# *RC* relaxation of 2D Hall currents

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A relaxation "scenario" for the decay of Hall current states in 2D systems is proposed. The decay time $\tau_{RC} = RC$ and its components, the effective resistance $R$ and the capacitance $C$, are derived. The possibility of an independent evaluation of $R$ and $C$ is discussed. The related question of testing the validity of the $RC$ relaxation mechanism is also discussed.

Zhitenev[1] has carried out an experimental study of the response of a magnetized 2D electron strip to the application of a $\theta$-shaped potential $V(t) = V\theta(t)$ between the points $\pm b$, as shown schematically in Fig. 1. According to the results,[1] the relaxation of the current between $\pm b$ is exponential in its late stages, with a relaxation time $\tau^*$ which is an oscillating function of the filling factor of the Landau levels, $\nu$. The time $\tau^*$ reaches a maximum at integer points $\nu$; i.e., we have $\tau^* \propto \sigma_{xx}^{-1}$. In addition, there is a shorter time $\tau_{mp}$, which reaches a minimum at integer points $\nu$. This time, which had been observed previously in Refs. 2 and 3, is not a relaxation time, since it is not proportional to $\sigma_{xx}^{-1}$.

Zhitenev[1] found an estimate of $\tau^*$, citing Govorov and Chaplik:[4]

$$\tau^* \simeq a^2/D, \quad D = 4\pi d\sigma_{xx}/\kappa \tag{1}$$

($\kappa$ is the dielectric constant). That estimate is qualitatively correct in the sense of the $\sigma_{xx}$ dependence of $\tau^*$. However, the role played by the geometric factors—the strip width $a$, the gap $b$, and the distance to the screening electrode, $d$, which appears in the definition of $D$—needs to be refined.

Our purpose in this letter is to derive $\tau_{RC} = \tau^*$ in the quasisteady approximation, in which $\tau_{RC} = RC$ is far larger than $2b/s$, where $2b$ is the distance between the contacts $\pm b$ (Fig. 1), and $s$ is the velocity of an edge magnetoplasmon. To determine $\tau_{RC}$ in this case we can use the $RC$ impedance approximation,[5] in which the effective resistance $R$ and the capacitance $C$ are calculated for a steady-state distribution of Hall currents in a sample of given geometry. We assume that $\tau_{RC}$ can be associated with the time $\tau^*$ observed in Ref. 1.

**1**. Our starting point is the energy conservation law, in which the Hall state is characterized by the electrostatic energy $CV^2/2$, which can vary only to the extent that there is a Joule heating $W$. We thus write

$$W + \frac{1}{2} CV^2 = 0, \quad W = \int j_i \frac{\partial \mu}{\partial x_i} dxdy,$$

$$j_i = \sigma_{ik} \frac{\partial \mu}{\partial x_k}. \tag{2}$$

  

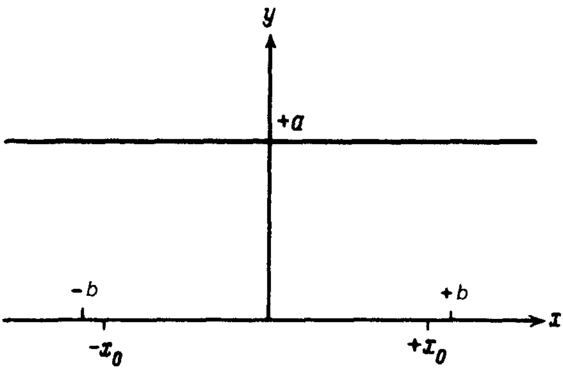



FIG. 1. A 2D conducting strip of width $a$ in a magnetic field normal to the plane of the figure. A potential difference $V(t)$ is applied to the points $\pm b$. The difference $(b - x_0)$ is a measure of the width of the conducting contacts.

Here $\mu$ is the electrochemical potential, $C$ is the capacitance of the Hall sample corresponding to the charge separation caused by the Hall voltage $V$, and $\sigma_{ik}$ are the components of the conductivity tensor. For simplicity we are assuming that the distance $d_0$, between the 2D system and the donor layer, is negligible and that

$$\sigma_{xx}/\sigma_{xy}=\epsilon\ll 1,\quad \nu\gg 1. \tag{3}$$

In my opinion, the use of the electrochemical potential $\mu$, rather than the electric potential $\varphi$, in the definition of the current $j_i$ in (2) resolves the contradiction between the microscopic theory[6] and the phenomenological theory[7] of the Hall current distribution in 2D systems.

2. As was mentioned above, the values of $R$ and $C$ can be calculated through the use of a steady-state current distribution. In this case the requirement

$$\mathrm{div}\mathbf{j}=0,$$

along with the assumption that $\sigma_{xx}$ be nonzero, leads to the equation

$$\Delta\mu=0 \tag{4}$$

for $\mu$ with the boundary conditions that the faces of the sample are impenetrable to the current. In the limit $\epsilon \ll 1$ [see (3)], these conditions become

$$\mu_0(xy)\,|_{y=a}=0,\quad \mu_0(xy)\,|_{y=0}=\begin{cases}0, & |x|>2b,\\ V, & |x|<2b.\end{cases} \tag{5}$$

The solution of (4) and (5) is

$$\mu_0(xy)=2V/\pi\begin{cases}\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi y}{a}\right)\left[1-e^{-n\pi b/a}\cosh\left(\frac{n\pi x}{b}\right)\right], & x<b,\\ \displaystyle\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi y}{a}\right)e^{-n\pi|x|/2b}\sinh\left(\frac{n\pi b}{a}\right), & |x|>b.\end{cases} \tag{6}$$

The quantity $\dot{W}$ can be written in terms of $\mu_0$ at logarithmic accuracy (explained below) as follows:

$$\dot{W}=\sigma_{xx}\int\left[\left(\frac{\partial\mu_0}{\partial x}\right)^2+\left(\frac{\partial\mu_0}{\partial y}\right)^2\right]dxdy=\sigma_{xx}V\int_{-b}^{+b}\left(\frac{\partial\mu_0}{\partial y}\right)\bigg|_{y=0}dx. \tag{7}$$

Going back to Eq. (2), and using (7) and (6), we find

$$\frac{dV}{dt}=-\frac{V}{RC},\quad V=V_0e^{-t/\tau_{RC}},\quad \tau_{RC}=RC,$$

$$R=\frac{a}{2\sigma_{xx}[x_0+(4a/\pi)\ln z]}=\begin{cases}\dfrac{a}{2\sigma_{xx}b}, & b\gg a,\\[2mm] \dfrac{\pi}{4\sigma_{xx}\ln z}, & b\ll a,\end{cases} \tag{8}$$

$$Z=(e^{\pi b/a}-e^{-\pi x_0/a})/(e^{\pi b/a}-e^{\pi x_0/a}),$$

$$|b-x_0|\ll b. \tag{8a}$$

Expression (8a) for $Z$ diverges as $x_0\rightarrow\pm b$, as we would expect in view of the jumps in the potential $\mu_0$ at these points. The appearance of a large logarithm here allows us to restrict expression (7) for $\dot{W}$ to the contribution from the segment $-b\leqslant x\leqslant +b$ alone.

3. A calculation of $C$ for the general case is more involved. It is necessary to evaluate the excess electron density due to the perturbation $V$ by analogy with Ref. 6 but under the additional condition that there is a coordinate dependence $\mu_0(xy)$ [see (6)] and with the introduction of edge electron states, which were omitted in Ref. 6. This will all be done in a separate paper; we content ourselves here with finding an upper estimate of $C$ under the assumption that we can use the classical approximation for this purpose Ref. 7. In that approximation, the difference between $\mu(xy)$ and the electric potential $\varphi(xy)$ drops out of the picture. A further simplification becomes possible thanks to the use of a screening electrode in Ref. 1. It thus becomes a simple matter to reconstruct the 2D charge density from the known distribution $\mu_0(xy)$. As a result, we find

$$C<C_0,\quad C_0=Q_0/V,$$

$$Q_0=\frac{\kappa}{4\pi d}\int_0^a dy\int_{-\infty}^{+\infty}dx\mu_0(xy),$$

$$C_0\simeq\begin{cases}\dfrac{\kappa ab}{\pi d}, & b\gg a,\\[2mm] \dfrac{2\kappa b^2}{\pi^2 d}\ln\dfrac{a}{2b}, & b\ll a,\end{cases} \tag{9}$$

where $\kappa$ is the dielectric constant between the plates, which are separated by a gap $d$, $d\ll a$, $d\ll b$.

In Zhitenev's experiments[1] it was possible to measure the capacitance $C$ independently, as the ratio of (a) the charge $Q$ which flowed onto the control electrode upon the appearance of the perturbation $V$ to (b) this potential. As a result, in estimating the time $\tau_{RC}$ in (8) we can use as $C$ its experimental value $C=Q/V$.

The combination $\tau_{RC} = RC_0$ with $R$ and $C_0$ from (8) and (9) is the same (within a factor of 2) as $\tau^*$ from (1) in the limit $b \gg a$. On the other hand, the experimental geometry of Ref. 1 corresponds primarily to the case $b \gtrsim a$. Unfortunately, no special study of the $b/a$ dependence of $\tau^*$ was made in Ref. 1.

**4.** The appearance of an electrostatic energy $CV^2/2$ under the condition $\operatorname{div} \mathbf{j}=0$ suggests that Coulomb-blockade effects are being seen in this system. In particular, a nonlinear behavior of the current-voltage characteristic is possible under the conditions $eV < T < e^2/2C$.

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