Possibility of a high-T_c superconductivity based on "Coulomb" mechanisms for Cooper pairing of carriers

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It is shown that Anderson's assertion that it is not possible to reach high superconducting transition temperatures T_c by means of nonphonon ("Coulomb") mechanisms for Cooper pairing is not correct in general. The reason is that local-field effects increase the charge of the quasiparticles. This increase compensates nearly completely for the decrease in the attraction between electrons due to the renormalization of the coupling constant λ by a factor of $(1+\lambda)$. As a result, it is possible to reach values $T_c > 100$ K.

1. Long before Bednorz and Müller[1] discovered high-T_c superconductivity in cuprate metal oxides, Cohen and Anderson[2] had asserted that it would be impossible to achieve high superconducting transition temperatures $T_c > 10$ K by any nonphonon mechanism for Cooper pairing involving the exchange of virtual Bose excitations of a Coulomb nature (excitons, plasmons, etc.). This assertion, which was recently reiterated by Anderson[3] even more categorically, is based on two extremely general positions: first, the Kramers–Kronig relations for the reciprocal of the dielectric constant of the system,[4] $\epsilon^{-1}(\mathbf{q}, \omega)$, according to which the dimensionless constant of the retarded electron–electron attraction, λ, is always smaller than the Coulomb repulsion constant μ_C if the static dielectric constant is positive, $\epsilon(\mathbf{q}, 0) > 0$; second, the theory of superconductivity with strong electron–phonon coupling,[5] according to which the attraction is renormalized (weakened) by a factor of $(1 + \lambda)$ near the Fermi surface. This renormalization leads in practice to low values $T_c < 10$ K.

However, local-field effects stemming from many-body Coulomb correlations in a charged Fermi liquid[4] were ignored in Refs. 2 and 3. These correlations renormalize the charge of the quasiparticle. Effects of this sort were analyzed in Refs. 6 and 7, where it was asserted that strong Coulomb correlations in a homogeneous Fermi liquid might lead to negative values of the static dielectric constant over a wide momentum range $q \neq 0$. As a result, the limitation on the value of λ would be relaxed, and λ could be greater than μ_C. However, the renormalization of the constants λ and μ_C due to the same local-field effects were ignored in Refs. 6 and 7. This renormalization was first taken into consideration in Ref. 8 for a "plasmon" superconductivity mechanism[9] in semimetals or degenerate semiconductors with overlapping broad and narrow bands ("light" and "heavy" valleys), by virtue of an exchange of quanta of low-frequency collective excitations of the charge density, i.e., acoustic plasmons.[10] As a result, it was possible to achieve values $T_c \geqslant 100$ K in the jellium model[4] through a numerical solution of the Éliashberg equation[5] for the gap in the quasiparticle spectrum.

In the present letter we show that Anderson's assertion,[2,3] which effectively blocks an entire direction for searching for possible mechanisms for high-T_c superconductivity, is not correct in general. The reason is that the nearly complete mutual cancellation of the local-field and strong-coupling effects for "Coulomb" Cooper-pairing mechanisms leads to a substantial intensification of the electron–electron attraction. Even under the conditions $\epsilon(\mathbf{q}, 0) > 0$ and $\lambda < \mu_C$, this effect makes it possible to reach fairly high T_c's, in contradiction of the conclusions reached in Refs. 2 and 3 (on the one hand) and 6 and 7 (on the other).

2. In the superconducting state of a charged Fermi liquid, the normal (Σ_1) and anomalous (Σ_2) eigenenergy parts, which are determined by the exchange of virtual bosons of a Coulomb nature (plasmons and excitons), take the following form when local-field effects are taken into account in the limit $T \to T_c$ (Refs. 5 and 6):

$$\Sigma_1(\mathbf{p},\omega) = T \sum_{\omega'} \int \frac{d^3p'}{(2\pi)^3} G(\mathbf{p}',\omega') \widetilde{V}_C(\mathbf{p}'-\mathbf{p},\ \omega'-\omega) \Gamma_C(\mathbf{p}',\omega';\ \mathbf{p}'-p,\omega'-\omega), \tag{1}$$

$$\Sigma_2(\mathbf{p},\omega) = T \sum_{\omega'} \int \frac{d^3p'}{(2\pi)^3} F(p',\omega') \widetilde{V}_C(\mathbf{p}'-p,\ \omega'-\omega) \Gamma_C^2(p',\omega';\ p'-p,\ \omega'-\omega). \tag{2}$$

Here G and F are the normal and anomalous Green's functions; ω and ω' are discrete Matsubara frequencies; $\widetilde{V}_C$ is a matrix element of the screened Coulomb interaction (retardation is taken into account), which is given in the approximation of a self-consistent field by[4]

$$\widetilde{V}_C(\mathbf{q},\omega) = V_C(\mathbf{q})/\epsilon(\mathbf{q},\omega) \equiv D_B(\mathbf{q},\omega) + v_c(\mathbf{q}), \tag{3}$$

where $D_B(\mathbf{q}, \omega) = V_C(\mathbf{q})[\epsilon^{-1}(\mathbf{q}, \omega) - 1]$ is the Green's function of collective Bose excitations; V_C is the matrix element of the unscreened Coulomb repulsion; and Γ_C is the Coulomb vertex. In the limit $q \to 0$, $\omega \to 0$, this vertex satisfies the Ward-Pitaevskiĭ identities[11,12] for a charged Fermi liquid with a uniform neutralizing background of the opposite sign:[13]

$$\Gamma_C^\omega(\mathbf{p}',\omega') \equiv 1 - \frac{\partial \Sigma_1(\mathbf{p}',\omega')}{\partial \omega'}, \quad \Gamma_C^q(\mathbf{p}',\omega') \equiv 1 - \frac{\partial \Sigma_1(\mathbf{p}',\omega')}{\partial \mu}, \tag{4}$$

where μ is the chemical potential of the Fermi liquid, and Γ_C^ω and Γ_C^q correspond to the limits $q/\omega \to 0$ as $\omega \to 0$ and $\omega/q \to 0$ as $q \to 0$.

If the characteristic energies of collective boson excitations of the charge density satisfy $\Omega_B \ll E_F$ (E_F is the Fermi energy), then it can be shown by analogy with the electron–phonon interaction[5] that near the Fermi surface ($\omega \to 0$ and $p \simeq k_F$, where k_F is the Fermi momentum) the part of Σ_1 which is odd in ω is given in the normal state by

$$f(\omega) \equiv \frac{1}{2}[\Sigma_1(k_F,\omega) - \Sigma_1(k_F,-\omega)] \simeq -\omega\lambda, \quad \lambda = 2\int_0^\infty \frac{d\omega'}{\omega'} S(\omega'). \tag{5}$$

Here λ is the dimensionless constant of the electron–plasmon or electron–exciton interaction, $S(\omega)$ is the spectral function of the bosons (plasmons or excitons), given by

$$S(\omega)=-\frac{1}{\pi}N(0)<\langle V_C(\mathbf{q})\mathrm{Im}\ \epsilon^{-1}(\mathbf{q},\omega)\Gamma_C(\mathbf{q},\omega)\rangle, \tag{6}$$

$N(0)$ is the density of states at the Fermi surface, and the angle brackets $\langle\ldots\rangle$ mean an average over q in the region $0\leqslant q\leqslant 2k_F$.

It follows from (4) and (5) that we have $\Gamma_C^\omega=1+\lambda$ for $p'=k_F$ and $\omega'\to 0$. Since the renormalization of the effective mass due to the Coulomb interaction, $(1+\partial\Sigma_1/\partial\zeta)$, where $\zeta=k^2/2m^*-\mu$, is cancelled nearly completely by the nonadiabatic renormalization of the quasiparticle spectrum, by a factor $Z_C(\omega)=1-f(\omega)/\omega$ (Ref. 14), it is quite accurate to set $\partial\Sigma_1/\partial_\mu=\partial\Sigma_1/\partial\omega=-\lambda$. At $p'=k_F$ and $\omega'=0$ we thus have

$$\Gamma_C^q\approx\Gamma_C^\omega\equiv Z_C(0)=1+\lambda. \tag{7}$$

On the other hand, a calculation of the average value $\bar{\Gamma}_C$ in the region $\omega\geqslant\Omega_B$, in which radiative (Coulomb) corrections of first order are taken into account within terms $\lambda\Omega_B/E_F\ll 1$, leads to the estimate[8]

$$\bar{\Gamma}_C\approx 1+\mu_C+O(\lambda\Omega_B/E_F). \tag{8}$$

Here μ_C is the constant of the screened Coulomb repulsion in the limit $\omega\to\infty$, in which we have $\epsilon(\mathbf{q},\omega)\to 1$ and $\Gamma_C\to 1$ (the second-order corrections are small under the condition $\mu_C<1$), averaged over the energy transfer ($\omega\leqslant E_F$) and over the momentum transfer ($q\leqslant 2k_F$). By virtue of the Kramers–Kronig relation for $\epsilon^{-1}(\mathbf{q},\omega)$ (Ref. 4), there is a relationship[2,7] between the constants λ and μ_C:

$$\mu_C-\lambda=N(0)\langle V_C(\mathbf{q})\epsilon^{-1}(\mathbf{q},0)\Gamma_C(\mathbf{q},0)\rangle. \tag{9}$$

According to this relation we have $\lambda<\mu_C$ if $\epsilon(\mathbf{q},0)>0$ or $\lambda>\mu_C$ if $\epsilon(\mathbf{q},0)<0$ over the wide range $q\neq 0$. In ionic crystals, the static dielectric constant of the lattice is large, $\epsilon_0\gg 1$, and the screening of charges by degenerate free carriers is strong in the limit $\omega\to 0$. We can thus assume $\epsilon^{-1}(q,0)\ll 1$ on the right side of (9) and set $\lambda\simeq\mu_C$. In this case the estimates of Γ_C in (4) and (8) are essentially the same.

The effective electron–electron attraction constant, which is responsible for the Coulomb pairing, can thus be approximated by [cf. (5) and (6)]

$$\tilde{\lambda}=-\frac{2}{\pi}N(0)\int\frac{d\omega}{\omega}\langle V_C(\mathbf{q})\mathrm{Im}\ \epsilon^{-1}(\mathbf{q},\omega)\Gamma_C^2(\mathbf{q},\omega)\rangle\approx\lambda(1+\mu_C), \tag{10}$$

where we are taking account of the "extra" Coulomb vertex Γ_C in Eq. (2) for the superconducting order parameter Σ_2.

On the other hand, the Coulomb repulsion constant in Eq. (2), renormalized by local-field effects, takes the following form when we incorporate (8):

$$\tilde{\mu}_C=N(0)\int_0^{E_F}\frac{d\omega}{E_F}\langle V_C(\mathbf{q})\Gamma_C^2(\mathbf{q},\omega)\rangle\approx\mu_C(1+\mu_C)^2. \tag{11}$$

3. We can show that the upper limit on λ, which follows from relation (9) in the case $\epsilon(q,0)>0$, does not rule out the attainment of high values $T_C\geqslant 100$ K, because of the renormalization of the interaction constants in (10) and (11) due to the local-field corrections. To estimate the superconducting transition temperature, we use an exponential formula of the McMillan type[15] in the intermediate-coupling approximation ($\lambda \leqslant 1$) and with a nonadiabatic renormalization (reduction) of the coupling constant by a factor $Z_C(0)=1+\lambda$ (cf. Ref. 16):

$$T_c \approx \Omega_B \exp\left\{-\frac{1+\lambda}{\lambda-\tilde{\mu}_C^*(1+\lambda)}\right\}. \tag{12}$$

Here μ_C^* is the Bogolyubov–Tolmachev–Morel–Anderson Coulomb pseudopotential,[17,18] given by

$$\tilde{\mu}_C^* = \tilde{\mu}_C[1+\tilde{\mu}_C \ln(E_F/\Omega_B)]^{-1}. \tag{13}$$

For the maximum value of T_C over Ω_B we find the following expression from (12) and (13), using (10) and (11):

$$T_c^{\max} \approx E_F \exp\left\{-\frac{4(1+\lambda)}{\lambda(1+\mu_C)}+\frac{1}{\mu_C(1+\mu_C)^2}\right\}. \tag{14}$$

The expression for $T_c^{\max}$without local-field corrections, on the other hand, is[7,16]

$$T_c^{\max} \approx E_F \exp\left\{-\frac{4(1+\lambda)}{\lambda}+\frac{1}{\mu_C}\right\}. \tag{15}$$

According to (14), under the condition $\lambda \simeq \mu_C$ there is a nearly complete cancellation of the renormalizations in the effective attraction constant near the Fermi surface. This cancellation stems from local-field and strong-coupling effects.

As a result, for $\lambda \simeq \mu_C \simeq 1$ and $E_F \simeq 1$ eV we find from (14) the estimate $T_c^{\max} \simeq 270$ K, while from (15) we find the far lower value $T_c^{\max} \simeq 10$ K. The estimate $T_c^{\max} \simeq 300$ K in Ref. 16 was derived from expression (15) with $\lambda \simeq 1, \mu_C \simeq 1/2$, and $E_F \simeq 10$ eV. The estimates $T_c^{\max} \simeq 1-10$ K with $E_F \simeq 1$–10 eV given in Refs. 3, 6, and 7 actually correspond to $\lambda \simeq \mu_C \simeq 1/2$. This case is characteristic of a screened Coulomb interaction in a metal with a high electron density.[6,16] However, the constant of the unscreened Coulomb repulsion, which may be far larger ($\mu_C \gg 1$) figures in Eq. (9).

In summary, the estimate of $T_c^{\max}$ in (14) indicates that it is possible in principle to achieve fairly high values $T_c > 100$ K, by virtue of an increase in the effective charge of the quasiparticles, $\bar{e} \simeq e(1+\lambda)^{1/2}$, and also an increase in the coupling constant $\Lambda = \lambda - \mu_C^*$ by virtue of local-field effects, even under the conditions $\epsilon(\mathbf{q}, 0) > 0$ and $\lambda < \mu_C$. These conclusions are in contradiction of Anderson's assertion[2,3] and in contradiction of the concept of negative values of the static dielectric constant.[6,7]

In the case of a plasmon mechanism for superconductivity,[8,9,11] an increase in T_c may be promoted by such factors as a hybridization of acoustic plasmons with optical phonons in ionic crystals,[19] the multivalued nature of the band spectrum (a multiply connected Fermi surface) of the current carriers,[20,21] and the quasi-2D electron spectrum and the "packet" structure of layered crystals such as the cuprate metal oxides BiSrCaCuO and TaBaCaCuO.[22]

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