Current-phase relation in layered cuprates

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The influence of interlayer tunneling in layered superconducting (SC) medium on the intralayer SC is shown to be essential in strongly anisotropic cuprates. Josephson-like transverse current is found to be a universal function of the phase difference Ω between adjacent layers, while a simple Josephson sinusoidal dependence on Ω is valid only at rather low temperature $T \ll T_{\rm ex}$, the 3D–2D crossover temperature.

In an ordinary Josephson junction formed by bulk banks SC order parameter, $|\psi|$, is weakly perturbed by a tunneling current and deeply in the banks may be assumed equal to the value $\langle \psi_0 \rangle$ of a separate bulk sample. In a layered compound with atomic thickness of layers $d \cong \xi_\perp$, SC correlation length in the z direction normal to the layer (as in a crystal HTSC, both effects (SC and tunneling) are of the same dimensionality and may essentially influence one another. If in the bulk case the perturbation of the order parameter modulus $|\psi|$ by a tunneling current j_z may be ignored in the layered system, in contrast, j_z may essentially reduce the density of SC pairs, n_s, by analogy with the current flowing through a thin SC film or wire.[1] In this work the set of variational equations for a layered SC is studied with allowance for the vector potential, A, and the order parameter phase, φ, the uniform density $n_s = |\omega|^2$, as a variational variable. An optimal current-dependent value of n_s is found in an array of S–I–S-junctions. It is assumed that a correlation between the phases φ_n, φ_{n+1} of the order parameters ψ_n, ψ_{n+1} of the adjacent SC layers is controlled by a uniform SC current flowing in the z direction.

Since the value of the Josephson current is proportional to n_s the persistent current in such a structure exhibits nontrivial dependence on the temperature and gauge-invariant phase difference

$$\Omega_n = \varphi_{n+1} - \varphi_n - \int_{z_n}^{z_{n+1}} dz A_z \quad (\hbar = 1). \tag{1}$$

Uniform suppression of the order parameter when current is applied and uniform phase difference between adjacent layers is established to reduce critical current density with respect to the Josephson critical current $j_c = 2eE_J(T)$, where $E_J(T)$ is the energy of the Josephson coupling of layers. In the most anisotropic Bi- and Tl-based HTSC, where the fluctuation region width $\tau_f \gg \tau_{\rm cr}$ ($\tau_{\rm cr} = 1 - T_{\rm cr}/T_{c0}$, T_{c0} is the mean field transition temperature) suppression of the interlayer coupling E_J due to fluctuations may reduce j_c more substantially than the mechanism mentioned above. But in less anisotropic single crystals of the 1-2-3 compounds, where $\tau_f \ll \tau_{\rm cr}$ (Ref. 2) fluctuation correction is on the order of τ_{KT}/τ ($\tau = 1 - T/T_{c0}$, $\tau_{KT} < \tau_f$ is dimensionless

Kosterlitz–Thouless temperature of the single layer) and may be ignored at $\tau > \tau_{cr}$. At temperatures $\tau \ll \tau_{cr}$ a linear dependence of the Josephson critical current $j_c \propto \tau$, can then be substituted by $j_c \propto \tau(1-\tau_{cr}/2\tau)$.

Quasi-2D behavior of a HTSC is usually described by the Ginzburg–Landau free energy functional in the Lawrence–Doniach form (GL–LD model), which reads[3]

$$F = d\sum_n \int d^2\rho \left\{ \alpha n_s + \beta n_s^2 + \frac{n_s}{4m}\left[\left| \bar{\nabla}_\parallel \varphi_n - \frac{2e}{c}\mathbf{A}_\parallel \right|^2 + \frac{2m}{Md^2}[1-\cos(\Omega_n)] \right] \right\} + \int \frac{h^2 dV}{8\pi}, \tag{2}$$

where d is the interlayer distance, m and M are the effective electron masses in the (ab) plane and the z direction, respectively; and $\mathbf{h}$ is the magnetic field.[1] The GL–LD model is valid at temperatures $T < T_{cr}$, where 3D–2D crossover temperature is defined by the equality $\xi_\perp(T_{cr}) = d/\sqrt{2}$.

The 2D vortex fluctuations thermally induced in the separate layers account for nonuniform phase difference between adjacent layers, which effectively reduces the interlayer coupling and the critical current, respectively.[4–6] In the layered systems, however, the fluctuation width of the SC transition is controlled by $\gamma = m/M$, the anisotropy parameter, and turns out to be finite, contrasted with the 2D case. The fluctuation mechanism for the suppression of the transverse critical current $j_{c\perp}$ in this case is essential only in the temperature region close to T_{kt}, the Kosterlitz–Thouless temperature of a separate SC sheet. In the low temperature region $\tau \ll \tau_{cr}$, the number of thermally induced 2D vortices of the Abrikosov type and Josephson-like vortex loops[5] is exponentially small and may be disregarded.

Thus, in strongly anisotropic layered single crystal HTSC (where it is assumed that the width of the fluctuation region $\tau_f > \tau_{cr}$) outside of the fluctuation region and in less anisotropic 1-2-3 compounds at $\tau > \tau_{cr}$ the suppression of the interlayer critical current may occur only due to a decrease in n_s compared with $n_{s0} = |\psi_0|^2 = -\alpha/\beta$.

To single out this effect, let us assume for simplicity that j_z is distributed uniformly in the (ab) plane, and that it is small enough to ignore its self-field.

Omitting in the absence of an external magnetic field the terms connected with the nonuniformity of the order parameter in the planes, we find that the minimum of the free energy (2) is given by

$$n_s = n_{s0}[1-(\xi_\perp/d)^2(1-\cos(\Omega_n))], \tag{3}$$

where Ω_n under the adopted conditions is the same for all junctions. At low Ω_n the SC density deviates by the value $\Omega_n^2 \simeq j_z^2$, as in a thin film (wire).[1] Variation of the energy (2) with respect to A_z (Ref. 3) yields, together with (3),

$$j_z = j_{c0}\left[1-\frac{\tau_{cr}}{2\tau}[1-\cos(\Omega_n)]\right]\sin(\Omega_n). \tag{4}$$

Here $j_{c0} = (en_{s0}\gamma/2md)$ is the value of the Josephson critical current of the bulk junction.

It can easily be seen that the maximum value of dependence (4) is achieved at $\Omega < \pi/2$, defined by

$$\cos(\Omega) = [\sqrt{(1-\tau_{cr}/2\tau)^2 + 2(\tau_{cr}/\tau)^2} - (1-\tau_{cr}/2\tau)](2\tau_{cr}/\tau)^{-1}, \tag{5}$$

and may be as small as $j_{z,\max}/j_{c0} = 3\sqrt{3}/8$ [at $\cos(\Omega) = 1/2$]. In the low temperature region $\tau \gg \tau_{cr}$, we have $\cos(\Omega) \cong \tau_{cr}/2\tau$, and we find the following expression from (4):

$$j_{z,\max} = j_{c0}[1 - \tau_{cr}/2\tau]. \tag{6}$$

It should be noted that the universal nature of the functional dependence (4) which contains no specific parameters of the material except the anisotropy-mediated τ_{cr} (the constant value j_{c0} is determined by the material). The parameter $\tau_{cr}/2\tau$ generally is not small, because the GL–LD model is valid up to $\tau \cong \tau_{cr}$. In the region $\tau < \tau_{cr}$ the above analysis loses its validity and the crossover to anisotropic 3D low critical current $j_{c\perp} \propto \tau^{3/2}$ should take place. Thus, the decrease in the critical current (6) may be considered as a pre-crossover phenomenon.

We note in conclusion that deviation of the current-phase relation from simple sinusoidal dependence and violation of the linear temperature dependence of the Josephson-like critical current may take place in a granular SC medium such as a ceramic HTSC. If the intergranular barrier transparency satisfies certain conditions,[7] this may occur due to homogeneous suppression of the order parameter inside grains by a usual depairing process[1] or due to the effect of the surface on the density n in the grains.

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