High-frequency size effects in thin metallic films

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The microwave absorption of low-conductivity metallic films on an insulating substrate has size-effect features. These features should be taken into account in an analysis of experimental data on the microwave absorption of thin films (~ 0.1 μm) of high-T_c superconductors.

1. An alternating magnetic field of frequency ω penetrates into a normal metal a distance equal to the skin layer $\delta = c/(2\pi\omega\sigma)^{1/2}$, where σ is the conductivity of the metal. However, an electromagnetic wave is effectively screened out by thin metal layers at much smaller thicknesses, $d \ll \delta$. In particular, a metallic film in vacuum serves as a good screen to a critical thickness $d_c = c/2\pi\sigma = 2\pi\delta^2/\lambda_0$, where λ_0 is the wavelength in vacuum.[1] At this thickness, the loss in the film is at a maximum. Here d_c is a fundamental distance which appears directly in Maxwell's equation $\mathrm{curl}\mathbf{H} = (4\pi/c)\sigma\mathbf{E} + (1/c)(\partial\mathbf{D}/\partial t)$.

For copper at room temperature we would have $\sigma_{\mathrm{Cu}} = 5\times 10^{17}$ s^{-1} and thus $d_{c,\mathrm{Cu}} \sim 10^{-8}$ cm. Recently, however, metals with a low specific conductivity $\sigma \leqslant \sigma_{\mathrm{Cu}}/100$ have been playing an increasing role in physics experiments and applications. For such metals, the dimension d_c increases to several hundred angstroms. In the present letter we wish to call attention to the circumstance that the critical thickness may increase to ~ 1000 Å for films deposited on an insulating substrate. In such a case, the critical thickness is comparable to the actual thicknesses of films being studied experimentally. This is true, for example, of thin films of high-T_c superconductors, which have their highest quality, with the lowest residual losses, at thicknesses[2] ~ 1000 Å. Since the thickness of these films is small, comparable to the critical thickness, effects due to a screening of the external field must be taken into account in an analysis of dissipative processes in such films.

2. We denote by P the losses in the film per unit area, by $P_i = cE_iH_i/8\pi$ the power of the electromagnetic wave incident on the film, and by E_i and H_i the amplitudes of the electric and magnetic fields in the incident wave. In the case of single-sided excitation of the film in vacuum, two structural features are found on a plot of the absorption P versus the film thickness d: a maximum at $d = d_c \ll \delta$ (Ref. 1) and a minimum at $d = \pi\delta/2$. The heights of these structural features, normalized to the losses (P_0) in a thick film ($d \gg \delta$) are quite different, as can be seen from curve *1* in Fig. 1.

The electric field in a thin film ($d \ll \delta$) is distributed uniformly. The field amplitude $E = E_i d_c(d + d_c)$ falls off monotonically with increasing thickness d. The losses in the film, $P = \sigma E^2 d/2$, increase at $d < d_c$ and decrease at $d > d_c$. This circumstance is the origin of the maximum on the loss curve at $d = d_c$. If $d < d_c$, the film becomes

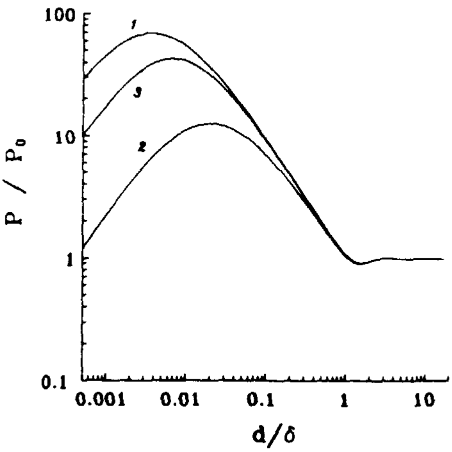

FIG. 1. The loss P/P_0 versus d/δ for a film of thickness d with a resistivity $\rho=1/\sigma=400$ $\mu\Omega\cdot$cm in an electromagnetic wave of frequency $f=30$ GHz. *1*—Film without substrate in vacuum or film on insulating substrate ($\epsilon=10$) of thickness D which is a multiple of half the wavelength in the insulator ($D=\lambda_D n/2$); *2*—the same film, on a substrate of thickness $D=\lambda_D(2n+1)/4$; *3*—an intermediate value, $D=0.4\lambda_D$.

nearly transparent to the incident electromagnetic wave; if $d > d_c$, most of the power of incident wave is reflected from the film.

We turn now to the absorption of a wave in a film on an insulating substrate of thickness D. The dielectric constant of the substrate is ϵ, and the losses in it are negligible. Figure 1 shows P/P_0 as a function of d/δ, as found from a standard solution of Maxwell's equations with the boundary conditions for a structure of the type in Fig. 1. For a film thickness $D=\lambda_D n/2$ ($n=0, 1, 2, \ldots$; $\lambda_D=\lambda_0/\epsilon^{1/2}$ is the wavelength in the insulator), the absorption of the film has the same maximum at $d=d_c$ (curve *1*). At $D=\lambda_D(2n+1)/4$, the height of the maximum decreases by a factor of several units, and the position of the maximum reaches $d=d_c(\epsilon+1)/2$ (curve *2*). For other values of D, i.e., values other than $\lambda_D n/4$, the height and position of this peak occupy intermediate positions (curve *3*).

For example, epitaxial YBaCuO films are deposited on insulating substrates with dielectric constants ϵ ranging from about 10 (MgO and Al_2O_3) to 24 ($LaAlO_3$), and for which the loss tangent satisfies $\tan\kappa \leqslant 10^{-5}$ (Ref. 3). The resistivity at room temperature is $\rho_{YBaCuO} \simeq 400$ $\mu\Omega\cdot$cm, so the critical thickness of such a film ranges from $d_c \simeq 200$ Å to $d_c(\epsilon+1)/2 \geqslant 1000$ Å, depending on the substrate size D.

An even larger critical thickness is found if the substrate is bounded on its back side by a metallic screen which reflects nearly all the radiation incident on it. In this case the maximum critical value is $d=\pi\delta/2$ at a substrate thickness $D=\lambda_D n/2$. The height of the maximum is $P(\pi\delta/2) \simeq 1.09 P_0$. Except for values of D in very small neighborhoods of $\lambda_D n/4$, equal to the skin thickness of the film, the dependence of the absorption P on the thickness d,

$$P(d) = P_0 \delta d/(d^2 + d d_c + d_{max}^2), \tag{1}$$

has a maximum whose position and height are given by

$$d = d_{max} = d_c(1+\epsilon \cot^2\alpha)^{1/2}/2, \quad P = P_{max} = P_0\delta/(d_c + 2d_{max}), \tag{2}$$

where $\alpha = 2\pi D/\lambda_D$. The curves of $P(d)$ coincide for any thickness D in the set

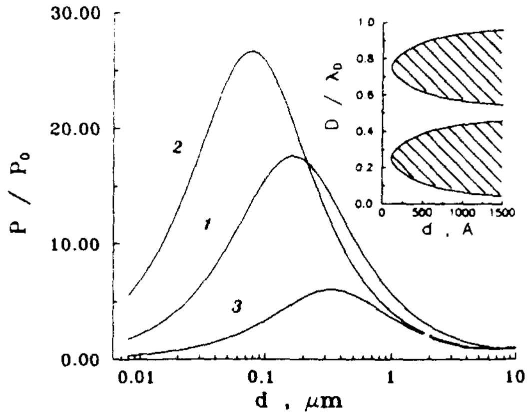

FIG. 2. Plot of P/P_0 versus d for various frequencies and substrate thicknesses ($\rho=400\ \mu\Omega\cdot\text{cm}$; $\epsilon=16$). *1*—$f=30$ GHz, $D_0=0.1$ mm; *2*—60, 0.1; *3*—60, 0.6. The lines in the inset correspond to the $d_{max}(D/\lambda_D)$ dependence from (2). The regions in which the film is transparent ($d<d_{max}$) are not hatched.

$$D_0=|D-\lambda_D n/2|,\quad n=0,1,2,...\quad 0<D_0<\lambda_D/4, \tag{3}$$

so that for $n=1$, for example, we would have

$$P(d,\lambda_D/2+D_0)=P(d,\lambda_D/2-D_0)=P(d,D_0). \tag{4}$$

A dependence of the amplitude and position of maximum (2) on the frequency of the incident wave and the substrate thickness is embodied in $\cot^2\alpha$. The curves of P/P_0 versus d in Fig. 2 give an idea of the typical size of P_{max} and d_{max}. At a given frequency, the position of the maximum in the absorption can be "regulated" through a suitable choice of the substrate thickness D.

Expressing P_0, δ , d_c, and d_{max} in (1) in terms of σ, we find that, as the film conductivity $\sigma(T)$ is varied, there is a maximum in the loss

$$P\propto\sigma/(\sigma^2+\sigma\sigma_0+\sigma_{max}^2),\quad \sigma_0=c/2\pi d,\quad \sigma_{max}=\sigma_0(1+\epsilon\cot^2\alpha)^{1/2}/2, \tag{5}$$

at $\sigma=\sigma_{max}$. It follows from (2) and (5) that the temperature dependence of the loss in a bulk sample will differ from that of the absorption $P(T)$ for a thin film in the transmission region (at $d<d_{max}$ in the inset in Fig. 2). The inset in Fig. 3 shows curves of $P(T)/P_0(300\text{ K})$ for three values of d, plotted under the assumption that the film resistivity $\rho=1/\sigma$ is proportional to the temperature: $\rho=AT$, where $A=1.5$ $\mu\Omega\cdot\text{cm/K}$. With decreasing film thickness, the decreasing function of the temperature is replaced by an increasing function. Outside the transmission region, the loss in the thin film is very high (dashed curve *4*).

These are the simplest manifestations of the screening of the field by a film in the normal state.

3. If the film goes into a superconducting state at $T=T_c$ in this structure as the temperature is lowered, then the conductivity of the film becomes complex: $\sigma_s=\sigma_{1s}-i\sigma_{2s}$. In the two-fluid model, for example, we would have

$$\sigma_{1s}=\sigma_n(T/T_c)^4,\quad \sigma_{2s}=c^2/4\pi\,\omega\,\lambda_L^2(T)=c^2[1-(T/T_c)^4]/4\pi\omega\,\lambda_L^2(0), \tag{6}$$

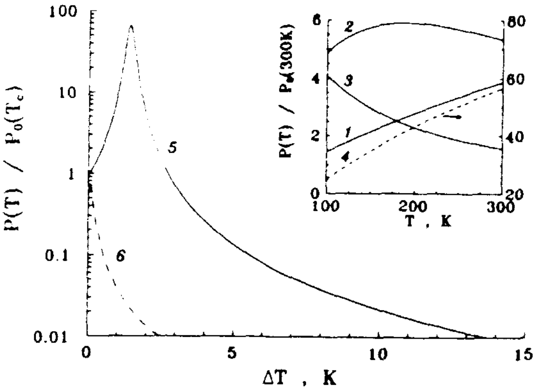

FIG. 3. The dependence $P(T)$ in the normal and superconducting states of the film at the frequency 60 GHz. Curves *1–3* in the inset are plotted for $d=10\,000$ Å, 2400, and 500 Å , respectively, for substrate parameter values $D=0.6$ mm and $\epsilon=16$. Dashed line *4* shows the temperature dependence of the loss for a thin film ($d=500$ Å) outside the transmission region ($D=0.3$ mm). In the main figure we have $\Delta T=T_c-T$, $T_c=90$ K. Curve *5* corresponds to $d=300$ Å, $D=0.5$ mm, and $\epsilon=24$. The loss in a film of this sort is considerably higher than that in a bulk sample (curve *6*, calculated from the two-fluid model).

where σ_n is the conductivity at $T=T_c$, and $\lambda_L(T)$ is the depth to which the field penetrates into the superconductor at $T_c<T_c$. Except at temperatures very close to T_c we have $\sigma_{1s}/\sigma_{2s}\ll 1$. As in the normal state, the absorption $P(d)$ at $T<T_c$ has a maximum $P_m=P_0\lambda_L/(d_m+d_{cs}\epsilon^{1/2}\cot\alpha)$ at

$$d=d_m=d_{cs}(1+\epsilon\cot^2\alpha)^{1/2},\quad d_{cs}=2\pi\lambda_L^2/\lambda_0. \tag{7}$$

Because of the difference in the conductivities σ_s and σ_n and the penetration depths λ_L and δ, however, the size-effect features in the normal and superconducting states of the film are manifested in different ways:

a) Equations (4) do not hold at $T<T_c$, and the height of the peak at substrate thickness

$$\lambda_D(2n+1)/4<D<\lambda_D(n+1)/2 \tag{8}$$

is much greater than at other values of D.

b) At $T<T_c/2$ we have $\lambda_L(T)\simeq\lambda_L(0)\ll\delta$. In (7) we thus have $d_m\ll d_{\max}$ from (2), and for realistic film thicknesses $d>d_m$ there can be only an increase in the loss $P(d)$ at $d\leqslant\lambda_L(0)$.

c) At $T>T_c/2$, as T_c is approached, the sharp increase in $\lambda_L(T)$ causes structural feature (7) on the plot of $P(T)$ to be seen as a narrow peak, which is noticeable at substrate thicknesses D in the intervals in (8). Curve *5* in Fig. 3 shows a plot of $P(T)/P_0(T_c)$ calculated in the two-fluid model, (6). The temperature T_m at the maximum of P is determined by the quantity $\lambda_L(T_m)\simeq(D_0d)^{1/2}$, where D_0 is found from (3). At the given value of D_0, the difference T_c-T_m increases with decreasing film thickness. It may be that the maximum observed on the temperature dependence of the microwave absorption at $T < T_c$ in the experiments of Ref. 4 for a thin ($d \simeq 350$ Å) BiSrCaCuO film on a MgO substrate may have been a manifestation of a size-effect feature. The width of the maximum is a few degrees, and its height is tens of times the background absorption signal at $T < T_c$.

4. The nature of the maximum on the microwave conductivity $\sigma_1(T)$ near T_c in YBaCuO thin films is presently the subject of an active debate in the literature.[5-8] In the experiments, the film, on an insulating substrate, is placed in a waveguide[5-7] or in a cavity as a terminating load.[8-10] We wish to point out that in determining the conductivity σ_1 from such measurements one must take account of the specific features of the screening of the field by the YBaCuO film even in the case in which the film thickness satisfies $d > \lambda_L(0) \simeq 0.14$ μm.

Let us assume that the film on the substrate covers the bottom of a cylindrical copper cavity. Using expressions (6), we can find the temperature dependence of the impedance of the film, $Z = R + iX = 4\pi\, E(0)/cH(0)$, at $T \leqslant T_c$ [$E(0)$ and $H(0)$ are the electric and magnetic fields at the film surface, formed as a result of repeated reflections]. For a film with $\rho(100\ \mathrm{K}) = 150\ \mu\Omega\cdot\mathrm{cm}$ and $d = 0.25$ μm, for example, regardless of the substrate thickness $D < \lambda_D/2$, the surface resistance $R(T) \propto P(T)$ falls off sharply at $T \leqslant T_c$ and has no structural features, in agreement with the experiments. If we now assume that Z is the measured film impedance, we can use the customary local relation $Z = 2[i\pi\omega/(\sigma_1 - i\sigma_2)]^{1/2}/c$ to calculate the conductivity σ_1. We then find that the function $\sigma_1(T)$ is monotonic: It increases starting at $T = T_c$, goes through a maximum at $T \simeq 0.9 T_c$, and then falls off, becoming the same as $\sigma_{1s}(T)$ from (6) at $T < 0.7\ T_c$. This behavior of $\sigma_1(T)$ near T_c does not correspond to the true temperature dependence of the conductivity σ_{1s} of the film in the model originally adopted for calculating Z [Eq. (6)], because in this example the standard relationship between Z and σ does not hold. In determining the correct relation between the conductivity of such a film and measured quantities, one must take account of the effects due to a screening of the field by the film under the actual experimental conditions in each specific case.

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