Low-frequency dynamics of quantum-wire arrays in a strong magnetic field

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The physical features of the low-frequency electromagnetic response of an array of quantum wires under conditions of the quantum Hall effect are discussed.

1. Until recently collective excitations in arrays of quantum wires (QWs) were investigated in the far-infrared region.[1-4] A resonance, corresponding to the excitation of a volume two-dimensional (2D) magnetoplasmon with frequency $\omega=(\omega_c^2+\omega_\perp^2)^{1/2}$, where ω_c is the cyclotron frequency, $\omega_\perp \propto W^{-1/2}$ is the plasmon frequency with $B=0$, and W is the width of the wire, was observed in the IR transmission spectra in a magnetic field. The theory of IR absorption spectra of QW arrays has been studied extensively; see, for example, Refs. 5 and 6 and the references cited there. In Ref. 7 an edge-type resonance, whose frequency decreases with increasing B, was observed with the help of an auxiliary grating orientated perpendicular to the wires.

The rf ($f<200$ MHz) electrodynamic response of an array of QWs was recently investigated.[8] The samples were produced by deep etching of GaAs–AlGaAs heterostructures with a 2D electron gas. Unique features of the system's response were observed at low temperatures in the vicinity of the Hall plateaus with small numbers ($\nu=2$, 4) in quantizing magnetic fields. Resonances corresponding to excitation of weakly damped waves, which could be interpreted as edge magnetoplasmons traveling along the perimeter of each QW, were observed in a small neighborhood of the plateaus. Some distance from the plateau centers the response was of a relaxational nature. The characteristic relaxation frequency either increased away from the center of a plateau or dropped sharply, depending on the direction of the external exciting field relative to the axis of the wires. In this letter we discuss the main physical features of the low-frequency response of QW arrays in quantizing magnetic fields and we give a qualitative explanation of the experimental results of Ref. 8.

2. Consider an array of QWs in a magnetic field oriented perpendicular to the plane of the structure (Fig. 1). The length of the wires is L, the period of the structure is a, and the total number of wires is N. We use a model in which the conductivity of the 2D electrons within each wire is described by a local tensor $\sigma_{\alpha\beta}(\omega)\delta(z)$, where $\{\alpha,\beta\}=\{x,y\}$, $\sigma_{xx}=\sigma_{yy}$, $\sigma_{xy}=-\sigma_{yx}$. For $L\gg W$ and $Na\gg L$ the temporal evolution of charge-density fluctuations in the system is determined from the equations [all quantities are assumed to be proportional to $\exp(-i\omega t)$]

$$-i\omega\rho_1+j_y=0, \quad -i\omega\rho_2+fj_x=0, \tag{1}$$

$$j_\alpha=\sigma_{\alpha\beta}E_\beta, \tag{2}$$

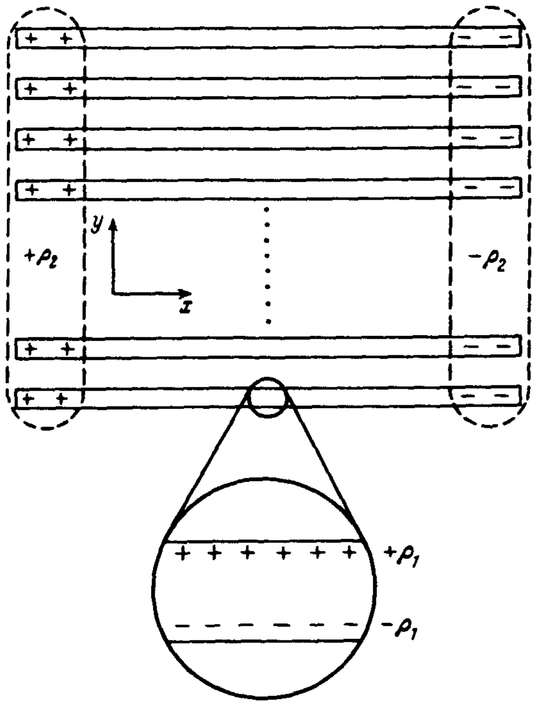

FIG. 1. Geometry of the experimental array of quantum wires.

where ρ_1 is the linear charge density near the lateral sides of a wire, ρ_2 is the average linear charge density near the end faces of the sample (see Fig. 1), j_α are the components of the local current density inside the wires, and $f=W/a$ is the geometric filling factor of the array. The electric field acting on the electrons inside the wires can be estimated as follows:

$$E_x=4\gamma_2\rho_2/\kappa L, \quad E_y=4\gamma_1\rho_1/\kappa W. \tag{3}$$

Here κ is the dielectric constant of the surrounding medium, and γ_1 and γ_2 are numbers of order unity (when the dipole interaction of the wires is taken into account, γ_1 and γ_2 can depend slightly on the filling factor f). Writing the system of equations (1)–(3) in the form

$$(-i\omega+4\gamma_1\sigma_{xx}/\kappa W)\rho_1-(4\gamma_2\sigma_{xy}/\kappa L)\rho_2=0,$$

$$(4\gamma_1 f\sigma_{xy}/\kappa W)\rho_1+(-i\omega+4\gamma_2 f\sigma_{xx}/\kappa L)\rho_2=0, \tag{4}$$

we obtain the dispersion law for the normal modes of the system:

$$\omega^2+i\omega(4\sigma_{xx}/\kappa)(\gamma_1/W+\gamma_2 f/L)-16\gamma_1\gamma_2 f(\sigma_{xx}^2+\sigma_{xy}^2)/\kappa^2 WL=0. \tag{5}$$

3. We will now analyze Eq. (5). Let us first consider the high-frequency limit ($\omega\tau\gg 1$), where Drude's model can be used for $\sigma_{\alpha\beta}(\omega)$ (here τ is the momentum relaxation time). Equation (5) can then be rewritten in the form

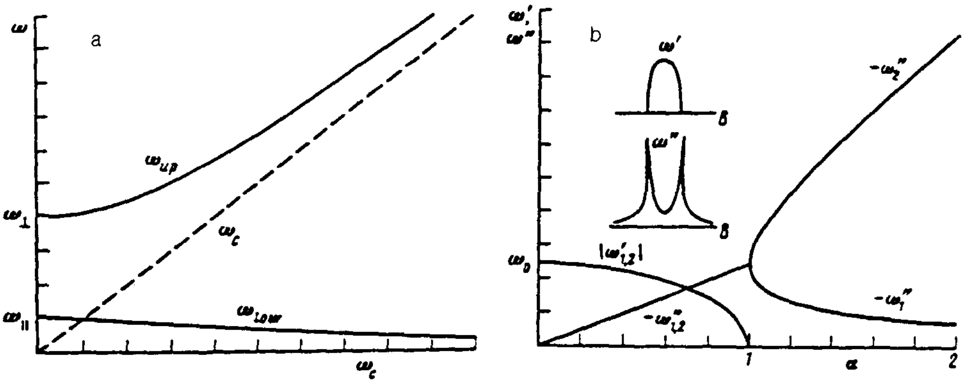

FIG. 2. a) Magnetic field dependence of the modes ω_{up} and ω_{low}; the dashed line is the cyclotron frequency. b) The frequency (ω') and damping (ω'') of the low-frequency mode ω_{low} (Eq. (8)) as a function of the parameter a. Inset: schematic representation of the magnetic field dependences of ω' and ω'' near a Hall plateau.

$$\omega^2(\omega^2-\omega_c^2)-\omega^2(\omega_\perp^2+\omega_\parallel^2)+\omega_\perp^2\omega_\parallel^2=0, \quad (6)$$

where $\omega_\perp^2=4\gamma_1 ne^2/m\kappa W$, and $\omega_\parallel^2=4\gamma_2 fne^2/m\kappa L$ are, respectively, the characteristic frequencies of plasma oscillations perpendicular to and parallel to the wires; n, e, and m are the density, the charge, and the effective mass of 2D electrons. The solution of Eq. (6) gives two branches of oscillations ω_{up} and ω_{low}, whose magnetic-field dependences are illustrated qualitatively in Fig. 2a. In the limit of narrow wires ($L\gg W$ or $\omega_\parallel^2\ll\omega_\perp^2$)

$$\omega_{up}^2=\omega_c^2+\omega_\perp^2, \quad \omega_{low}^2=\omega_\perp^2\omega_\parallel^2/(\omega_\perp^2+\omega_c^2). \quad (7)$$

The upper branch ω_{up} gives the spectrum of volume 2D magnetoplasmons. The frequency ω_{low} of the lower mode decreases with increasing B, just as the frequency of edge magnetoplasmons. Equation (6) agrees qualitatively with the equation obtained in Ref. 9 for a different anisotropic 2D electron system—an elliptic disk—in the oblate-ellipsoid model.

We will now consider the opposite limiting case, in which the excitation frequency ω is so low that the frequency dispersion of the conductivity can be completely ignored: $\sigma_{\alpha\beta}(\omega)=\sigma_{\alpha\beta}(0)$. In this limit, for $W\ll L$, Eq. (5) has the solution

$$\omega_{1,2}=\omega_0[\pm(1-\alpha^2)^{1/2}-i\alpha], \quad (8)$$

where

$$\omega_0^2=16\gamma_1\gamma_2 f(\sigma_{xx}'^2+\sigma_{xy}^2)/\kappa^2 WL, \quad \alpha^2=(\gamma_1/\gamma_2)(L/4fW)[\sigma_{xx}'^2/(\sigma_{xx}'^2+\sigma_{xy}^2)], \quad (9)$$

and $\sigma_{xx}'=\sigma_{xx}(0)$. Figure 2b are plots of the frequency ω' and the damping ω'' of the natural modes (8) as a function of the parameter a. In the limit $a\ll 1$, which corresponds to a very narrow neighborhood of the center of the Hall plateau ($\sigma_{xx}'^2/\sigma_{xy}^2\ll 4fW/L\ll 1$), the system contains a weakly damped mode with a frequency

$\omega'_{1,2}=\pm\omega_0$, with a damping $\omega''_{1,2}=-\omega_{0a}$, and with an elliptic polarization of the local electric field inside the wires $(E_x/E_y)_{1,2}=(\pm i\mathrm{sign}\sigma_{xy})(\gamma_2 Wf/\gamma_1 L)^{1/2}$. This mode is the low-frequency limit of the branch ω_{low} of the edge type [Eq. (7)]. For $a>1$ the response of the system is relaxational, and in the limit $a\gg 1$ we have

$$\omega_1\approx -i\omega_0/2a=-i4\gamma_2 f/\kappa L\rho'_{xx},\quad \omega_2\approx -i\omega_0\cdot 2\alpha=-i4\gamma_1\sigma'_{xx}/\kappa W, \tag{10}$$

where $\rho'_{xx}=\sigma'_{xx}/(\sigma'^2_{xx}+\sigma^{\ 2}_{xy})$. The solutions (10) have an obvious physical meaning: The mode ω_2 corresponds to transverse charge relaxation over a distance $\simeq W$ with the Maxwellian rate $\simeq\sigma'_{xx}/\kappa$; the mode ω_1 corresponds to longitudinal charge relaxation over a distance $\simeq L$ with Maxwellian rate $\simeq f/\kappa\rho'_{xx}$. The boundary between the relaxational mode and the vibrational mode is determined by the condition $a=1$, or $\sigma'^2_{xx}/\sigma^2_{xy}\approx 4fW/L$.

According to Ref. 10, the charge localization scale of the edge plasmons at the edge of a macroscopic sample is determined by the length

$$l=2\pi i\sigma_{xx}(\omega)/\omega\kappa=l_0+2\pi i\sigma'_{xx}/\omega\kappa, \tag{11}$$

where the contribution $l_0\equiv-2\pi\sigma''_{xx}(\omega)/\omega\kappa$ is linked with the polarizability of 2D electrons, $\chi'_{xx}=-\sigma''_{xx}(\omega)/\omega$. Let us determine the response of the array of QWs, taking into account $\sigma''_{xx}(\omega)$. We substitute $\sigma_{xx}(\omega)$ into the dispersion relation (5) in the following form:

$$\sigma_{xx}(\omega)=\sigma'_{xx}-i\omega\kappa l_0/2\pi. \tag{12}$$

In the low-frequency limit l_0 and σ'_{xx} may be assumed to be independent of the frequency, since σ'_{xx} is even and σ''_{xx} is odd. In this case the spectrum of excitations of the system is described by expression (8), in which the wire width W is replaced by $W^*=W+2\gamma_1 l_0/\pi$:

$$\omega_0^2=16\gamma_1\gamma_2 f\sigma^2_{xy}/\kappa^2 LW^*,\quad \alpha^2=(\gamma_1/\gamma_2)\sigma'^2_{xx}/\sigma^2_{xy})(L/4fW^*) \tag{13}$$

(we assumed here that $\sigma'^2_{xx}\ll\sigma^2_{xy}$). Thus the imaginary part of $\sigma_{xx}(\omega)$ can be justifiably ignored only when $l_0\ll W$. When the length l_0 is taken into account, the characteristic frequency ω_0 of the response decreases.

The magnetic-field dependence, described by Eq. (8), of the frequency and damping of characteristic modes of the array of wires (the inset in Fig. 2b) agrees qualitatively with the experimentally observed dependence.[8] The length l_0 can be estimated by comparing with Eq. (13) the resonance frequency measured in Ref. 8 at the center of the $\nu=2$ plateau. Setting $L=4.5$ mm, $a=1$ μm, $f\simeq 0.2$, $\kappa\simeq 12.8$, $\gamma_1\simeq\gamma_2\simeq 1$, and $\omega_0/2\pi\simeq 70$ MHz, we obtain $l_0\simeq 19$ μm, which agrees in order of magnitude with the estimate obtained in Ref. 11 ($l_0\simeq 10$ μm). Thus, under the experimental conditions of Ref. 8 we have $l_0\gg W$ and the excitation field inside a wire is uniform. In a system of QWs the excitation frequency is a power-law function of l_0, and not a logarithmic function, as in the case of macroscopic samples.[10] This circumstance can make it easier to determine the length l_0 (and the polarizability χ'_{xx} of a 2D system) in the quantum Hall effect regime.

4. In studying the low-frequency response of QW arrays new features can be expected to appear when retardation is taken into account. In Refs. 12 and 13 it was shown that in samples with high electron mobility the Maxwellian relaxation length $2\pi\sigma/\kappa$ (σ is the conductivity) in the absence of a magnetic field can exceed the speed of light c, which accounts for the significant change in the way the change spreads[12] and fourth change in the spectrum of plasma oscillations.[13] In a macroscopic sample with $B\neq 0$ the finiteness of the speed of light is no longer important, since the parameter $2\pi\sigma'_{xx}/\kappa c$, which determines the rate of Maxwellian spreading in a magnetic field, decreases with increasing B. In the case of an array of wires the situation is different, since the longitudinal Maxwellian relaxation rate $2\pi f/\rho'_{xx}\kappa$ diverges as the center of a plateau is approached, and when

$$\sigma'_{xx}/\sigma_{xy} < 2\pi f\sigma_{xy}/\kappa c = \nu f e^2/\hbar c\kappa \approx \nu f/137\kappa, \tag{14}$$

it becomes greater than the speed of light. If $\nu f e^2/\hbar c\kappa \ll 2(fW^*/L)^{1/2}$, as is the case in Ref. 8, the results obtained above remain valid. In the opposite limit $\nu f e^2/\hbar c\kappa > 2(fW^*/L)^{1/2}$, the existence domain of the weakly damped mode ω_0 widens.

Thus the low-frequency electromagnetic response of a system of QWs in quantizing magnetic fields depends on the relative values of three small parameters: σ'_{xx}/σ_{xy}, $2(fW^*/L)^{1/2}$, and $\nu f e^2/\hbar c\kappa$. Any ratios between these quantities can be realized experimentally, which gives rise to new interesting possibilities for studying the low-frequency response of an array of QWs.

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