Pole approximation for decays of neutral K mesons

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A description of the K^0–$\bar{K}^0$ system without the use of the Weisskopf–Wigner approximation is discussed. Without any assumptions regarding the structure of the Hamiltonian, it is shown that an exponential time dependence is associated with two combinations, K_S and K_L. If there is CP violation, the structure of K_S and K_L contains an apparent violation of CPT invariance.

Despite a major effort, the only manifestations of CP violation which have been found so far are those which stem from K^0–$\bar{K}^0$ mixing. There is accordingly greater interest in a detailed study of the K-meson system and its time evolution. It is expected that new experimental data, highly accurate with a large statistical base, will be obtained in the near future both from hadron interactions (CPLEAR, Tevatron, etc.) and e^+e^- collisions (the φ factory). At the same time, a major campaign is being waged on this problem on the theoretical front.

The Weisskopf–Wigner approximation is the standard basis for describing the evolution of neutral K mesons.[1,2] Recently, however, this approximation has also come under debate. The criticism[3,4] is based primarily on the requirement that the results be mathematically self-consistent. The conclusions can be summarized as follows:

1) The Weisskopf–Wigner approximation is inapplicable in principle in the case of CP violation.

2) It is impossible to construct states K_S and K_L in such a manner that they evolve independently, without regenerating each other.

3) The amplitudes $P_{KK}(t)$ and $P_{\bar{K}\bar{K}}(t)$ for the appearance of $K^0(\bar{K}^0)$ at the time t from an initially pure $K^0(\bar{K}^0)$ state are equal if CPT invariance is valid, while the corresponding amplitudes $P_{\bar{K}K}(t)$ and $P_{K\bar{K}}(t)$ for the appearance of $\bar{K}^0(K^0)$ not only differ in magnitude but also differ in t dependence in the case of CP violation (the t dependence is the same for the two in the Weisskopf–Wigner approximation). These problems were later discussed[5] on the basis of a specific quantum-field model of the Lee-model type.[6] In the present letter we examine these problems from the standpoint of general quantum-mechanical considerations, independent of any specific model.

We take the approach of Ref. 7 (see also Ref. 8). An arbitrary state $|\Psi_0\rangle$ given at $t=0$ subsequently evolves in accordance with

$$|\Psi_t\rangle = e^{-iHt}|\Psi_0\rangle = \frac{i}{2\pi}\int_{-\infty}^{\infty} dE\,\frac{e^{-iEt}}{E-H+i\epsilon}\,|\Psi_0\rangle, \tag{1}$$

where H is the complete Hamiltonian. Since there are transitions accompanied by a change in strangeness here, each state generally contains components with different values of S. If we adopt an initial state with a certain strangeness, $S=-1$ or $S=+1$ (or a corresponding superposition of the two), and if we are then interested in not the entire state but only its components with $|S|=1$, then we can retain integral expression (1) for their evolution, if we replace the complete Hamiltonian H by an effective one. This effective Hamiltonian $\tilde{H}$ incorporates the possibility of a process which is stretched out in time, consisting of an "instantaneous" change in the initial strangeness $S=\pm 1$, a "complete" evolution" in intermediate channels (but without a return to $S=\pm 1$), and, finally, an "instantaneous" return to $S=+1$ or $S=-1$. This process generates two important properties:

1) $\tilde{H}$, in contrast with H, is non-Hermitian, reflecting a "leakage" of probability into other channels.

2) $\tilde{H}$ acquires an explicit E dependence because the process is "nonlocal" in time.

Restricting the discussion to one-meson states K^0 and $\bar{K}^0$, we find $\tilde{H}$ as a 2×2 matrix. In the case of CP invariance we would have

$$\tilde{H}_{11}(E)=\tilde{H}_{22}(E), \quad \tilde{H}_{12}(E)=\tilde{H}_{21}(E), \tag{2}$$

where the subscripts 1 and 2 correspond to K^0 and $\bar{K}^0$. If, on the other hand, CP is violated, but CPT is not, then we would have

$$\tilde{H}_{11}(E)=\tilde{H}_{22}(E), \quad \tilde{H}_{12}(E)\neq\tilde{H}_{21}(E). \tag{3}$$

The Weisskopf–Wigner approximation[1,2] is understood as one of ignoring the E dependence:

$$\tilde{H}(E)\approx\tilde{H}(m_0), \tag{4}$$

where m_0 is the mass of $K^0(\bar{K}^0)$ when decays and $K^0\rightleftharpoons\bar{K}^0$ transitions are ignored. In this approximation, two combinations are singled out:

$$|K_S\rangle=p_S|K^0\rangle+q_S|\bar{K}^0\rangle, \quad |K_L\rangle=p_L|K^0\rangle-q_L|\bar{K}^0\rangle. \tag{5}$$

These combinations have a purely exponential time dependence. In the case of CPT invariance we would have

$$q_S/p_S=q_L/p_L. \tag{6}$$

A test of this property is usually identified with a test of CPT invariance in a K-meson system.

We now lift restrictions (4). We write the 2×2 matrix $\tilde{H}(E)$ in the form

$$\tilde{H}(E)=U^{-1}(E)\Lambda(E)U(E), \tag{7}$$

where $\Lambda(E)$ is a diagonal matrix with the eigenvalues

$$\lambda_{S,L}(E)=\frac{1}{2}\operatorname{Tr}\tilde{H}\pm\frac{1}{2}(a-b), \tag{8}$$

and $a(E)$ and $b(E)$ are the two roots of the equation

$$x^2+(\tilde{H}_{22}-\tilde{H}_{11})x-\tilde{H}_{12}\tilde{H}_{21}=0. \tag{9}$$

The matrix U is not single-valued. One can choose, for example,

$$U(E)=\begin{pmatrix} a & \tilde{H}_{12} \\ b & \tilde{H}_{12} \end{pmatrix}.$$

We now find

$$[E-\tilde{H}(E)+i\epsilon]^{-1}=[E-\lambda_S(E)+i\epsilon]^{-1}U^{-1}(E)\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}U(E)$$

$$+[E-\lambda_L(E)+i\epsilon]^{-1}U^{-1}(E)\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}U(E). \tag{10}$$

Abandoning the Weisskopf–Wigner approximation, (4), but restricting (1) and (10) to the contributions of poles which lead to an exponential t dependence, we again find the two combinations in (5), with the complex masses

$$M_{S,L}=m_{S,L}-\frac{i}{2}\Gamma_{S,L}, \tag{11}$$

which are solutions of the equations

$$M_{S,L}-\lambda_{S,L}(M_{S,L})=0. \tag{12}$$

In this pole approximation we have

$$q_S/p_S=-b/\tilde{H}_{12}|_{E=M_S}, \quad q_L/p_L=a/\tilde{H}_{12}|_{E=M_L}. \tag{13}$$

CPT invariance leads to

$$a(E)=-b(E), \tag{14}$$

and if the E dependence is ignored, we arrive at (6) again. However, incorporating an E dependence renders it invalid and leads to an apparent violation of CPT invariance. This situation is similar to the case of T reversal: It is known that the decay amplitude may contain *T-odd* correlations even in the case of T invariance if a final-state interaction is taken into account.

Here are the consequences of the pole approximation.

1) The amplitudes $P_{KK}(t)$ and $P_{\bar{K}\bar{K}}(t)$ are the same in the case of CPT invariance. The ratio $r=P_{\bar{K}K}(t)/P_{K\bar{K}}(t)$ varies with t if CP is broken. The deviation of $r(t)$ from a constant value decreases with increasing t as $\exp[-\frac{1}{2}t(\Gamma_S-\Gamma_L)]$. These properties are in qualitative agreement with the general assertions of Khalfin.[3,4]

2. In the pole approximation, there exist two combinations, K_S and K_L, which have an independent exponential time evolution. This situation is at odds with the corresponding assertion by Khalfin.[3,4] However, we recall that in the case of CP violation the combinations K_S and K_L are not orthogonal. They furthermore contain an apparent CPT violation, as was explained above [see (13)]. Furthermore, if CP is violated, then the two independent combinations K_S and K_L can be distinguished only if the nonpole contributions with a nonexponential t dependence are ignored. When these terms are taken into account, this assertion of Khalfin becomes correct.

Even crude estimates show that the differences between the pole approximation and the Weisskopf–Wigner approximation are quantitatively small, probably amounting to no more than nonexponential corrections. It would seem to be difficult to detect them (and a "violation" of CPT invariance) experimentally. Nevertheless, their existence and their properties are of interest from the fundamental standpoint.

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