Many-body correlations in conformal field theories

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(Submitted 25 February 1993)
Pis'ma Zh. Eksp. Teor. Fiz. **57**, No. 10, 610–613 (25 May 1993)

The general structure of Green's functions in conformal field theories leads to conclusions regarding the nature of many-body correlations in such theories. The corresponding equations serve as an alternative to the widely used expressions of the bound-pair approximation.

The observation of significant fluctuations in the number of particles in small regions of the phase volume in hadron production processes, on the one hand, and the subsequent detailed structure of these fluctuations (see the review article[1]), on the other, indicate a possible manifestation of power-law scaling. One hypothesis[2] regarding the dynamic nature of this behavior of the fluctuations is that the fluctuation field has a conformal symmetry. In the present letter we show that by adopting this assumption one can derive a general relation between many-body and two-body correlation functions. When applied to the problem of fluctuations in the multiple production of particles, this formula leads to a relationship between the many-body and two-body cumulants which is an alternative to the familiar approximation of bound pairs. This new relationship gives a fairly good description of the experimental data, and it leads to new predictions regarding the behavior of higher-rank cumulants.

According to the general theory of conformal fields,[3–5] Green's functions must have a power-law behavior as a function of certain Lorentz-invariant combinations of the coordinates in theories which have conformal symmetry. For example, if we introduce the coordinates of particles j in momentum space, $\boldsymbol{\kappa}_j$, and if we express them in terms of the rapidity and transverse momenta, then (for example) two- and three-point correlation functions G in a conformal theory can be found within normalization constants N_q in the form[3–5]

$$G_2(1,2)=\langle\varphi(1)\varphi(2)\rangle=N_2(\kappa_{12}^2)^{-d}, \tag{1}$$

$$G_3(1,2,3)=\langle\varphi(1)\varphi(2)\varphi(3)\rangle=N_3(\kappa_{12}^2\kappa_{23}^2\kappa_{31}^2)^{-d/2}, \tag{2}$$

where $\varphi(j)$ is the fluctuation field at point j, d is its dimensionality, and $\kappa_{12}^2 = (\boldsymbol{\kappa}_1 - \boldsymbol{\kappa}_2)^2$. The higher-order correlation functions contain (in place of the normalization constants N_q) functions of the dimensionless ratios $\boldsymbol{\kappa}_{ij}^2$. Regardless of how the coordinates $\boldsymbol{\kappa}_j$ are defined, it is easy to find the following relation from (1) and (2):

$$G_3(1,2,3)=\frac{N_3}{N_2^{3/2}}G_2^{1/2}(1,2)G_2^{1/2}(2,3)G_2^{1/2}(3,1). \tag{3}$$

In general, for a q-particle correlation function we have

$$G_q(1,...,q)=\Pi_{i\neq k}^{N} G_2^{1/(q-1)}(i,k) f(\mu_p), \tag{4}$$

where $N=q(q-1)/2$. In the case $q>3$, the ratio of normalization constants is replaced by a function of the dimensionless ratios κ_{ij}^2 [they are designated by μ_p, where $p=q(q-3)/2$ is the number of such variables]. We wish to stress that the relationship between the many-body correlation function and the two-body correlation functions given by (4) is a consequence of simply the conformal symmetry of the fluctuation field. It does not require any additional assumptions. The general structure of the irreducible Green's functions in (4) leads to a corresponding relation for the normalized cumulant correlations, given by

$$k_q=C_q(1,...,q)/\rho(1)...\rho(q) \tag{5}$$

or, in integral form,

$$K_q(\delta y)\sim\int_{\Omega(\delta y)} dy_1...dy_q k_q, \tag{6}$$

where the integration is over the volumes specified by the rapidity interval δy. Here

$$C_q(1,...,q)=\rho_q(1,...,q)-\rho(1,...,q), \tag{7}$$

$$\rho_q(1,...,q)=\frac{1}{\sigma_{inel}}\frac{d\sigma}{dy_1 dy_2...dy_q}, \tag{8}$$

where ρ_q are inclusive q-body densities (the lower-order correlations have been subtracted).

The cumulant correlation functions are related to the factorial moments F_q, which are ordinarily used, by[1]

$$F_2=1+K_2, \tag{9}$$

$$F_3=1+3K_2+K_3, \text{ etc.} \tag{10}$$

The cumulant correlation functions k_q in (5) are given in a conformal field theory by irreducible q-body correlation functions G if the fluctuation field φ is understood as fluctuations of the number density of particles in individual events, as was proposed in Ref. 2, by analogy with the fluctuation theory of phase transitions, which is customarily used.[6] These functions must then be expressed in terms of two-body cumulants by analogy with (4). The integral cumulants in (6) can be calculated only under the assumption of translational invariance in rapidity space. This assumption has been used, for example, in a study of the bound-pair approximation in particle physics[7] and also earlier, in a description of correlations between galaxies.[8] In this case the functions f depend only weakly on their arguments and can be replaced by constants, and the integrals of the two-body functions over small volumes of the relative rapidity reduce to their average values. From (4)–(6) and (9) we then find

$$K_q(\delta y)\approx N_q[K_2(\delta y)]^{q/2}=N_q(F_2-1)^{q/2}. \tag{11}$$

This expression differs from the corresponding behavior which arises in the bound-pair approximation, which we mentioned above, i.e.,

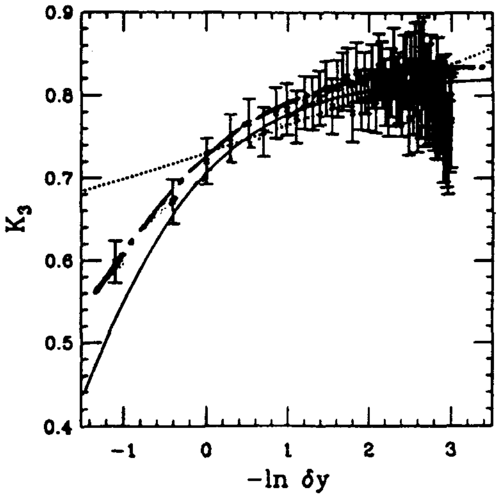

FIG. 1. The cumulant moment K_3. Crosses—Experimental data of the UA1 group; solid curve—result derived in the bound-pair approximation for $A_3 = 1.84$; dot-dashed curve—a consequence of conformal theories, (11), with $N_3 = 1.55$; points—power-law fit of the correlation functions.[9]

$$K_q(\delta y) = A_q[K_2(\delta y)]^{q-1}, \tag{12}$$

where A_q are independent of the rapidity intervals δy.

In each case, the exponent increases linearly with the rank of the momenta, but the slope of the rise is twice as large in the case of the bound-pair approximation. This distinction might become an important criterion for experimentally distinguishing one of these possibilities, if it were found possible to find limitations of some sort on the behavior of A_q and N_q as a function of the rank of the correlation function. At present, we have no such limitations, and we must work with the functional dependence of K_q on δy, expressed in terms of the $K_2(\delta y)$ behavior by means of Eqs. (11) and (12). An analysis of this sort was carried out for the bound-pair approximation [Eq. (12)] in Ref. 9.

Figures 1 and 2 show experimental data from the UA1 Collaboration on *pp* collisions at an energy $\sqrt{s} = 630$ GeV for the cumulant correlation functions K_3 and K_4 (crosses), their description in the bound-pair approximation (solid curve) (the experimental data and theoretical curves were taken from Ref. 9), and a fit of Eq. (11), derived in conformal theories, to the experimental data (dot-dashed curve). We see that the experimental data currently available are hardly adequate for giving preference to any of these approaches. Methods are being developed[10] for representing experimental data as an integral over a band. These methods are leading to a more accurate description of the moments and may be of assistance in choosing between two possibilities. The alternative to the bound-pair method suggested by the conformal symmetry shows that a good description of experimental data in this method is not by itself a sufficient basis for drawing the conclusion that the multiplicative formula which has been proposed is correct. The theoretical foundations of conformal symmetry are of course far stronger than the phenomenology of the bound-pair approxi-

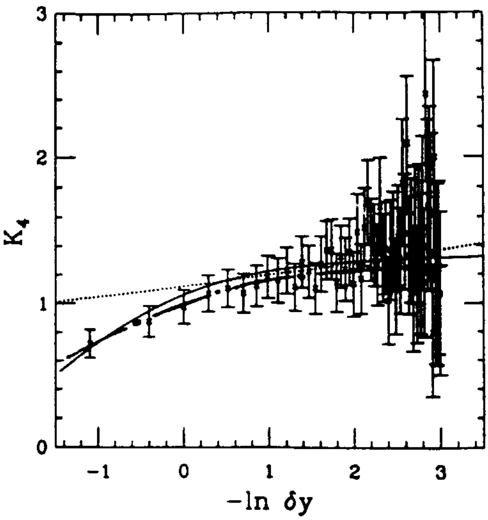

FIG. 2. The cumulant moment K_4. The notation corresponds to that in Fig. 1 (here $A_4 = 4.4$ and $N_4 = 2.7$).

mation. Incidentally, the existence of this symmetry in particle physics has not been rigorously proved; it requires a stronger foundation.

A common feature of Eqs. (11) and (12) is that the many-body correlations reduce to definite (although different) products of two-particle correlation functions. It is interesting in this connection to note that the application of quantum chromodynamics (QCD) to hard e^+e^--annihilation processes has led[11] to expressions of this sort for the exponents of factorial moments which do not reduce to multiplicative forms as in (11) and (12). At large values of the rank of the moment, q, in QCD there is also a linear increase in the exponent with q, with a slope of approximately 1/2 (specifically, the slope is $1-\gamma_0$, where $\gamma_0 = \sqrt{6\alpha/\pi} \approx 0.5$ is the anomalous dimensionality of QCD). At small values $q<5$ (which are experimentally accessible), on the other hand, the growth is faster. The requirement that the distributions be stable leads to a limitation on the growth of the exponents in expressions like (11) and (12). Specifically, the growth cannot be faster than quadratic.[12] The replacement of $q/2$ or $q-1$ in these expressions by $q(q-1)/2$ is also consistent with experimental data.

In summary, an alternative form of the many-body correlation function, (11), has been proposed. This form of the function stems from the conformal symmetry [see (4)] and differs from the widely used bound-pair approximation. At the moment, the experimental data do not support a choice between these possibilities. It does seem that QCD indicates a new dependence of a nonmultiplicative type for many-body correlations in hard processes of multiple production of particles. The nature of the genuinely many-body correlations requires further research.

This study was carried out with the financial support of the Russian Fund for Fundamental Research (Project 93-02-3815).

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Translated by D. Parsons