Spin-dependent current fluctuations in quantum wells

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A new type of spin noise is predicted in nanostructures in a magnetic field.

Electron transport in nanostructures (quantum wells, wires, and dots) has several distinguishing features. The most obvious one is the appearance of steps on the current-voltage characteristics. These steps reflect the discrete nature of the electron spectrum, caused by either size effects[1] or an interelectron interaction (the Coulomb blockade).[2] The distinguishing features of the electron spectrum should evidently also be manifested in the spectrum of current fluctuations (noise) in such nanostructures. A low-frequency noise can be described (as transport can) in terms of a transmission coefficient. The distinguishing features of the transmission coefficient are seen most vividly in the suppression of noise at values of the current corresponding to the plateau on the current-voltage characteristics.[3,4] Long-lived states in nanostructures (local phonons and spin degrees of freedom) should also be manifested as structural features in the noise spectrum at corresponding frequencies.[5,6]

In this letter we wish to call attention to a specific spin noise (when there is a static magnetic field) which is manifested at the frequency of the Zeeman splitting. This spin noise is fairly general and should be seen both in the absence of an electron–electron interaction and under Coulomb blockade conditions. The physical reason for the noise can be explained in the following example. We consider a quantum well with two barriers which border metal electrodes. We assume that there is one quantum-size level in the well (this situation is typical of the states of the conduction band in $Ga_{1-x}Al_xAs/GaAs$ quantum wells with $x=0.3$ and a well width ~ 40 Å;[7]). In the absence of a magnetic field the level is doubly degenerate. In a static magnetic field the degeneracy is lifted for the spin-up and spin-down states. If there is no spin–orbit interaction in the metal banks, then as an electron tunnels from spin-up and spin-down states, its spin does not change, even in the electrodes. If instead there is a spin–orbit interaction, then the state with spin up in the well splits into components with spin up and spin down in the electrode. Similar arguments apply to the spin-down state in the well. After the splitting of the states, there are components in the electrode with spin up and down which are separated by the Zeeman energy. They "interfere" at the final frequency; this interference should also be manifested as a structural feature in the spectrum of current fluctuations.

In several heavy metals which are used as contacts (Au, W, Pt, and Pb), the spin–orbit interaction is not negligible ($\lambda_{so}\sim 1$ eV), and the effect is not particularly small.

We wish to calculate the spectrum of current fluctuations, which we express in terms of the Fourier transform of the current correlation function:

$$S(\Omega)=\int dte^{i\Omega t}\langle\hat{I}(t)\hat{I}(0)+\hat{I}(0)\hat{I}(t)\rangle \quad (1)$$

[$\hat{I}(t)$ is the current operator]. A Hamiltonian for the system which incorporates the basic features of the problem is

$$\hat{H}=\sum_{k\sigma}(\epsilon_{Rk\sigma}b_{k\sigma}^{+}b_{k\sigma}+\epsilon_{Lk\sigma}a_{k\sigma}^{+}a_{k\sigma})+\sum_{\sigma}\epsilon_{0\sigma}c_{\sigma}^{+}c_{\sigma}$$
$$+\sum_{k\sigma}(T_{kL}c_{\sigma}^{+}a_{k\sigma}+T_{kR}c_{\sigma}^{+}b_{k\sigma}+\text{H.a.})+\sum_{k,k}\lambda_{so}^{\sigma-\sigma}(k,k')a_{k\sigma}^{+}a_{k-\sigma}, \quad (2)$$

where the operators c_{σ}^{+}, $a_{k\sigma}^{+}$, and $b_{k\sigma}^{+}$ create electrons in the well, in the right electrode, and in the left electrode; $\epsilon_{L,Rk\sigma}=\epsilon_{L,Rk}+\sigma g_{0,L,R}\mu_B H$; and $g_{0,L,R}$ are the electron g-factors in the well and the contacts, respectively. For definiteness, we take the spin-orbit interaction $\lambda_{so}^{\sigma-\sigma}(k,k')$ into account in the right electrode. We assume that the magnetic field H is weak enough that we can ignore orbital effects. The third term in this Hamiltonian describes the coupling with the banks by tunneling.

The tunneling current operator is written in the standard form:

$$\hat{I}(t)=\frac{ie}{2}\sum_{k\sigma}(T_{kL}c_{\sigma}^{+}a_{k\sigma}+T_{kR}c_{\sigma}^{+}b_{k\sigma}-\text{H.a.}). \quad (3)$$

The current fluctuation spectrum can be expressed in terms of Keldysh Green's functions, as in Ref. 8:

$$S(\Omega)=\frac{e^2}{4}\int \text{Tr}\,\{(\hat{A}+\hat{B})(\omega+\Omega)\hat{G}(\omega)+\hat{G}(\omega+\Omega)(\hat{A}+\hat{B})(\omega)+\hat{G}(\omega+\Omega)$$
$$\times[(\hat{A}-\hat{B})(\omega)\hat{G}(\omega)(\hat{A}-\hat{B})(\omega)]+[(\hat{A}-\hat{B})(\omega+\Omega)\hat{G}(\omega+\Omega)$$
$$\times(\hat{A}-\hat{B})(\omega+\Omega)]\hat{G}(\omega)-[(\hat{A}-\hat{B})(\omega+\Omega)\hat{G}(\omega+\Omega)][(\hat{A}-\hat{B})(\omega)]$$
$$\times\hat{G}(\omega)-[\hat{G}(\omega+\Omega)(\hat{A}-\hat{B})(\omega+\Omega)][\hat{G}(\omega)(\hat{A}-\hat{B})(\omega)]\}d\omega/2\pi. \quad (4)$$

The trace in (4) implies a summation over the spin and Keldysh indices of the Green's functions. Over the Keldysh indices in the correlation function it is understood in the following way:

$$\text{Tr}\,\{[\hat{A}...][\hat{G}...]\}=[\hat{A}...]^{++}[\hat{G}...]^{++}+[\hat{A}...]^{-+}[\hat{G}...]^{+-}+[\hat{A}...]^{+-}[\hat{G}...]^{-+}$$
$$+[\hat{A}...]^{--}[\hat{G}...]^{--}. \quad (5)$$

The Green's functions in (4) have the following form (in contrast with the case of the spin–orbit interaction, which will be dealt with later by perturbation theory):

$$A^{R,A}(\omega)=\sum_{k}|T_k|^2/(\omega-\epsilon_k\pm i0),$$
$$A(\omega)=\gamma_L\begin{cases}f_L(\omega)\\ f_L(\omega)-1,\end{cases}\quad \gamma_L=\pi\sum_{k}|T_k|^2\delta(\omega-\epsilon_k). \quad (6)$$

There are corresponding expressions for the Green's functions of the bank on the right. In the limit of a constant density of states near the Fermi level in the electrodes, the decay constants describing the rate of escape into the banks, γ_L and γ_R, can be assumed independent of the energy.

The Green's functions of the electrons in the well are

$$G^{R,A}(\omega)=1/(\omega-\epsilon_0\pm i0),\quad G_\sigma^<(\omega)=ip_\sigma(\omega)\begin{cases}F(\omega)\\ F(\omega)-1,\end{cases}$$

$$\rho_\sigma(\omega)=\gamma/[(\omega-\epsilon_{0\sigma})^2+\gamma^2],$$

$$F(\omega)=[\gamma_L f_L(\omega)+\gamma_R f_R(\omega)]/\gamma,\quad \gamma=\gamma_L+\gamma_R. \tag{7}$$

In the limit $\Omega\to 0$, expression (4) becomes the Nyquist formula for the noise spectrum, which is expressed in terms of the static conductivity [$H=0$; $f_L(\omega)=f_R(\omega)=f(\omega)$ is the equilibrium Fermi distribution function]:

$$\sigma=(e^2\gamma_L\gamma_R/\pi\gamma)\int df(\omega)/d\omega\rho(\omega)d\omega. \tag{8}$$

If there is a bias voltage across the electrodes, the following expression can be found from (4) for the shot noise, whose density is proportional to the current:[8]

$$S(\Omega)=\frac{e^2\gamma_L\gamma_R}{\pi\gamma}\left\{1-\frac{1}{2}\frac{4\gamma_L\gamma_R}{\Omega^2+\gamma^2}\right\}. \tag{9}$$

When there is a spin–orbit interaction, it is sufficient to consider those corrections to the Green's functions $\hat{A}$ which are off-diagonal in terms of the spin. Since these corrections contain an additional power (T^2) of the tunneling matrix element, they can be ignored in the equations for the Green's functions for the well. The increment in the spectral density of the excess noise when the spin-orbit interaction and the static magnetic field are taken into account is

$$\delta S(\Omega)=\Big(|\gamma_{\uparrow\downarrow}|^2/\pi\int \mathrm{Im}\,[G_\uparrow^R(\omega)]\mathrm{Im}\,[G_\downarrow^R(\omega+\Omega)]$$

$$\times\{F(\omega)[1-F(\omega+\Omega)]+F(\omega+\Omega)[1-F(\omega)]\}/(d\omega/2\pi$$

$$\approx(e^2\gamma_L\gamma_R|\gamma_{\uparrow\downarrow}|^2)/\{\gamma\pi[(\Omega-\omega_L)^2+\gamma^2]\}, \tag{10}$$

where

$$\gamma_{\uparrow\downarrow}=\frac{1}{\pi}\,\mathrm{Im}\,\{\hat{A}_\uparrow(\omega)\hat{\lambda}_{\uparrow\downarrow}\hat{A}(\omega)_\downarrow\} \tag{11}$$

is the energy of the Zeeman splitting, and $\omega_L=g_0\mu_B H$. In deriving (10) we assumed that when there is an external voltage across the banks, the chemical potential in one of the banks is above the level $\epsilon_{0\pm\sigma}$, while that in the other is below it (we are assuming that the temperature is zero). We are also assuming that the voltage is greater than the width of the level in the well. In the absence of an external voltage, the spin noise should be low, since the level in the well lies either below or above the chemical potentials in both banks, and $\delta S(\Omega)$ contains an exponential small parameter [$\exp\{-(\mu-\epsilon_0)/T\}$ at $\mu>\epsilon_0$]. The spin-dependent contribution with respect to the excess noise is determined at $\Omega=\omega_L$ by the ratio $|\gamma_{\uparrow\downarrow}|^2/\gamma^2$, as follows from (10). This ratio can be estimated from (11):

$$|\gamma_{\uparrow\downarrow}|^2/\gamma^2 \approx (\lambda_{so}/w)^2, \tag{12}$$

where w is the width of the band gap in the electrode. For heavy metals such as W, Au, and Pt this width is $w \approx 5$ eV, and the spin–orbit interaction is $\lambda_{so} \approx 1$ eV (for d electrons, $\lambda_{so}/w \approx 1/5$). The spin-dependent contribution for these metals against the background of the ordinary shot noise is determined by the quantity from (12), and $\delta S(\omega_L)/S(\omega_L)$ amounts to a few percent.

One should thus observe a sharp peak in the noise spectrum of the quantum well in a magnetic field, at the frequency corresponding to the Zeeman energy. The noise is of a single-frequency nature and is unrelated to the electron–electron interaction, in contrast with the spin noise discussed in Ref. 6. This contribution to the noise persists even under conditions such that the repulsion of electrons in the well becomes important (Coulomb blockade conditions). In the calculations in this case the one-electron Green's functions in (4) should be replaced by Green's functions incorporating the Coulomb interaction, as in a calculation of the dynamic conductivity.[9]

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