"Hot"-electron laser using a band–band transition

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(Submitted 12 April 1993)
Pis'ma Zh. Eksp. Teor. Fiz. **57**, No. 11, 676–679 (10 June 1993)

The interaction between an electron beam injected into a superlattice and an electromagnetic field is analyzed. Bragg reflection of electrons from the superlattice is taken into account. The IR gain for the electromagnetic field is derived. The possibility of experimentally observing the effect is discussed for realistic values of the parameters of the superlattice and of the injected electron beam.

Kazarinov and Suris[1] were the first to discuss the idea of a laser operating on the basis of "cold" ballistic electrons with $E \lesssim 0.1$ eV, in which electrons with an energy below the height of the potential barrier of the superlattice, U_0, would undergo transitions between quasibands. The possibility of generating "hot" ballistic electrons, with $E \simeq 0.1$–0.3 eV, injected into GaAs, was first demonstrated by Heiblum *et al.*[2] and Levy *et al.*[3] The mean free path of these electrons could reach $(2–3) \times 10^{-5}$ cm.

Malov and Zaretsky[4] have suggested developing a laser which uses hot ballistic electrons in a superlattice under the assumption that the latter is short; equivalently, one would be dealing with the motion of electrons within a single band. The single-band model is valid if the reflection coefficient of the superlattice, R_n, satisfies

$$R_n = \frac{U_0^2 n}{4(E-U_0)E} \sin^2 \frac{d}{\hbar} \sqrt{2m(E-U_0)} \ll 1, \qquad (1)$$

where d is the period of the superlattice, n is the number of periods in the superlattice, and E is the energy of an electron.

Superlattices with a number of periods, on the order of ten, and with an electron energy $E \gg U_0$ were discussed in Ref. 4; condition (1) was therefore satisfied. For $E = 2U_0$ and $n \approx 100$, condition (1) is not satisfied, and there may be a Bragg reflection of electrons from the superlattice. In the present letter we analyze a hot-ballistic-electron laser under the condition that there is a Bragg reflection of electrons from the superlattice or, equivalently, under the condition that the energy of a hot electron is close to the bottom of one of the quasibands of the superlattice. In this case, regardless of the value of U_0, the interaction of the electron with the superlattice is not weak, as was assumed in Ref. 4. If the photon energy is greater than the width of the quasigap, "vertical" transitions can occur between the edges of neighboring quasibands, corresponding to a stimulated emission. If the lower quasiband is not filled, there would be essentially no absorption.

Let us consider the motion of an electron in a superlattice with a potential

$$U(x) = U_0 \cos qx, \qquad (2)$$

where $q=2\pi/d$, in the field of an electromagnetic wave which is polarized along the axis of the superlattice,

$$\mathscr{E}=\mathscr{E}_0\cos\omega t.$$

Here $\mathscr{E}_0$ and ω are respectively the amplitude and frequency of the electromagnetic wave.

We consider the problem in the case in which the momentum of an electron is close to the boundary of the Brillouin zone, $q/2$, i.e., the case of Bragg reflection. We write the electron wave function in the superlattice field in the two-component approximation in accordance with Ref. 5, and in contrast with Ref. 1, where the strong-coupling approximation was used, and where the basis functions were Wannier functions

$$\Psi=\left[c_1\exp\left(i\frac{p}{\hbar}x\right)+c_2\exp\left(i\frac{p-q}{\hbar}x\right)\exp\right]\left(-i\frac{E}{\hbar}t\right), \tag{3}$$

where

$$c_1=\left(1+\frac{\mu_p}{(1+\mu_p^2)^{1/2}}\right)/2L^{1/2},\quad c_2=\left(1-\frac{\mu_p}{(1+\mu_p^2)^{1/2}}\right)/2L^{1/2},$$

$$\mu_p=\hbar^2(2pq-q^2)/2m^*U_0,$$

L is a normalization length, and m^* is the effective mass of an electron.

The quantity E in (3) is the energy of an electron near the bottom of the upper quasiband:

$$E=\hbar^2\frac{p^2+(p-q)^2}{4m^*}+\frac{U_0(1+\mu_p^2)^{1/2}}{2}. \tag{4}$$

The wave function of an electron near the upper edge of the lower quasiband is similar in form (here and below, the prime means that the quantity belongs to the lower quasiband):

$$\Psi'=\left[c_1'\exp\left(i\frac{p'}{\hbar}x\right)+c_2'\exp\left(i\frac{p'-q}{\hbar}x\right)\right]\exp\left(-i\frac{E'}{\hbar}t\right), \tag{5}$$

where

$$c_1'=\left(1-\frac{\mu_p'}{(1+\mu_{p'}^2)^{1/2}}\right)/2L^{1/2},\quad c_2'=-\left(1+\frac{\mu_p'}{(1+\mu_{p'}^2)^{1/2}}\right)/2L^{1/2},$$

$$\mu_p'=\hbar^2(2p'q-q^2)/2m^*U_0, \tag{6}$$

$$E'=\frac{\hbar^2(p'2+(p'-q)^2}{4m^*}-\frac{U_0(1+\mu_p^2)^{1/2}}{2}.$$

For the transition of an electron between quasibands, under the condition $p'=p=q/2$, the energy gap is U_0. The condition under which the two-component approximation [(3) and (5)] is applicable, is that two parameters be small:

$2m^*U_0/\hbar^2q^2 \ll 1$ and $\mu_p \ll 1$. The matrix element of the quasiband–quasiband transition describing the stimulated emission of a photon by an electron is

$$M = \frac{ie\hbar q\mathscr{E}_0}{4m^*\omega L(1+\mu_p^2)^{1/2}} \frac{2\sin(p'-p)L/2}{p'-p}. \tag{7}$$

If the lattice is long, i.e., if $L \gg 1/\Delta p$, where Δp is the momentum spread in the initial beam, we can write the following equation in this approximation:

$$\left(\frac{2\sin(p'-p)L/2}{p'-p}\right)^2 = 2\pi L\delta(p'-p).$$

These are "vertical" transitions. It is now a simple matter to derive an expression for the transition frequency:

$$dW = \frac{\pi}{8} \frac{e^2\hbar\mathscr{E}_0^2q^2}{(m^*)^2\omega^2(1+\mu_p^2)^2} \delta(p'-p)\delta(\hbar\omega - E - E')dp'. \tag{8}$$

Integrating (8) over dp', we find

$$W = \frac{\pi}{8} \frac{e^2\hbar\mathscr{E}_0^2q^2}{(m^*)^2\omega^2(1+\mu_p^2)^2} \delta[\hbar\omega - U_0(1+\mu_p^2)^{1/2}]. \tag{9}$$

It follows from expression (9) that the emission frequency in the case $p=q/2$ is given by

$$\hbar\omega = U_0(1+\mu_p^2)^{1/2}. \tag{10}$$

Dividing (9) by the photon flux density $c\mathscr{E}_0^2/8\pi\hbar\omega\sqrt{\epsilon_1}$, and taking an average of the resulting expression over the initial electron distribution $f(p)$, we find the cross section for stimulated emission:

$$\sigma = \pi^2 \frac{e^2}{c} f(p) L \frac{U_0}{\hbar\omega} \sqrt{\epsilon_1}, \tag{11}$$

where L is the length of the superlattice, and ϵ_1 is the dielectric constant.

From (11) we easily find the gain:

$$G = \pi^2 N \frac{e^2}{m^*c^2} Lbpf(p) \frac{U_0}{\hbar\omega} \frac{c}{v} \sqrt{\epsilon_1}, \tag{12}$$

where b is the transverse dimension in the radiation propagation direction ($b \gg L$), N is the density of electrons, and v is their velocity. Expression (12) for the gain differs substantially from that in Ref. 4 in that it does not contain the small parameter $(U_0/E)^2$.

The probability for spontaneous emission per electron in the course of interband transitions is

$$W = \frac{e^2}{3\hbar c}\left(\frac{\hbar q^2}{m^*c}\right)^2 \omega_0 \approx \frac{e^2}{\hbar c}\left(\frac{v}{c}\right)^2 \omega_0, \tag{13}$$

where

$$\omega_0 \approx \frac{U_0}{\hbar} \int \frac{f(p)}{(1+\mu_p^2)^{1/2}} dp$$

is the average photon emission frequency, which is equal in order of magnitude to $U_0/\hbar$.

Actually, the number of electrons (A) which can radiate in the superlattice depends on the current density j_0 and the parameters of the superlattice:

$$A = j_0 SL/ev, \tag{14}$$

where S is the cross-sectional area of the superlattice. The photon emission intensity is then given by

$$\eta = AW. \tag{15}$$

Let us plug in some numbers. The quantity η can be estimated for the real parameter values of a superlattice and of the injected electron current[3] ($S \sim 10^{-6}$ cm^2, $j_0 \approx 10^2$ A/cm^2, $L \approx 3 \times 10^{-5}$ cm, $v \approx 10^8$ cm/s, $\omega \approx 10^{14}$ s^{-1}): $\eta \approx 5 \times 10^9$ s^{-1}.

To estimate the gain G we assume $U \simeq \hbar\omega = 0.1$ eV, $d \approx 60$ Å ($q \approx 10^7$ cm^{-1}), $p/\Delta p \approx 10$, $\epsilon_1 \approx 12$, $N \approx 10^{14}$ cm^{-3}, $L \approx 3 \times 10^{-5}$ cm, and $v \approx 10^8$ cm/s. We find

$$G = 10^3 b. \tag{16}$$

The gain per pass is larger than one in this case; with $b > 10^{-3}$ cm, the gain would be fairly high even in the absence of mirrors.

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Translated by D. Parsons