Calculation of multiplicity and charge distributions of fragments produced in multifragmentation of nuclear systems

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A partitioning of an integer into sums of integer terms is used to calculate the multiplicity and charge distributions of fragments produced in the multifragmentation of nuclear systems. Comparison of the resulting distributions with experimental distributions shows that this calculation method is suitable for analyzing experimental results.

Multifragmentation is observed experimentally in collisions of heavy ions at advanced accelerators. This process is being studied theoretically by both macroscopic and microscopic approaches.[1] So far, however, the reasons for the occurrence of this process have not been completely clarified. Multifragmentation is also observed in the interaction of relativistic protons with heavy nuclei, occurring along with the production of two fission fragments in the same nuclear reaction. The relationship with the fission process is seen in the circumstance that the bombardment of ^{238}U nuclei by protons with energies of 1–9 GeV results in the appearance, along with the two nuclear-stable fragments, of a third fragment, which is nuclear-unstable and which is comparable in mass to the first two. This third fragment undergoes a multifragmentation process during the dispersal of the fragments as the result of repulsive Coulomb forces.[2] It is interesting to compare the characteristics of the multifragmentation processes which occur under different conditions. A large number of charged fragments form in a many-body decay process such as multifragmentation. The multiplicity and the charge distributions of the fragments which are formed can serve as the primary characteristics of this many-body decay. Calculations of these characteristics are extremely complicated in all physical models; they depend on the particular features of each model. As a first step in this research it is thus worthwhile to base the calculations on some obvious principle which would make it possible to find the distributions of multiplicity and charges for a given excitation of the nuclear system. Such a principle does exist; it is the conservation of the integer nature of the electric charge in the course of any nuclear decay. Mathematically, this principle can be expressed in an extremely simple way:

$$Z_0=\sum_{i=1}^{n} z_i. \tag{1}$$

The primary charge Z_0 of the nuclear system is partitioned into a sum of integer terms z_i, the number of which, n, is the multiplicity of the charged particles for the given partitioning. All the z_i, as well as Z_0 and n, are positive integers. A highly excited

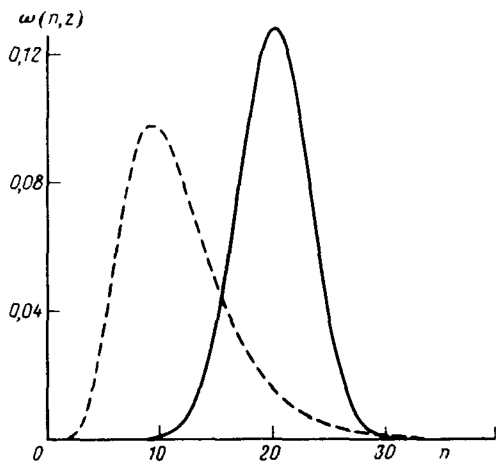

FIG. 1. Normalized distributions of the multiplicity of charged fragments during the decay of a nuclear system with $Z_0 = 39$. Solid curve—For ordered partitionings of the number 39 into integer terms; dashed—the same, for disordered partitionings.

nuclear system can decay with any set of z_i and n values which satisfy condition (1). Accordingly, it is reasonable to start with the *a priori* assumption that the appearance of all the sums in (1) is equiprobable. Working from that assumption alone, we can derive multiplicity and charge distributions for all partitionings.

In this letter we wish to calculate these distributions explicitly for various values of Z_0. The problem reduces to one of analyzing the partitionings, which can be either ordered or disordered, of a positive integer.[3] For the number 3, for example, the partitionings {3; 2+1; 1+2; 1+1+1} are "ordered," while the partitionings {3; 2+1; 1+1+1} are "disordered." In ordered partitionings, the order of the various terms is taken into account, and the partitionings 2+1 and 1+2 are treated as different; in disordered partitionings, in contrast, no distinction is made between the two.

For ordered partitionings, the normalized multiplicity distribution is expressed in terms of binomial coefficients,

$$\omega(n,Z_0)=2^{1-Z_0}C_{Z_0-1}^{n-1}, \tag{2}$$

while the normalized charge distribution is written

$$\omega(z,Z_0)=2^{-z}(Z_0-z+3)(Z_0+1)^{-1}. \tag{3}$$

There are no analytic expressions for the corresponding distributions for disordered partitionings; numerical methods are used to calculate them. Figure 1 shows normalized multiplicity distributions for ordered partitionings (the solid curve) and for disordered ones (the dashed curve). Figure 2 shows normalized charge distributions for ordered and disordered partitionings, again by the solid and dashed curves, respectively. The curves in these figures are plotted for the values $Z_0=39$. This value was selected so that the results could be compared with experimental results[4] on the bombardment of a $^{45}_{21}$Sc target by $^{40}_{18}$Ar ions accelerated to an energy in the range 35–115 MeV/nucleon. The charge distributions of fragments with charges $1\leqslant z\leqslant 12$ were measured by a 4π detector for eight values of the energy near the boundaries of the specified interval and in its interior. The charge distributions measured for the

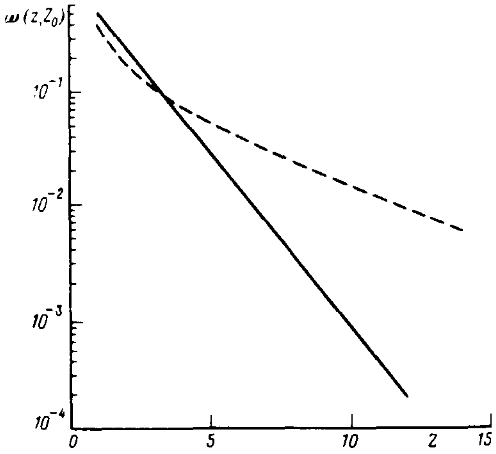

FIG. 2. Normalized charge distributions of fragments during the decay of a nuclear system with $Z_0=39$. Solid curve—For ordered partitionings of the number 39 into integer terms; dashed—the same, for disordered partitionings.

interval $3\leqslant z\leqslant 12$ were approximated by an exponential function $A_\lambda e^{-\lambda z}$ and by a power law $A_\tau z^{-\tau}$. It was found that the exponential expression corresponds to the upper part of the energy interval, and the power law to the lower part. The values found for the coefficient λ and the power τ increase with the energy.

Examining the theoretical charge distributions in Fig. 2 as the result of numerical simulations, we approximated them by the least-squares method, again by exponential and power-law functions, for the same charge interval, $3\leqslant z\leqslant 12$. The approximation by a power law was carried out under the assumption that all ten points being approximated have a 10% error, while a 1% error was assumed in the approximation by the exponential function. It turns out that for ordered partitionings the charge distribution can be approximated better by an exponential law, while the power law is better for the disordered partitionings. The values found for the parameters, $\lambda=0.722\pm0.001$ and $\tau=1.82\pm0.07$, lead to values of χ^2 per degree of freedom of 0.9/8 and 7.2/8, respectively. The calculated parameter values agree satisfactorily with their experimental values: λ toward the upper boundary of the energy interval and τ toward the lower one. An agreement between the theoretical and experimental results in the same boundary regions of this energy interval is also found for the average multiplicities: 20 and 21 for the ordered partitionings and 11.6 and 14 for the disordered ones. We should conclude from these comparisons that the method proposed here for calculating multiplicities and charge distributions is suitable for real nuclear systems which undergo multifragmentation. It would be interesting to look at some other experimental results and to compare them with theoretical distributions for other values of Z_0. That is not possible at the moment, however, since in all other experiments of which we are aware the nuclear systems which underwent multifragmentation usually had numerous values of Z_0. The experiments of Ref. 4 were particularly welcome in that collisions of nearly symmetric nuclei of a beam and a target were used; the beam and target nuclei differed only very slightly in charges and mass numbers. As a result, the charge Z_0 of the decaying nuclear system was effectively fixed.

The calculation method proposed here can also be used to calculate correlations between the multiplicity and charge distributions of the fragments which are produced.

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