Transition from self-organization to chaos in two-dimensional magnetic-bubble lattices whose boundaries have the shape of Cassinian ovals

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The dynamic properties of magnetic bubbles, which form in two-dimensional lattices with symmetry space groups *P6* and *Pab2* in the process of self-organization in a pulsed magnetization field, were studied by the method of high-speed photography. Division of domains, which determines the scenarios of the transition from self-organization to chaos, was observed. At all stages of motion the shape of the domain walls was found to be described well by the Cassinian ovals.

In Refs. 1–3 we reported the observation of new types of self-organization of the magnetic moment in iron garnet films with a large uniaxial anisotropy constant β_μ under the action of monopolar pulses of a magnetizing field. It was established that the anomalous quasielliptic mode of pulsational oscillations of magnetic bubbles, which is characterized by a periodic change in the orientation of the major symmetry axis of the domains by $\pi/2$, plays the key role in the appearance of self-organization. Above a critical height $\tilde{H}=\tilde{H}_{\rm cr}$ (or length $\tau_p=\tau_p^{(\rm cr)}$) of the magnetization pulse, self-organization stops and a transition occurs to chaotic motion of the domains.

In this letter we report the results of an experimental study of the transition from self-organization to chaotic motion of domains. These results were obtained by the method of high-speed photography with double-pulse illumination and exposure time $\simeq 10$ nsec. An 8.2-μm-thick iron-garnet film with the composition $(YBi)_3(FeGa)_5O_{12}$ on a $Gd_3Fe_5O_{12}$ substrate with (111) orientation was employed in the experiments. The labryinthal domain structure (DS) had a period of 54.3 μm, uniaxial anisotropy constant $\beta_u \simeq 10^3$, magnetization $M \simeq 5$ G, and bubble collapse field $=18.6$ Oe. The magnetic field pulses of width $\tau_p=4$ msec, generated by a flat coil with an inner diameter of 1 mm, had an approximately triangular shape with rise time τ_r and cutoff τ_f of about 2 μsec; the field $\tilde{H}$ was directed along the normal to the film.

The experiments were performed as follows. First, a dynamically stable DS with symmetry space group *Cmm*2 or *P6mm* (for static DS—*Pab*2 or *P*6) was created in the film in the self-organization regime with pulse height $|\tilde{H}| < |\tilde{H}_{\rm cr}|$ and "frozen" (by shutting off the pulse generator). Next, a single magnetization pulse with a height $|\tilde{H}| = |\tilde{H}_{\rm cr}| + \delta H$, which exceeded only slightly $(dH \ll |\tilde{H}_{\rm cr}|)$ the threshold of the transition from periodic to chaotic domain motion, was generated; two illumination pulses were applied with time delay $\tau_1 = var$ and $\tau_2 = var$ in order to record the instantaneous shape of the dynamic DS at selected times.[1)] After each photograph was taken, the procedure was repeated for other values of τ_2. The results presented below

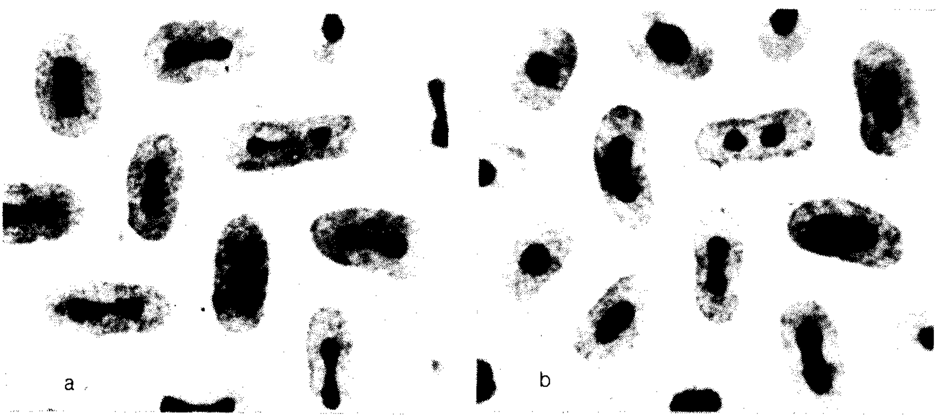

FIG. 1. Photographs of domain structures with initial symmetry *Pab*2 at the time of rupture (a) and after rupture (*b*) of domain walls. $\tilde{H}=100$ Oe, $\tau_1=0.8$ μsec, $\tau_2=4.0$ μsec (*a*) and $\tau_2=4.4$ μsec (*b*).

pertain to DS with *Pab*2 symmetry; similar experimental data were obtained for structures with *P*6 symmetry.

Analysis of the shape of the dynamic DS for $|\tilde{H}| > |\tilde{H}_{cr}|$ and different τ_2 shows that the transition from self-organization to chaotic motion occurs as a result of the division of dumbbell-shaped domains by rupture of the domain walls (DW). This is illustrated in the photographs in Fig. 1. These photographs were taken with $|\tilde{H}|=100$ Oe, $|\tilde{H}_{cr}|\approx 90$ Oe, $\tau_1=0.8$ μsec, and $\tau_2=4.0$ μsec (Fig. 1a), and 4.4 μsec (Fig. 1b); the period of the DS was ≈ 90 μm. Under ideal conditions (uniform magnetization field, no defects, etc.) rupture of the DW would occur simultaneously in all domains and the transition to chaos would be accompanied by doubling of the number of domains,[2)] i.e., doubling of the wave vector **k** of the two-dimensional lattice. This scenario of the transition to chaos is analogous to Feigenbaum's scenario,[4] but in our case period doubling occurs not in frequency space but rather in **k** space.

At all stages of the motion the shape of the DW is described well by Cassini ovals which are described by the equation[5]

$$(x^2+y^2)^2-2c^2(x^2-y^2)+(c^4-a^4)=0, \tag{1}$$

where the parameter c determines the distance between the foci $F_1=(+c,\ 0)$ and $F_2=(-c,\ 0)$, the product of the distances from which up to any point M on the curves is a constant; $(MF_1MF_2)^{1/2}=a$. For $a>c\sqrt{2}$ and $c<a<c\sqrt{2}$ the curves are, respectively, convex ovals and ovals with a neck ("dumbbells"); the condition $a=c$ determines the separatrix self-intersecting curve (lemniscate), for which $(x^2+y^2)^2=2c^2(x^2-y^2)$. For $a<c$ Eq. (1) describes two closed curves, which lie inside the separatrix.

Statistical analysis of a large number of experimental data yielded the time dependences of the major and minor axes of the domains, from which the parameters a and c were calculated for the approximating Cassinian ovals. The computational

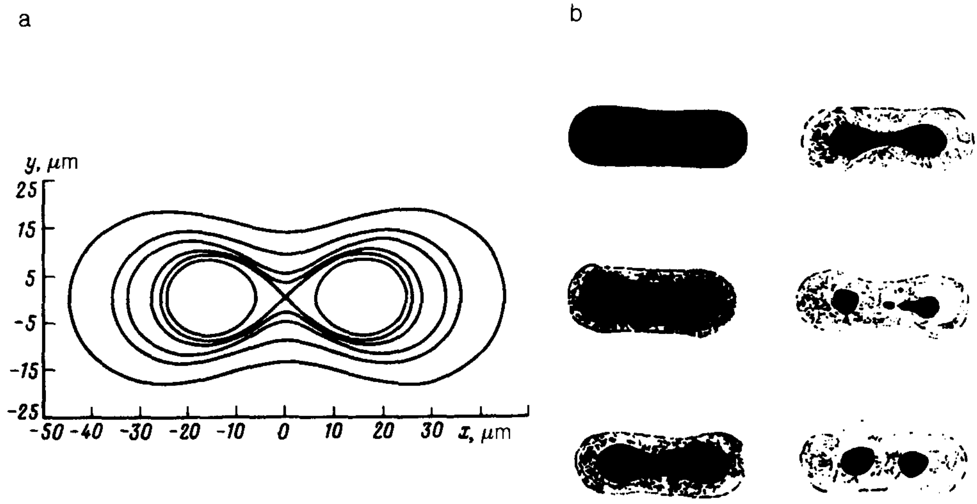

FIG. 2. Series of Cassinian ovals (a) approximating the shape of actually observed domains (b) at the times $\tau_2=0$, 2.0, 2.6, 3.2, 4.0, and 4.4 μsec with $\tilde{H}=100$ Oe. The values of (a,c) at the corresponding times are (in microns) (33.2, 30.37), (26.2, 24.7), (23.6, 23.1), (19.9, 19.7), (18.4, 18.4), (17.0, 18.0); the length of the major axis of an initial domain is ≈ 90 μm.

results shown in Fig. 2a demonstrate that the approximating curves agree well with the shape of the domain walls which were observed at different times and which are shown in Fig. 2b. This makes it possible to use in the theoretical description of the compression and division of domains curvilinear coordinates, in which the confocal Cassinian ovals play the role of a family of coordinate lines.[3)] It is easy to show that the second family of (orthogonal) curves is described by the single-parameter equation

$$(x^2-y^2)-2\zeta xy-c^2=0, \qquad (2)$$

where ζ is the parameter of the family of curves which assumes any real values. Curves orthogonal to the Cassinian ovals are equilateral hyperbolas which intersect at the points F_1 and F_2. The corresponding system of orthogonal curvilinear coordinates is constructed in Fig. 3 in the $(x/c, y/c)$ plane.

We note that the grid shown in Fig. 3 can be obtained by means of a conformal mapping of the type[6]

$$w=u+iv=\ln(z^2-c^2), \qquad (3)$$

where $z=x+y+iy$, and w is a complex potential. The lines $u=\text{const}$ are mapped into the (x,y) plane in the form of Cassinian ovals (equipotentials), and the lines $v=\text{const}$ are mapped in the form of orthogonal curves (streamlines). The parameter ζ from Eq. (2) is given by the relation $\zeta=\cot(v)$. The existence of the mapping (3), which makes it possible to describe by means of an analytic function the shape of the domains arising in dynamical self-organization processes, confirms the existence of a close relationship between synergetics and homotopic topology.[7]

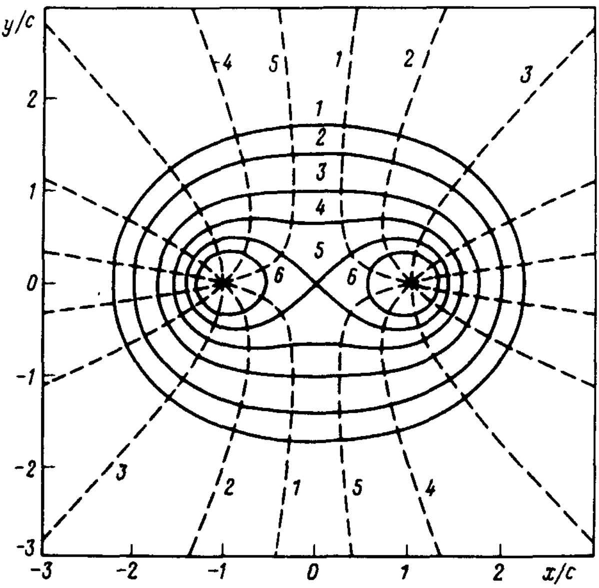

FIG. 3. System of orthogonal curvilinear coordinates with equipotentials in the form of Cassinian ovals with $c=1$ (solid lines) and streamlines (dashed lines) perpendicular to the equipotentials. For the equipotentials 1–6, respectively, $a=2$, 1.73, 1.41, 1.19, and 0.84; for the streamlines 1–5, respectively, $\zeta=-2$, -0.5, 0, $+0.5$, and $+2$.

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[1)]The value of τ_1 was usually chosen to be small ($\lesssim 1$ μsec), so that the form of the dynamical DS, recorded by the first illumination pulse, differed insignificantly from the shape of the static DS; see Ref. 3.

[2)]This pertains to structures with *P2ab* symmetry; for DS with *P6* symmetry the number of domains at the transition to chaos increases not by a factor of 2, but rather by a factor of 7/4, since circular bubbles do not undergo division.

[3)]The Cassinian ovals, which approximate the shape of a DW at different times, are not confocal (see Fig. 2), so that the distance between the foci must be taken into account in the dynamic equations as an additional generalized coordinate.

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