Spectrum of epithermal electrons accelerated by a collisionless shock wave

A. M. Bykov and Yu. A. Uvarov
A. F. Ioffe Physicotechnical Institute, Russian Academy of Sciences, 194021 St. Petersburg, Russia

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A model is proposed for the energy spectrum of epithermal electrons near the front of a strong, quasilongitudinal, supercritical, collisionless shock wave. The spectrum is calculated. The relative number of electrons injected from the thermal background into the nonthermal distribution depends exponentially on the Mach number of the shock wave.

1. Collisionless shock waves are a universal source of nonthermal charged particles in low-density plasmas.[1] These waves play a particularly important role in the problem of the shaping of the energy spectra of charged particles in astrophysical objects of various types.[2,3] If these shock waves are sufficiently strong (if they have an Alfvén Mach number M greater than a few units), the dissipation due to the anomalous resistance of electrons is inadequate, and the front structure is governed by kinetic instabilities of ions. Such shock waves are generally called "supercritical." Numerical simulation of the structure of collisionless shock waves by hybrid codes which treat the protons as particles and the electrons as a fluid has successfully described the basic features of quasilongitudinal supercritical shock waves (i.e., those for which the normal to the front and the local magnetic field make an angle less than $\pi/4$). The front structure and other features in the case of quasilongitudinal waves are quite different from those in the case of quasitransverse waves.[1,2] In the present letter we are concerned with quasilongitudinal shock waves. It has been shown elsewhere[4,5] that the front of a supercritical quasilongitudinal shock wave is an extremely large transition region filled with magnetic-field fluctuations which have amplitudes $\delta B/B \sim 1$ and characteristic frequencies below the ion gyrofrequency. These fluctuations are generated by instabilities of mutually penetrating, multiple-flux motions of ions. The front width Δ of a quasilongitudinal shock wave reaches values equal to a few tens of times the inertial length of an ion (which is c/ω_{pi}).

A simulation using hybrid codes has yielded a result of fundamental importance: A group of nonthermal reflected ions emerges during the relaxation of the fluctuations at the front of a quasilongitudinal supercritical shock wave.[4,5] These reflected ions have a gyroradius greater than the front width of the shock wave. They are subsequently accelerated very effectively by the converging plasma streams carrying MHD fluctuations, by the Fermi mechanism.[1,6] Electrons with gyroradii larger than the front width are also accelerated effectively by the first-order Fermi mechanism near a quasilongitudinal shock wave.[7] A nonrelativistic electron, however, must have an energy larger than that of a corresponding proton by a factor of (m_p/m_e) if it is to be injected into Fermi acceleration. The electron injection problem essentially reduces to the

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formation of a nonthermal electron energy distribution up to energies on the order of $(m_p/m_e)T_1$, where T_1 is the plasma temperature in the unperturbed region. Here and below, the subscripts 1 and 2 refer to the region of the plasma stream incident on the front and the region of the plasma stream directed away from the front.

The fluid description of the electrons in the hybrid codes makes it impossible in principle to obtain information on nonthermal electrons. On the other hand, the primary sources of electromagnetic radiation from astrophysical objects at frequencies from the rf range to the γ-ray range are specifically nonthermal electrons. In this letter we accordingly propose a model which makes it possible to calculate the energy spectrum of nonthermal electrons near a quasilongitudinal shock wave and to calculate the relative number of electrons which are injected from a thermal background into first-order Fermi acceleration by converging fluxes of magnetic inhomogeneities.

2. Let us calculate the electron distribution function near the front of a strong quasilongitudinal shock wave which is propagating through a plasma with $\beta = 4\pi P/B^2 \sim 1$ (P is the plasma pressure). If the Alfvén Mach number of the shock wave satisfies the condition $M < (\beta m_p/m_e)^{1/2}$, the velocities of the thermal electrons will be greater than the velocity of the shock front. The angular distribution of the electrons will be approximately isotropic in this case. We treat the front of the supercritical shock wave as a transition region of finite width Δ (between the incoming and outgoing plasma streams), in which there are large fluctuations in the magnetic field. The typical correlation length of the fluctuations, λ, is comparable to the ion gyroradii and much larger than the gyroradii of electrons with energies near the thermal region. It is important to note that these fluctuations are observed directly in quasilongitudinal shock waves and are seen clearly in numerical simulations.[5,8] Since the distribution of the fluctuations in the transition region seems to be nearly isotropic,[1,8] the electron motion at scales greater than λ can be treated as a spatial diffusion. The transport range of the magnetized electrons is in this case roughly equal to the correlation length of the strong fluctuations of the magnetic field, λ, and it is independent of the energy. The longitudinal diffusion coefficient for electrons with gyroradii smaller than λ is $\kappa_{zz} \approx v\lambda/3$, where v is the electron velocity, and the z axis is along the normal to the shock front.

The vortical fluctuation electric fields induced by the random fluxes of ions in the transition region are responsible for a stochastic acceleration of electrons. The acceleration of electrons by the potential electric field in the transition region can be ignored for a strong supercritical quasilongitudinal shock wave, since the potential jump at the front is less than $0.05 m_p u_1^2$ for $M \sim 5$, and it falls off with increasing M (see the review article[8]). A potential field of this magnitude would not let electrons acquire an energy on the order of $(m_p/m_e)T_1$.

Let us look at the problem in a reference frame in which the transition region of the front is at rest. We ignore slow effects associated with a cyclic reshaping of the front.[8] The isotropic part of the quasisteady electron momentum distribution $N(z,p)$, normalized to a phase-volume element, satisfies the following equation in the transition region:[9]

$$\kappa_{zz}\frac{\partial N}{\partial z^2}-u\frac{\partial N}{\partial z}+\frac{p}{3}\frac{\partial N}{\partial p}\frac{\partial u}{\partial z}+\frac{1}{p^2}\frac{\partial}{\partial p}p^4 D\frac{\partial N}{\partial p}=0. \tag{1}$$

Here $u(z)$ is the smooth profile of the hydrodynamic velocity (which is determined by the ions), averaged over the fluctuations. The diffusion coefficient in velocity space is $D(p)\approx\delta u^2/9\kappa_{zz}$,[9] where δu is the amplitude of the velocity fluctuations of the ion fluxes in the transition region. For nonrelativistic electrons, κ_{zz} is proportional to the momentum: $\kappa_{zz}\propto p$.

At the boundaries of the transition region, at the points $z=0$ and $z=\Delta$, the distribution function $N(z,p)$ and the flux in phase space join with the corresponding quantities in the incoming and outgoing flows. Outside the transition region (away from the shock front), at distances greater than λ but smaller than the length scale for the inelastic relaxation of an electron, the functions $N_{1,2}(z,p)$ satisfy diffusion equations which are found from (1) by omitting the last two terms. At $z\to-\infty$ in the incoming flow the function $N_1(z,p)$ converts into an unperturbed Maxwellian distribution function $n_{\rm Maxw}(p)$ with an electron density n_e and a temperature T_1. At $z\to\infty$, we assume that $N_2(z,p)$ is bounded.

To solve the problem of the injection of nonthermal electrons into first-order Fermi acceleration, it is sufficient (as mentioned above) to calculate the distribution function $N(z,p)$ at $z=0$ for electrons with energies on the order of $(m_p/m_e)T_1$. When the average velocity in the transition region has a profile $u(z)=u_1-(u_1-u_2)z/\Delta$, we can study the problem analytically. The system of equations and of boundary conditions written above reduces to an equation for the auxiliary function $g(\mu)$, in terms of which we express the quantity $N(0,p)$, in which we are interested. In dimensionless variables, these equations are

$$N(0,\xi)=\int_0^\infty d\mu g(\mu)\cos(\mu)\exp(\omega(\mu)\xi), \tag{2}$$

$$\int_0^\infty d\mu g(\mu)\cos(\mu)\exp(\omega(\mu)\xi)-\frac{\xi}{\Gamma}\int_0^\infty d\mu g(\mu)\mu\sin(\mu)\exp(\omega(\mu)\xi)$$
$$=n_{\rm Maxw}(\xi), \tag{3}$$

where the dimensionless momentum ξ is $\xi=p/(2m_eT_1)^{1/2}$. The function $\omega(\mu)$ is defined by

$$\omega(\mu)=\frac{3}{8\alpha\Gamma}\left[1-\left(1+\frac{64}{9}\alpha\mu^2\right)^{1/2}\right], \tag{4}$$

where we are using the parameters $\Gamma=u_1\Delta/v_{T_e}\lambda$ and $\alpha=\delta u^2/u_1^2$.

The asymptotic solution of Eq. (3) at $\xi\gg\Gamma\gg1$ is

$$N(0,\xi)\approx n_e\frac{\pi^{3/2}C\xi}{6\Gamma\psi(\alpha)}\exp\left[\frac{3\xi}{8\alpha\gamma}[1-\psi^2(\alpha)]\right], \tag{5}$$

where

$$\psi(\alpha)=\left(1+\frac{16\pi^2}{9}\alpha\right)^{1/4}. \tag{6}$$

The dimensionless constant C generally depends on the particular profile of the average velocity, $u(z)$, in the transition region. Estimates yield values $0.1 \leqslant C < 1$.

3. In the model proposed here it has thus been shown that fluctuations of the magnetic and vortical electric fields which are induced by instabilities of mutually penetrating fluxes of ions in the transition region at the front of a quasilongitudinal supercritical shock wave give rise to a nonthermal electron distribution as in (5). The distribution function of the epithermal electrons, (5), at the boundary between the incoming flow and the transition region of the shock front can be used to estimate the relative number (η_e) of electrons which are injected into acceleration by the converging fluxes of magnetic inhomogeneities:

$$\eta_e \approx \left.\frac{N(0,\xi)}{n_e}\right|_{\xi=(m_p/m_e)^{1/2}}. \tag{7}$$

Since the parameter Γ satisfies $\Gamma \approx M(m_e/m_p\beta)^{1/2}(\Delta/\lambda)$, the relative number of electrons which are injected, η_e, depends strongly on the Mach number of the shock wave according to (5) and (7). We should point out that the quantity (Δ/λ), i.e., the number of strong-fluctuation correlation lengths in the transition region at the front, also depends on the Mach number of the shock wave. Adopting some typical parameter values, $(\Delta/\lambda) \sim 30$ and $\alpha \sim 3$ for $M \sim 10$, we find the estimate $\eta_e \sim 10^{-5}$ from (5). Knowing η_e, we can calculate the absolute values of the fluxes of nonthermal electrons which are accelerated by the converging fluxes of magnetic inhomogeneities, taking account of the nonlinear effect of the nonthermal ions on the structure of the shock front.[7]

The strong dependence of the rate of electron injection into first-order Fermi acceleration on the Mach number of the shock wave distinguishes the injection of electrons from that of ions in a substantial way. There is no such strong dependence on the Mach number in the case of ion injection.[1,4] The model proposed in this letter predicts a substantial increase in the ratio of the flux of nonthermal electrons to that of nonthermal ions with increasing Mach number of the shock wave in the case of strong quasilongitudinal shock waves.

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