Note on the problem of interacting one-dimensional Hubbard chains

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Pairs or arrays of essentially one-dimensional chains with hopping matrix elements which transfer electrons between chains have been the subject of an extensive and controversial literature, which is quoted most recently in a paper by Yakovenko,[1] entitled appropriately "Once Again about Interchain Hopping." A previous contribution by me[2] has been widely criticized and even more widely misunderstood. Since some new ideas which seem to reconcile many of the various approaches have appeared, it seems appropriate to add another explanatory note.

Many artificial models of spin-chains and interacting electron chains have been proposed. It is important to note that most of these models have few interesting physical realizations, since real substances contain real electrons which have a very restricted set of interactions. This fact was brought out by Haldane[3] in his "Luttinger liquid" idea, and is expressed most recently by Metzner and di Castro[4] by showing that most of the coupling constants are connected by conservation laws and Luttinger–Ward identities[5] (although the specific identities are mostly available already in the review by Solyom.[5]) In the absence of umklapp terms and magnetic fields, these authors show that there is only a one-parameter set of physical models for repulsive interactions.[6] The multichain problem is physically even simpler: The physical interactions between chains are entirely a consequence of momentum-conserving interchain hopping, with the Hamiltonian

$$\mathscr{H}^{\text{int}} = t_\perp \sum_{ijk\sigma} c^+_{k\sigma}(i) c_{k\sigma}(j) \tag{1}$$

and magnetic or other interactions between chains are higher-order consequences of $t_\perp$. The chain Hamiltonians will, for definiteness, be taken as of Hubbard form

$$\sum_{k\sigma} \epsilon_k c^+_{k\sigma} c_{k\sigma} + U \sum_i U_{i\uparrow} U_{i\downarrow} , \tag{2}$$

and $\epsilon_k \simeq t_\parallel \cos k$.

It is this simplicity which I proposed to exploit in Ref. 2 to bring out the basic physics by examining the orders of limits in perturbation theory in $t_\perp$. I identified two limits, (1) $t_\perp / U \gg 1$ and (2) $t_\perp / U \ll 1$, and showed that these two limits showed qualitatively different behavior, in that for (1) $t_\perp$ may be treated as part of the unperturbed free-particle Hamiltonian, which has a "two-dimensional" Fermi surface; while in (2), the Fermi surface remains one-dimensional. The transition (which is *not*

a crossover) at $T=0$ from limit (1) to limit (2) is a unique type of phase transition in which the order parameter is the *shape of the Fermi surface*, i.e., whether it is one- or two-dimensional in k-space.

This problem is, in most of the literature, treated by renormalization-group methods. One introduces an energy cutoff, Λ or, alternatively, a length scale, b, which is continuously carried to the limit $\Lambda = 1/b \to 0$. Rescaling is handled as in the theory of phase transitions by rescaling the sample so that at each stage the same number of degrees of freedom remains within the cutoff Λ, so that the statistical mechanics controlled by the effective Hamiltonian at each scale, $\mathscr{H}(b)$ can remain comparable and $\mathscr{H}$ goes to a fixed limit $\mathscr{H}^*$. In this procedure Fermi velocities remain finite but $t_\parallel$ grows proportionately to b, i.e., $t_\parallel \to \infty$; and hence also $k_F \to \infty$. This corresponds to the fact that the two Fermi points get farther and farther apart relative to the momenta remaining in the problem.

Thus in a sense $\mathscr{H}^*$, the fixed point Hamiltonian has two meanings. If we write it in terms of Fermi velocities:

$$\mathscr{H}^* \sim \sum_k v_F k c_k^* c_k,$$

it is scale-invariant; but $t_\parallel$, as well as k_F, has dimension 1 and grows with scale. The paradox comes about because of the crucial role of Luttinger's theorem: k_F is a counting variable which counts the total density of particles, including those outside the energy scale Λ.

All this is reasonably easy to deal with until we introduce $t_\perp$, which does not have trivial dimension; the essence of my calculation, as well as of Yakovenko's, is that its dimension is

$$D(t_\perp) \simeq 1 - 4\alpha,$$

where 2α is the Fermi surface exponent. Thus *for all interacting* cases, spinless or not,

$$t_\perp / t_\parallel \to 0,$$

and is an irrelevant variable; so also, if we scale x and y dimensions together for a many-chain problem, is $v_{F\perp}$. The point is that $t_\perp$ is a determinant of the shape (and in some cases size) of the Fermi surface and in itself has no other physical meaning, so its criterion for relevance is the dimensionality of $t_\perp$ *divided* by that of $t_\parallel$.

The determinant $t_\perp$ has many higher order effects: antiferromagnetic and Josephson couplings, among others. These are not such that they alter the shape or size of the Fermi surface so that they are properly treated by renormalization group theory in the usual way, and when any of them diverge to ∞ relative to Λ that signals a change in the ground state in the appropriate direction. As Yakovenko remarks, there is, in general, at least one relevant interaction which therefore determines the ground state of the coupled chain problem, not, as he conjectures, for sufficiently strong coupling U, but for all couplings U, so long as $U \gg t_\perp$.

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[1] V. M. Yakovenko, JETP Lett. **56**, 510 (1992).
[2] P. W. Anderson, Phys. Rev. Lett. **67**, 3845 (1991).
[3] F. D. M. Haldane, Phys. Rev. Lett. **45**, 1358 (1980), J. Phys. **C14**, 2585 (1981).
[4] W. Metzner and C. di Castro, Preprint (1993).
[5] J. Solyom, Adv. Phys. **28**, 201 (1979).
[6] A recent Letter by Parola and Fabrizio, Phys. Rev. Lett. **70**, 226 (1993), violates these identities by setting the Fermi surface singularity constant $\alpha=0$. No interacting electron system has $\alpha=0$; among other things, $\alpha=0$ implies finite Z, and it is the vanishing of Z which is most relevant to interchain coupling.

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