Toward a theory for the formation and decay of metastable states of a kaonic helium atom

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The original populations of the recently discovered metastable states of hadronic helium atoms can be explained by a theory derived previously for the formation of mesic atoms. Calculations of the original populations of the longest-lived states of kaonic helium agree well with experimental data. The lifetimes are longer by a factor of 1.5 or 2 than those measured in liquid helium, possibly because of a "quenching" of these states in collisions with atoms of the medium.

1. It was shown in the experiments of Refs. 1–4 that negative hadrons stopped in helium go into metastable states with a probability of 2–4%. These metastable states include one group of π^- states, two groups of K^- states, and four groups of $\bar{p}$ states. The populations and lifetimes of these groups were determined. The possible existence of metastable states of neutral hadronic atoms of helium was discussed a long time ago.[5] According to the calculations of Ref. 6, states with a main quantum number $n > (\mu/m)^{1/2}$, where μ and m are the reduced masses of the hadron and the electron, and with an orbital angular momentum $l=n-1$ (circular orbits) are long-lived. Auger transitions from these states are strongly suppressed by the high multipolarity, while radiative $E1$ transitions are weak because of the low photon energy. Those calculations, however, do not give the probabilities for the filling of levels, i.e., quantities which are crucial to an experimental study of metastable states and the time-varying distributions of the products of the retarded nuclear absorption and decay of hadrons.

In this letter it is shown that the original populations of the metastable states can be derived from a quantitative theory which has recently been derived for the production of hadronic atoms of helium.[7,8] The original populations of the longest-lived states of kaonic helium are calculated, as are the lifetimes of these states.

It follows from the calculations of inelastic mesic-atom cross sections and an analysis of the kinetics of the formation of mesic atoms[8] that the original populations of the states of mesic helium can be described by

$$\rho_{nl}=\frac{1}{1}\,\theta(E_{nl}-E_0)\theta(E_0+I-E_{nl})\,|dE_{nl}/dn|\,\theta(l_n-l)(2l+1)/(l_n+1)^2, \qquad (1)$$

where E_{nl} is the energy of state nl of the mesic atom, E_0 and I are the energy and ionization potential of the He atom, $l_n=\min\{L_n, (n-1)\}$, and L_n is the largest value of l which satisfies the condition $(l+1/2)^2\leqslant 2\mu R_0^2(E_{nl}-E_0-V_0)$. The parameters

R_0 and $V_0 = -W_0$ can be calculated theoretically. They represent, respectively, the inelastic-interaction radius of the meson with the atom and the potential energy at point R_0. The distribution with respect to n begins at $E_{nl} = E_0$ and falls off with increasing n as $|dE_{nl}/dn|$. The distribution with respect to l is similar to a statistical distribution, except that it is "cut off" at $l_n \leqslant n-1$. In order to find the initial populations of the metastable states from (1), it is necessary to first calculate the energy levels and wave functions of the states with large angular momenta, derive the rates of radiative and Auger transitions, and determine which of the states are metastable. The calculations by Russell[6] are insufficient here, since Russell dealt with only a few states, and only states with $l = n-1$.

2. To calculate the energy levels of the neutral mesic atom, we use a variational wave function in the form of the product of a 1s electron wave function with a variational effective charge and a meson wave function:

$$\Phi_{nlm}(\mathbf{r}) = Ae^{-\beta\tau} r^{\,l} P(r) Y_{lm}(\Omega), \tag{2}$$

where β is a variational parameter, $P(r) = 1$ for $l = n-1$, $P(r) = 1-(\beta+\bar{\beta})r/(2n-1)$ for $l = n-2$, and $\bar{\beta}$ corresponds to the lowest-lying state with the same value of l.

The values found for the energies agree within a relative error of 10^{-4} with the existing results[6] (for $l = n-1$, $n = 27, 28, 29$). The energies of the states ($l = n-1$ and $n-2$) depend only weakly on l; at $n \geqslant 30$ they are described within an error of less than 2% by the expression $E_{nl} = -2-\mu/(2n^2)$.

3. The lifetime of the nl kaonic state is determined by the rates of radiative and Auger transitions. The rates of radiative $E1$ transitions, Λ_γ, are calculated from the customary expressions,[9] with allowance for the changes in the electron wave functions upon the transition of the meson. The rates of Auger transitions, Λ_A, are calculated by a perturbation theory in the interaction of the electron and the kaon. The initial states are described by the variational wave functions which were found, while the final states are described by hydrogen-like kaon wave functions in a field $Z = 2$ and by Coulomb electron wave functions in the continuum in a field $Z-1$. Auger transitions to states in which the final energy of the electron is nonnegative, $\epsilon = E_{nl} + \mu Z^2/2n_f^2 \geqslant 0$, are allowed by energy considerations. The smallest change in the main quantum number of the kaon, $\Delta n = n - n_f$, is equal to one at small values of n, but it increases rapidly with increasing n. The minimum transition multipolarity is $L_0 = \Delta n$ at $l = n-1$ and $L_0 = \Delta n - 1$ at $l = n-2$. The results calculated for the transition rates Λ_γ and Λ_A for kaonic helium are shown in Figs. 1 ($l = n-1$) and 2 ($l = n-2$). The rates of the radiative transitions fall off monotonically with increasing n. At $l = n-2$, the probabilities of the two branches of $E1$ transitions ($n_f = n-1$ and $n-2$) are comparable. Figure 2 shows their total contribution. The rates of Auger transitions behave in a nonmonotonic way. While the minimum multipolarity L_0 remains unchanged, the rate Λ_A depends weakly on n; when L_0 increases by one, however, Λ_A decreases by about two orders of magnitude.

The condition under which a state qualifies as metastable is related to the possibility of observing the decay and retarded absorption of a kaon. According to Ref. 1, the average time for the transition of metastable kaonic helium to the ordinary state is $\tau = 41 \pm 6$ ns, and the relative number of decays of stopped K^- mesons is $f = 1.9$

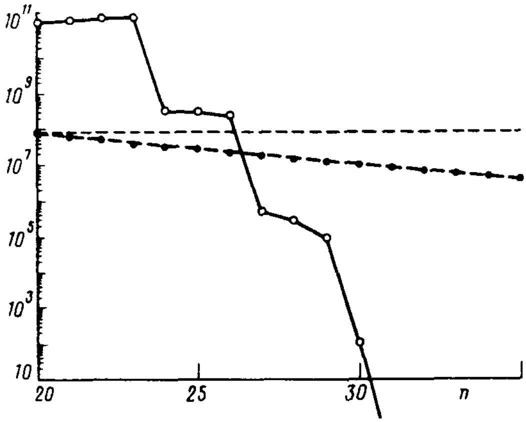

FIG. 1. Rates of transitions from circular-orbit states ($l=n-1$) of kaonic helium. Solid line—Auger transitions; dashed line—radiative transitions; dotted line—rate of spontaneous decay of the kaon.

$\pm 0.3\%$. A subsequent analysis[4] distinguished two groups of kaonic states, one having a lifetime (with respect to transitions) of about 60 ns and a population of $4.1\pm 0.3\%$, while the other has a lifetime of 1.8 ± 0.2 ns and a population of $1.3\pm 0.3\%$.

It can be seen from Figs. 1 and 2 that states with $l=n-1$ at $n\geqslant 27$ ($L_0\geqslant 4$) and with $l=n-2$ at $n\geqslant 31$ ($L_0\geqslant 5$) are metastable. For these states the relation $\Lambda_A<\Lambda_\gamma$ holds, and the overall transition rates are much lower than the kaon decay rate $\lambda_0=0.808\times 10^8\ \mathrm{s}^{-1}$. For lower orbits ($n<27$ at $l=n-1$ and $n<31$ at $l=n-2$) the transition rates are greater than λ_0.

4. The total populations of the states, $P(nl)$, consist of the original populations ρ_{nl} and the sum of the contributions from transitions from higher-lying states:

$$P(nl)=\rho_{nl}+\Sigma P(n'l')\Lambda(n'l'\rightarrow nl)/[\lambda_0+\Lambda(n'l')]. \tag{3}$$

For metastable states, $P(nl)$ is dominated by the original populations ρ_{nl}. To calculate these quantities we use (1) and the values of E_{nl} derived above. For the derivative in (1) we use the estimate

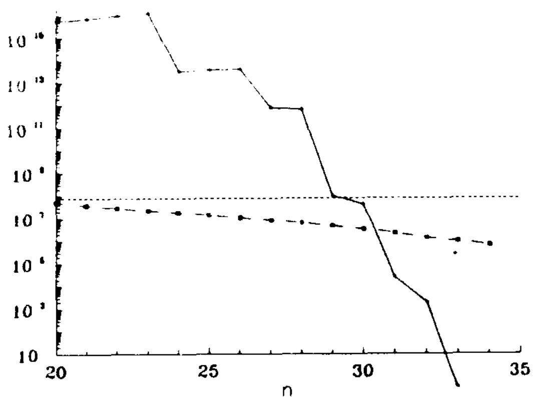

FIG. 2. The same as in Fig. 1, for nearly circular orbits ($l=n-2$).

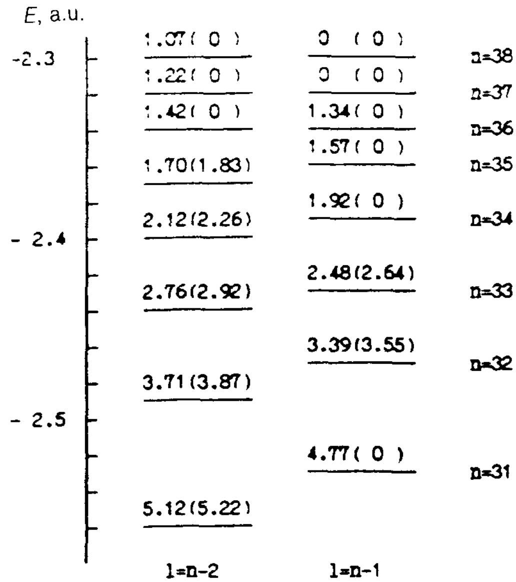

FIG. 3. Level scheme and original populations of the metastable states of kaonic helium. The positions of the levels (in atomic units) are shown on the scale at the left. The first number above the line representing a level is the population of the level (in units of 10^{-3}) as found in version *a*; of the calculations; the number in parentheses is the population according to version *b*. The values of *n* and *l* are shown at the right and at the bottom, respectively.

$$dE_{nl}/dn \simeq (E_{n+1,l+1} - E_{n-1,l-1})/2, \tag{4}$$

where we are ignoring the weak l dependence of the energy. Calculations have been carried out for two sets of parameter values:[8] a) $R_0 = 1.0$, $W_0 = 0.184$ and b) $R_0 = 0.88$, $W_0 = 0.325$ (both quantities are expressed in atomic units).

Figure 3 shows the level scheme and the original populations of the metastable states of kaonic helium with $l = n-1$ and $l = n-2$. The higher- and lower-lying metastable states with the same values of l, which are not shown in this diagram, are not populated in either version of the calculation. The total population of the specified states amounts to 3.5% in version *a* and 2.2% in version *b*. The first of these values agrees well with the population of long-lived group of states of kaonic helium found from the experiments of Ref. 4. The second version of the calculations agrees better with the first published value[1] of the relative number of K^- decays in helium. The calculated lifetimes of the individual states are greater than 100 ns, or 1.5–2 times the measured value. This result may be taken as an indication that the metastable states of kaonic helium are "quenched" by collisions with atoms of the medium. This assumption is in qualitative agreement with the recent data found in the HELIUMTRAP experiment,[10] according to which the average lifetime of the long-lived component of the $\bar{p}$-atom of helium varies from 3.8 to 2.3 μs, depending on the helium pressure and temperature.

The observed component of kaonic helium which is "short-lived" ($\tau = 1.8$ ns) can be compared (Figs. 1 and 2) with states with $l = n - 1$, $n = 24$, 25, 26 and $l = n - 2$, $n = 29$ and 30. According to our calculations, however, these states are not filled. In order to explain the experimental results on this component, it will apparently be necessary to analyze states with smaller angular momenta ($l \leqslant n - 3$) and possibly to take into account the blurring of the population boundaries along the E and l scales. Another possible explanation for the data in this part of the time-varying spectrum might stem from a nonexponential behavior of the decay curve due to a cascade of transitions between metastable states.[11]

On the whole, the theory draws a satisfactory picture of the formation and decay of the main component of the metastable states of kaonic helium. At the same time, the theory fails to resolve some uncertainties concerning the model for the formation of mesic atoms and the effect of collisions with neighboring atoms on the destruction of the metastable states.

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[1]T. Yamazaki, M. Aoki, M. Iwasaki *et al.*, Phys. Rev. Lett. **63**, 1590 (1989).
[2]S. N. Nakamura, M. Iwasaki, H. Outa *et al.*, Phys. Rev. **A45**, 6202 (1992).
[3]M. Iwasaki, S. N. Nakamura, K. Shigaki *et al.*, Phys. Rev. Lett. **67**, 1246 (1991).
[4]T. Yamazaki, Preprint INS-Rep.-952, Inst. of Nucl. Study, Univ. of Tokyo, Oct. 1992.
[5]G. T. Condo, Phys. Lett. **9**, 65 (1964).
[6]J. E. Russell, Phys. Rev. Lett. **23**, 63 (1969); Phys. Rev. **188**, 187 (1969); Phys. Rev. **A1**, 721 (1970); Phys. Rev. **A6**, 2488 (1972).
[7]J. S. Cohen, R. L. Martin, and W. R. Wadt, Phys. Rev. **A27**, 1821 (1983).
[8]V. K. Dolinov, G. Ya. Korenman, I. V. Moskalenko *et al.*, Muon Cat. Fusion **4**, 169 (1989).
[9]V. B. Berestetskiĭ, E. M. Lifshitz, and L. P. Pitaevskiĭ, *Quantum Electrodynamics* (Nauka, Moscow, 1989).
[10]E. Widmann, H. Daniel, J. Eades *et al.*, Preprint INS-Rep-961, Inst. of Nucl. Study, Univ. of Tokyo, Dec. 1992.
[11]B. L. Drujinin, A. E. Kudryavtsev, and V. E. Markushin, Preprint ITEP 2-93, Moscow, 1993.

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