Possibility of observing a negative electron temperature in semiconductor structures

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(Submitted 26 April 1993)
Pis'ma Zh. Eksp. Teor. Fiz. **57**, No. 10, 655–658 (25 May 1993)

The relaxation of nonequilibrium charge carriers in one miniband in a superlattice in a magnetic field is analyzed. The nonequilibrium carriers can be described by means of the electron temperature. A condition under which this temperature is negative is found.

The concept of a negative temperature has been used for a long time[1] to describe nonequilibrium systems whose energy spectrum is bounded and discrete with equidistant levels. The only system with a bounded but continuous spectrum, for which a distribution with a negative temperature has been discussed, is a system of spins with a dipole–dipole interaction.[2,3] In this letter we look at the possibility of a distribution with a negative electron temperature in a system with a continuous spectrum: a semiconducting superlattice in a magnetic field. A negative electron temperature can be introduced if the electron energy spectrum is bounded,

$$\epsilon_{\min} < \epsilon(k) < \epsilon_{\max}, \tag{1}$$

and the temperature relaxation time in the electron subsystem, τ_{ee}, is much shorter than the time scale of the scattering of electrons by phonons, τ_{ep}:

$$\tau_{ee} \ll \tau_{ep}. \tag{2}$$

We show below that electrons in semiconducting superlattices in a quantizing magnetic field can satisfy conditions (1) and (2).

Electron spectrum in a superlattice. The electron spectrum in a superlattice is distinguished by narrow minibands which correspond to a motion of electrons across the layers, which are spaced at a period d. A magnetic field parallel to the axis of the superlattice quantizes the motion in the plane of the layers, and the electron spectrum becomes

$$\epsilon = \hbar\omega_c(n+1/2) + (\Delta/2)(1+\cos k_z d), \tag{3}$$

where Δ is the width of a miniband; $n=0, 1, 2, \ldots$ are integers; and $\omega_c = eH/(m^*c)$ (we are ignoring spin splitting). We write the wave functions of the electrons in a magnetic field $\mathbf{A}=(0,Hx,0)$ as follows:

$$\Psi = (1/L)\exp(ik_y y + ik_z z)\varphi(z)\chi(x,k_y), \tag{4}$$

$$\chi(x,k_y) = \frac{1}{(\pi l_m)^{1/4}} \exp\left[-\frac{(x-x_0)^2}{2l_m^2}\right] H_0\left(\frac{x-x_0}{l_m}\right),$$

where H_0 are the Hermite polynomials, $x_0 = -k_y l_m^2$, $\varphi(z) = \varphi(z + d)$, l_m is the magnetic length, and L is the size of the system.

If the magnetic field is sufficiently strong ($\hbar\omega_c > \Delta$), gaps $E_g = \hbar\omega_c - \Delta$ appear between the minibands (characterized by various values of the index n) in the spectrum,[4] and the properties of the superlattice change substantially. In this letter we are interested in the relaxation of nonequilibrium electrons within one miniband. We assume that the carrier recombination time satisfies $\tau_r \gg \tau_{ee}$, and that electron scattering processes do not lead to changes in the index of a miniband. This condition can hold if the maximum energy of the acoustic phonons, with which the electrons interact in the magnetic field, satisfies $\hbar s/l_m \ll E_g$ (Ref. 5) and if the probability for a transition between minibands as a result of many-electron collisions is small. The spectrum of electrons in the superlattice thus satisfies condition (1).

Calculation of the electron temperature. Since the capacity of each band depends on the magnetic field H, the extent to which the magnetic minibands in the superlattice are filled can be varied by varying the field.[6] Under the condition

$$H=H_s=\pi\hbar cNd/(ek), \quad k=1, 2, 3,...,$$

an integer number of minibands is filled (this is an insulating state). If, at $H=H_s$, we inject (by optical pumping, for example) N_e electrons with a total energy E into the top magnetic miniband (the conduction band), the electrons will relax to a distribution with a temperature T_e. We can find T_e by solving the system of equations

$$N_e= \int_0^{\Delta} g(\epsilon) f(\epsilon,\mu,T_e) d\epsilon,$$
$$E= \int_0^{\Delta} \epsilon g(\epsilon) f(\epsilon,\mu,T_e) d\epsilon, \tag{5}$$

where

$$g(\epsilon)=\frac{1}{2\pi^2 l_m^2 d} \sum_n \left\{ \frac{\Delta^2}{4} - \left[\epsilon - \hbar\omega_c \left(n+\frac{1}{2} \right) - \frac{\Delta}{2} \right]^2 \right\}^{-1/2}$$

is the electron density of states in the magnetic field, and $f(\epsilon,\mu,T_e)$ is a Fermi–Dirac distribution with an electron temperature T_e and a chemical potential μ. We find that if the energy (per electron) is greater than $\Delta/2$, then the electron distribution is characterized by a negative T_e (the electrons are concentrated at the top of the magnetic band), while if the energy is less than half the band width, then T_e is greater than zero (Fig. 1). For the case $T_e<0$ we will reckon the value of μ and also the Fermi energy ϵ_F from the top of the magnetic miniband.

Electron–electron interaction. We know that the time τ_{ee} in a magnetic field is determined by the interaction of electrons in the presence of a third body.[7] In this letter we incorporate the interaction of electrons in a field of point defects with a potential $U(r)=U_0\Sigma\delta(r-r_i)$, where r_i is the coordinate of an impurity. We can estimate τ_{ee} from

$$\tau_{ee}^{-1} \simeq \alpha^{3/2} \tau_H^{-1}, \tag{6}$$

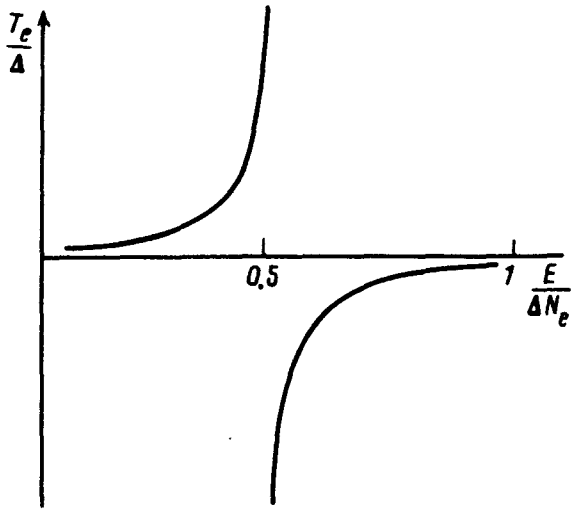

FIG. 1. The electron temperature T_e versus the average energy of the injected electrons, E/N_e (reckoned from the bottom of the miniband).

where τ_H is the time scale of the relaxation of longitudinal momentum during scattering by impurities, and α is a gas parameter, which characterizes the ratio of the average potential energy of an electron to the kinetic energy:

$$\alpha = N_e^{1/3} e^2/(\kappa\Delta),$$
$$\tau_H^{-1} = U_0^2 N g(z)/(2\pi^2 l_m^2 d\Delta\hbar), \quad g(z) = (z-z^2)^{-1/2}, \quad z = \epsilon_F/\Delta. \tag{7}$$

Here N_i is the concentration of the point defects, and κ is the dielectric constant of the medium. Expression (6) is valid if $\alpha < 1$.[7]

Electron–phonon interaction. In the approximation of uniform elastic properties of the superlattice, under the conditions

$$1 \gg \frac{\epsilon_F}{\Delta} \gg \frac{\hbar s}{l_m\Delta} > \frac{\hbar s}{d\Delta} \gg \frac{|T_e|}{\Delta}, \tag{8}$$

which are easy to satisfy in practice (ϵ_F is reckoned from the top of the miniband), the following expression has been derived for the reciprocal of the time scale for the acoustic-phonon relaxation of an electron:

$$1/\tau_{ep} = (1/\tau_0)(\epsilon_F/\Delta)^{1/2}, \tag{9}$$

where $\tau_0^{-1} = \pi^3 C_0^2 (l_m/d)/(4\rho s l_m^4 \Delta)$ is the nominal time scale of the scattering by acoustic phonons, C_0 is the strain-energy constant, s is the sound velocity, and ρ is the density of the crystal.

Stimulated processes were ignored in the calculation of τ_{ep}. This simplification is legitimate if the temperature of the crystal lattice, T_L, is close to zero and if the phonon mean free path l_p is greater than the dimensions of the sample, L:

$$l_p = s\tau_q > L, \tag{10}$$

where

$$\tau_q^{-1}=\frac{C_0^2 q^2 \hbar}{8\pi^2 \rho \Delta^2 l_m^2 d^2 q}\exp\left[-\frac{(q_\perp l_m)^2}{2}\right]g(z). \tag{11}$$

As $q_z \to 0$ and $\tau_q \to 0$, the maximum value of τ_q is reached at the maximum values $q_z=(4/d)(\epsilon_F/\Delta)^{1/2}$ and $q_\perp \simeq \pi/l_m \gg q_z$. Condition (10) holds for typical superlattice properties, as we will see by going through some numerical estimates.

For the typical superlattice properties $d\simeq 3\times 10^{-6}$ cm and $\Delta\simeq 10$ meV, in a magnetic field $H\simeq 8$ T we would have $l_m\simeq 10^{-6}$ cm. Estimating τ_{ee} from (6) with $N_i=10^{16}$ cm $^{-3}$ and $N_e=5\times 10^{16}$ cm^{-3}, we find $\tau_{ee}\simeq 10^{-12}$ s. The value of τ_{ep} for $C_0\simeq 10$ eV, $\rho=5\times 31$ g/cm^3, and $s\simeq 5\times 10^5$ cm/s is 10^{-9} s. We also have $\tau_r \simeq 10^{-7}$ s. The typical size of the system found from (10) is $L\simeq 2\times 10^{-1}$ cm.

It can be seen from these estimates that inequality (2) holds. We can thus describe the electrons by a Fermi distribution function with an effective temperature T_e.

Lifetime of the quasiequilibrium distribution. The relaxation time of a quasiequilibrium distribution with a negative electron temperature is estimated from Refs. 5 and 8

$$\tau_l=(E_1-E_2)/Q,$$

where E_1 and E_2 are the energies of the injected electrons at, respectively, the temperature T_e and an electron temperature equal to the temperature of the crystal lattice, T_L, and Q is the rate at which the nonequililbrium electrons lose energy.

Under the conditions $T_e\simeq -0$ and $T_L\simeq 0$, the expression for τ_l is

$$\tau_l=\tau_0 l_m \Delta g(z)/(\hbar s). \tag{12}$$

An estimate of the lifetime of a quasiequilibrium state with T_e close to -0, for a superlattice with the properties used above yields $\tau_l\simeq 5\times 10^{-7}$ s.

We note in conclusion that a distribution with a negative electron temperature might have several interesting properties, such as a negative absolute conductivity and the ability to amplify sound and electromagnetic waves. We wish to thank M. I. Kaganov and V. A. Volkov for useful discussions.

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Translated by D. Parsons