Isovector form factor of the K meson

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A 1D integral representation for the isovector form factor of the K meson is derived in terms of the π-meson form factor and the amplitude for πK backscattering. Normalization of the form factor at the origin yields sum rules for the amplitude and makes it possible to find the difference between the s-wave scattering lengths for πK scattering: $a_0^{3/2}-a_0^{1/2}\approx 0.22\mu^{-1}$, where μ is the mass of the π meson. The isovector form factor of the K meson turns out to be numerically close to half the π-meson form factor. This result agrees qualitatively with the vector dominance model.

Experimental research on the pion, kaon, and nucleon form factors (see Ref. 1 and the papers cited there) is stimulating the development of phenomenological models which are capable of describing the behavior of the form factors at low and intermediate energies and which have the correct QCD asymptotic behavior. In the present letter we discuss the problem of the isovector form factor of the K meson.

The unitarity relation for the isovector form factor of the K meson near the threshold is

$$\mathrm{Im}\ F_K^v(t)=\frac{t-4\mu^2}{4\sqrt{t-4m^2}}\ h_1^1(t)F_\pi^*(t). \tag{1}$$

Here m and μ are the masses of the K and π mesons, $F_\pi(t)$ is the π-meson form factor, and $h_1^1(t)$ is the analytic continuation into the region $t<4m^2$ of the p-wave amplitude for πK scattering in the t channel with isospin $I=1$.

The invariant relativistic amplitude $A_1^t(s, u, t)$ with $I=1$ is antisymmetric with respect to the variables s and u, so the quantity $\alpha\ (s, u, t)=A_1^t(s, u, t)/2(s-u)$ has no additional singularities. Near $t=4\mu^2$ in the amplitude

$$A_1^t(s,u,t)=8\pi\ \sqrt{t}\,\Sigma(2l+1)h_l^1(t)P_l(\cos\theta_t),$$

where θ_t is the scattering angle in the t channel, we can ignore the higher-order partial waves ($l=3, 5, \ldots$) and express $h_1^1(t)$ in terms of the quantity $\alpha(\nu, -1)=\alpha(s, u, t)$ with $\cos\theta_t=\cos\theta_s=-1$ (ν is the square of the momentum in the *c.m.* frame in the s channel). In this case we have $t=-4\nu$ and $h_1^1(t)=[(t-4m^2)(t-4\mu^2)]^{1/2}\alpha(\nu, -1)/(12\pi\sqrt{t})$. In the Frazer-Fulco-Gounaris-Sakurai model[2] of the pion form factor we have the relation

$$\frac{(t-4\mu^2)^{3/2}}{64\pi\sqrt{t}}\ |F\pi(t)|^2=\mathrm{Im}\ F_\pi(t+i0)D(0), \tag{2}$$

where $D(0)=0.0134\ \mathrm{GeV}^{2}$ is the value of the D function at the origin. Condition (1) means that the phase of the amplitude $h_1^1(t)$ and therefore that of the quantity $\alpha(\nu, -1)$ are the same as the phase of the pion form factor. Accordingly, if the function $\alpha(\nu, -1)$ has two cuts, $(-\infty, -\mu^2)$ and $(0, +\infty)$, in the complex ν plane, as follows from the Mandelshtam representation,[3] then the ratio $\alpha(\nu, -1)/F_\pi(t)$ has only one cut, specifically, the one on the right. In the model of Ref. 2 the form factor has no zeros in the complex ν plane, so the dispersion relation can be written

$$\frac{\alpha(\nu,-1)}{F_\pi(\nu)}=\frac{\alpha(0,-1)}{F_\pi(0)}+\frac{\nu}{\pi}\int_0^{+\infty}\frac{\mathrm{Im}\,\alpha(\nu',-1)}{\nu'(\nu'-\nu)}\frac{d\nu'}{F_\pi(\nu')}. \tag{3}$$

Using (2), we find $\mathrm{Im}\,F_K^v(\nu)\propto \mathrm{Im}\,F_\pi(\nu)\alpha(\nu, -1)/F_\pi(\nu)$. Using the residue-free dispersion relation, we find an integral representation for the isovector form factor of the K meson:

$$F_K^v(\nu)=\frac{4D(0)}{3}F_\pi(\nu)$$
$$\times\left(\frac{\alpha(0,-1)}{F_\pi(0)}-\frac{1}{\pi}\int_0^{+\infty}d\nu'\frac{\mathrm{Im}\alpha(\nu',-1)}{\nu'}\frac{\nu/F_\pi(\nu')-\nu'/F_\pi(\nu)}{\nu-\nu'}\right). \tag{4}$$

This representation incorporates unitarity, crossing symmetry, and analyticity. Asymptotically we have $F_K^v(\nu)\propto F_\pi(\nu)\propto 1/\nu$, in agreement with the quark counting rules.

The normalization condition at the origin yields a sum rule for the amplitude for πK backscattering:

$$\frac{1}{2}=\frac{4D(0)}{3}\left(\alpha(0,-1)-\frac{1}{\pi}\int_0^{+\infty}d\nu'\,\frac{\mathrm{Im}\,\alpha(\nu',-1)}{\nu'}\right). \tag{5}$$

We find a simple result by assuming $\alpha(\nu,-1)/F_\pi(\nu)=\mathrm{const}$. This hypothesis is equivalent to ρ-meson dominance in the amplitude for πK scattering, since the behavior of the pion form factor is governed by the ρ-meson pole. Taking the normalization at the origin into account, we find from the condition $\mathrm{Im}F_K^v(\nu)\propto \mathrm{Im}\,F_\pi(\nu)\alpha(\nu,-1)/F_\pi(\nu)=\mathrm{Im}F_\pi(\nu)\mathrm{const}$ the result $F_K^v(\nu)=F_\pi(\nu)/2$. This result is valid in the $SU(3)_f$ limit; it is also exact in the limit of a zero width of the ρ meson. Substituting $F_\pi(\nu)=(1+4\nu/m_\rho^2)^{-1}$ into (4), we find that the integral term is independent of ν.

The quantity $\alpha(0, -1)$ is expressed in terms of the difference between the s-wave πK scattering lengths in the channels with isospins $I=1/2$ and $3/2$. The phase shifts[4] in the channels $l=0$ with $I=1/2$, $3/2$ and $l=1$ with $I=1/2$ were interpolated smoothly and used to evaluate the integral term in (5). The result can be summarized as follows: $0.50=2.52(a_0^{3/2}-a_0^{1/2})\mu+(-0.33+0.06+0.24+\ldots)$. Hence we have $a_0^{3/2}-a_0^{1/2}\approx 0.22\mu^{-1}$. To evaluate the error here, we incorporated the contributions of some more massive resonances, with $l=0$–4 and $I=1/2$. They contribute an error

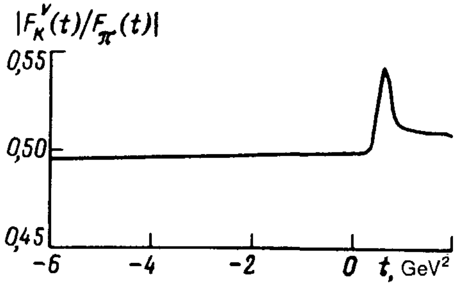

FIG. 1.

$\approx 0.02\mu^{-1}$, showing that the low-energy approximation is self-consistent. The chiral Lagrangians in the tree approximation yield $a_0^{3/2}-a_0^{1/2}=0.21\mu^{-1}$; experiments[5] have yielded $a_0^{3/2}-a_0^{1/2}=(0.14-0.37)\mu^{-1}$.

Taking the same partial waves into account, we have calculated the isovector form factor of the K meson from representation (4). The result is shown in Fig. 1. The deviation of this form factor from half the pion form factor does not exceed 10% near the ρ peak, while it is negligible in the spatial region, in qualitative agreement with the vector-dominance model.

We do not as yet have enough experimental data on the form factors of the K^+ and K^0 mesons in the timelike region to determine the isovector part. In the resonance region, according to the vector-dominance model, we have $|F(t)| = O(m/\Gamma)$, where m and Γ are the mass and width of the corresponding vector meson. In the isoscalar channel we have $|F_K^s(m_\omega^2)| \approx 100$, while in the isovector channel we have $|F_K^v(m_\rho^2)| \approx 10$. The resonance region is thus dominated by the isoscalar component. The assumption $F_K^v(t) \approx F_K^s(t)$, which holds exactly at the origin, agrees with the data of Amendolia *et al.*[1] near $t=0$ at $t<0$.

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