it is assumed that γ_c does not depend on z), the electron charge is $-e$, and $\{A, B\}_{\rm symm} = (AB + BA)/2$. Averaging expression (1) over the fast motion of the electron in the direction of size quantization, in first-order perturbation theory we obtain the spin Hamiltonian

$$\hat{V}_s=\frac{\gamma_c}{\hbar^3}(\hat{p}_z^2)_{nn}\left[-\hat{\sigma}_x\left(p_x+\frac{e}{c}H_y z_{nn}\right)+\hat{\sigma}_y\left(p_y-\frac{e}{c}H_x z_{nn}\right)\right]$$

$$+\frac{\gamma_c}{\hbar^3}[(\hat{p}_z^2)_{nn}z_{nn}-(\hat{p}_z^2 z)_{nn}](\hat{\sigma}_x H_y+\hat{\sigma}_y H_x), \quad (2)$$

where n is the number of the quantum-size level, and $A_{nn}=\langle n|A|n\rangle$.

The result (2) can be explained graphically. It is known[5] that when spin is ignored, the dispersion curve of a conduction electron in a magnetic field oriented parallel to the plane of the QW is a parabola which is displaced in the direction **p** by the vector $(e/c)z_{nn}(-H_y,H_x)$. The electron-momentum-dependent spin splitting of this parabola is described by the first term in Eq. (2). The second term in Eq. (2), as one can easily see, arises only in an asymmetric well in the presence of the magnetic field and does not depend on the electron momentum. This term can be expressed in terms of the off-diagonal components of the g-factor in the form $(\mu_B/2)(\hat{\sigma}_x g_{xy}^{(n)}H_y+\hat{\sigma}_y g_{yx}^{(n)}H_x)$, where

$$g_{xy}^{(n)}=g_{yx}^{(n)}=\frac{2\gamma_c e}{\hbar^3 c\mu_B}[(\hat{p}_z^2)_{nn}z_{nn}-(\hat{p}_z^2 z)_{nn}], \quad (3)$$

(the Bohr magnetron $\mu_B>0$).

In the derivation of Eq. (3) the vector potential was taken in the form $\mathbf{A}=(H_y z, -H_x z, 0)$. It can be shown, however, that expression (3) is invariant under a gauge transformation of the vector potential.

It is evident from Eq. (3) that the magnitude and sign of g_{xy} are determined by the specific form of the asymmetric potential. In order to make quantitative estimates we calculate g_{xy} in an infinitely deep triangular well, in which the potential energy $U(z)=\infty$ for $z<0$ and increases linearly: $U(z)=Fz$ for $z\geqslant 0$. Calculation shows that in such a well for an electron in the ground state $(n=0)$ we have

$$g_{yx}=g_{xy}=1.24\frac{\gamma_c e}{\hbar^2 c\mu_B}(m\hbar F)^{1/3}, \quad (4)$$

where m is the effective mass of the electron.

Let us now estimate from Eq. (4) the quantity g_{xy} in a real asymmetric triangular QW GaAs/$Al_{0.3}Ga_{0.7}As$, one wall of which is vertical and the other is inclined (the inclined wall is grown by changing the aluminum concentration linearly from zero to $X_{Al}=0.3$). For the electric field $F=10^5$ eV/cm (Ref. 6), which is easily realized in such wells, and $\gamma_c=2.45P\times10^{-23}$ eV $\cdot$ Å^3 (Ref. 7), $m=0.067m_0$, which are characteristic of GaAs, we have $g_{xy}=0.17$. The diagonal components of $\hat{g}$ are bounded by values of the g-factor in the bulk materials g_0(GaAs) $=-0.44$ and $g_0(Al_{0.3}Ga_{0.7}As)=0.4$ (Ref. 8). Thus g_{xy} *and* g_{xx} are comparable in absolute magnitude, so that the Larmor precession frequency $\omega=(\mu_B/\hbar)|\hat{g}\mathbf{H}|$ of the electron spins should depend on the orientation of the external magnetic field **H** in the plane of the QW. This dependence of ω on the orientation of the field **H** can manifest itself, for example, as a shift in the ESR frequency, as well as a change in the width of the magnetic depolarization curve for optically polarized electrons (the Hanle effect[3]).

In Ref. 9 it is shown that the g-factor electrons in an inversion layer, formed near the GaAs/AlGaAs heterojunction, can be measured by using ESR which can be detected from the change in the resistance of the layer. Since the exchange interaction of the electrons does not influence the ESR frequency,[10] the g-factor determined in this manner is a single-electron factor and therefore Eq. (4) can be used to estimate the effect of the component g_{xy} on the ESR frequency.

We call attention to the fact that an analysis of the spin splitting of the conduction band in an asymmetric QW must include in the Hamiltonian of the electron in addition to the term (1), a term, due to the potential gradient, of the type[11]

$$\hat{V}' = \alpha \frac{dU}{dz} \left[\vec{\nu} \left(p + \frac{e}{c} A \right) \right] \vec{\sigma}, \tag{5}$$

where a is a parameter, and $\vec{\nu}$ is a unit vector oriented in the direction of the z axis. However, a calculation similar to the one performed above shows that the interaction $\hat{V}'$ contributes only to the diagonal elements of the g-factor:

$$g_{xx}'^{(n)} = g_{yy}'^{(n)} = \frac{2\alpha e}{\mu_B c} \left(z \frac{dU}{dz} \right)_{nn}, \tag{6}$$

where a is assumed to be independent of z.

A complete analysis of the values of the diagonal elements of the g-factor in asymmetric QWs falls outside the scope of this work. It should be noted, however, that the approach proposed in this letter for determining the contributions of spin-orbit splitting of the conduction band to the components of the g-factor is not limited to asymmetric QWs, considered here and grown in the [001] direction from semiconductors with zinc-blende structure. This approach can be extended to the case of asymmetric QWs grown in an arbitrary direction from crystals of arbitrary symmetry.

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[1)] We disregarded the component $(H_{eff})_z = -\beta p_z p_y^2$ compared with $(H_{eff})_y = \beta p_z^2 p_y$, since motion perpendicular to the plane of the well is assumed to be fast.

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