Localized quasiparticle states in superconducting contacts with a paramagnetic tunnel barrier

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A localized quasiparticle state is predicted in a superconducting contact with a paramagnetic tunnel barrier in the absence of Josephson currents and magnetic fields. The energy of the state would be in the gap and would be degenerate in the orientation of the quasiparticle spin. The results of this study can be used to interpret tunneling experiments on $Pb/Ho(OH)_3/Pb$ and $Pb/Er(OH)_3/Pb$ systems.

Some new types of quasiparticle states in superconducting systems with Josephson coupling have recently been discussed in the literature. These states arise because of interactions which are invariant under time reversal. They are characterized by an energy in the gap region and by a wave function which is localized near a barrier. As examples we might cite quasiparticle states induced by a superconducting current in ordinary $S/I/S$ contacts[1,2] (S is a superconductor, and I an insulator) and S/I superlattices[3] and the polarized states in $S/I(F)/S$ contacts[4,5] [$I(F)$ is a ferromagnetic insulator].

It was shown in Refs. 4 and 5 that a localized state exists in an S–$I(F)$–S contact in the absence of Josephson currents and external magnetic fields for a single direction of the quasiparticle spin. Its polarization is determined by the sign of exchange interaction with the barrier, and the energy is given by Ref. 5

$$E_{0\alpha}(t)=\Delta_\infty[1-2T_S(t)], \quad |t|\gg\max\{[\Delta_\infty/E_F)^{1/2},[T_S(1)]^{1/4}\}. \tag{1}$$

Here Δ_∞ is the gap parameter in the interior of the superconducting banks, $T_S(t)$ is the exchange part of the tunneling probability, which depends on the cosine of the angle of incidence on the barrier [$T_S(1)\ll 1$], E_F is the Fermi energy, and $\alpha=(+,-)$ is a spin index. The results of Refs. 4 and 5 have been used to explain the features observed[6] in the tunneling characteristics of $Pb/Ho(OH)_3/Pb$ system at voltages less than twice the gap of bulk Pb [$Ho(OH)_3$ is a ferromagnetic insulator at $T<2.5$ K]. {See Ref. 7 for a discussion of the Josephson currents, external magnetic fields, and a generalization to the case of an $S/I(F)$ superlattice.}

Interestingly, these features of the behavior of $Pb/Ho(OH)_3/Pb$ persist to $T<4.5$ K according to Ref. 6. Furthermore, a broadening of a peak in the differential conductivity of a $Pb/Er(OH)_3/Pb$ tunnel junction was observed in the same study[6] at $1<T<4.5$ K [$Er(OH)_3$ is paramagnetic at $T>1$ K]. This broadening was in comparison with the peak of a reference system with a nonmagnetic $Pb/Lu(OH)_3/Pb$ barrier. It is thus natural to ask whether a localized state can exist near a paramagnetic tunnel barrier (the average exchange field is zero in the barrier region). In the present letter we show that such a state does indeed exist and that it has several features which distinguish it from the polarized state in an $S/I(F)/S$ contact.

As in Ref. 5, we start from an equation for the Matsubara Green's functions:

$$\left[i\omega\tau_0\sigma_0+\left(E_F+\frac{1}{2m}\nabla^2\right)\tau_3\sigma_0-\Delta(\mathbf{r})\tau_2\sigma_2-U(\mathbf{r})\right]G_\omega(\mathbf{r},\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}')\tau_0\sigma_0. \qquad (2)$$

Here m is the mass of an electron, $\omega=\pi T(2n+1)$ (n is an integer) is the Matsubara frequency, $\tau_k\sigma_l$ is the direct product of Pauli matrices in Gor'kov–Nambu space (τ_k) and in spin space (σ_l), and τ_0 and σ_0 are the corresponding unit matrices ($\hbar=1$). We assume that the banks of the contact are at the clean limit and that the temperature is low in comparison with the transition temperature. The potential of the barrier is described by

$$U(\mathbf{r})=\left[V\tau_3\sigma_0+\sum_i I_{\mathbf{S}_i}\delta(\boldsymbol{\rho}-\boldsymbol{\rho}_i)\right]\delta(x),$$

$$I_{\mathbf{S}_i}=\frac{J}{2}[(\tau_0+\tau_3)(\boldsymbol{\sigma}\mathbf{S}_i)-(\tau_0-\tau_3)(\boldsymbol{\sigma}^*\mathbf{S}_i)], \qquad (3)$$

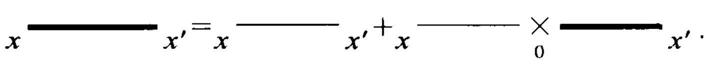
where V is the nonexchange part of the potential, $\boldsymbol{\rho}=(y,z)$, J is the exchange integral, and $\mathbf{S}_i$ is the spin of the magnetic atom at the point $\mathbf{r}_i=(0,\boldsymbol{\rho}_i)$. In this case we are ignoring the quantum nature of the spin $\mathbf{S}_i$, treating it as a classical vector. This approach is valid in the limit $S\gg 1$, $J\to 0$, with $SJ=\text{const}$ $(S=|\mathbf{S}_i|)$.[8] Discarding the self-consistency condition, we adopt the approximation $\Delta(\mathbf{r})=\Delta_\infty$ for the pairing potential. The legitimacy of this approximation is demonstrated below.

Using this approximation, we can easily transform from (2) to the integral equation

$$G_\omega(x,x';\mathbf{p}_\perp,\mathbf{p}'_\perp)=G^{(0)}_\omega(x,x';\mathbf{p}_\perp)\delta(\mathbf{p}_\perp-\mathbf{p}'_\perp)+\frac{1}{(2\pi)^2}G^{(0)}_\omega(x,0;\mathbf{p}_\perp)$$

$$\times\sum_i I_{\mathbf{S}_i}\exp(-i\mathbf{p}_\perp\boldsymbol{\rho}_i)\int d^2\mathbf{p}''_\perp\, G_\omega(0,x';\mathbf{p}''_\perp,\mathbf{p}'_\perp)\exp(i\mathbf{p}''_\perp\boldsymbol{\rho}_i), \qquad (4)$$

where we have taken Fourier transforms in the coordinates x and y. The Green's function $G^{(0)}_\omega(x,x';\mathbf{p}_\perp)$ in (4) gives us the solution of (2) at $J=0$. This Green's function is written out explicitly in. Ref. 5. With Eq. (4) we associate the following diagram, whose notation is obvious:



(5)

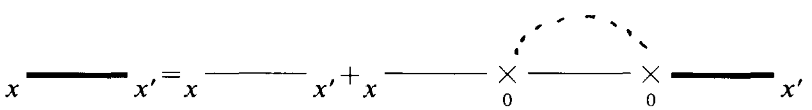
For simplicity we assume for the time being that the magnetic atoms are distributed at random in the plane of the barrier. If there is a ferromagnetic order, Eq. (4) can be solved immediately in the mean-field approximation for the spin system. It can also be described by means of diagram (5), but in this case expectation values of the Green's functions, $\langle G_\omega \rangle$ (these functions are diagonal in terms of the transverse momentum) would correspond to the heavy lines, and the factor would have to be associated with the cross.

$$\left\langle \sum_i I_{S_i} \exp(-i(\mathbf{p}_\perp - \mathbf{p}'_\perp))\rho_i] \right\rangle = (2\pi)^2 cJS\delta(\mathbf{p}_\perp - \mathbf{p}'_\perp)\tau_3\sigma_3,$$

where c is the concentration of moments. The function $\langle G_\omega \rangle$ is of course the same, aside from differences in the notation for the constants, as the Green's function of an $S/I(F)/S$ contact, which was found in Ref. 5 by a slightly different method. An analytic continuation of $\langle G_{-iE}(0, x; \mathbf{p}_\perp) \rangle$ in the limit of a low barrier transmission ($V \gg v_0$, JS, where v_0 is the Fermi velocity) has poles at the energies in (1), with $\alpha = \operatorname{sgn} J$ and

$$T_S(t) = \frac{(cJS)^2 v_0^2 t^2}{V^4}. \tag{6}$$

If the magnetic moments are in the paramagnetic phase (the expectation value of the spin at point $\mathbf{r}_i$ is zero) in the unaveraged equation represented by diagram (5), we must first distinguish a subclass of principal diagrams. Since the only nonzero contributions in an averaging over the angular variables come from even powers of the spin at the same point $\mathbf{r}_i$, we find, in the low-transmission limit,



(7)

The heavy lines here correspond to the average Green's functions $\langle G_\omega \rangle$, and the dashed line connects crosses corresponding to a single spin. In deriving (7) we ignored the diagrams which describe the averaging of powers of the spin greater than the second. We also ignored diagrams with intersecting dashed lines, since their contribution is small to the extent that the ratio JS/V is small (the unperturbed Green's functions $G_\omega^{(0)}$ with one coordinate in the plane of the barrier contain a factor V^{-1}). For the same reason, a thickening of the line between the two crosses in the second term in (7) contributes nothing new (the self-consistent approximation for the mass operator).

It is an elementary matter to solve Eq. (7). Of interest for our purposes is the quantity

$$\langle G_\omega(0,x;\mathbf{p}_\perp)\rangle = \left[\tau_0\sigma_0 - (2\pi)^{-1} cJ^2S^2 G_\omega^{(0)}(0,0;\mathbf{p}_\perp) \times \int d^2\mathbf{p}'_\perp \, G_\omega^{(0)}(0,0;\mathbf{p}'_\perp)\right]^{-1} G_\omega^{(0)}(0,x;\mathbf{p}_\perp).$$

The energy of the localized state is found from the equation

$$\det\left[\tau_0\sigma_0 - (2\pi)^{-1} cJ^2S^2 G_{-iE}^{(0)}(0,0;\mathbf{p}_\perp) \int d^2\mathbf{p}'_\perp \, G_{-iE}^{(0)}(0,0;\mathbf{p}'_\perp)\right] = 0. \tag{8}$$

Substitution into (8) of the expressions for the unperturbed Green's functions from Ref. 5, taken in the semiclassical approximation $[|t|\gg\{(\Delta_\infty/E_F)^{1/2}]$, leads to

$$E_{0\pm}(t)=\Delta_\infty[1-2T'_S(t)],\quad |t|\gg\{(\Delta_\infty/E_F)^{1/2},[T'_s(1)]^{1/3}\},\tag{9}$$

where

$$T'_S(t)=\frac{1}{3}\frac{cJ^2S^2v_0^2p_0^2}{V^4}\tag{10}$$

(p_0 is the Fermi momentum). As expected, in the absence of an average exchange field, for the given sign of J a localized state is formed for both orientations of the quasiparticle spin. It is clear from the spatial behavior of $\langle G_{-iE}\rangle$ that the smooth exponential decay of state (9) in the interior of the superconducting banks occurs at distances on the order of $[|t|/T'_S(1)]^{1/2}\xi_0$ from the barrier (ξ_0 is the BCS coherence length). Following the arguments of Ref. 5, we can show that the condition $|t|\gg[T'_S(1)]^{1/3}$ allows us to ignore the corrections to (9) for the dependence of the pairing potential on the coordinates. [The self-consistent solution $\Delta(x)$ near the barrier is characterized by a symmetric dip with an effective width $\sim 2\xi_0$ and a depth $\sim 2T'_S(1)\Delta_\infty$].

If the concentration of magnetic moments is close to one, the exchange part of the tunneling transmission of the paramagnetic barrier along the direction of the normal, $T'_S(1)$, is on the order of the exchange part of the tunneling transmission $T_S(1)$ of the barrier in an $S/I(F)/S$ contact (Eq. (6)]. However, the angular distribution of $T'_S(t)$ is different from that of $T_S(t)$. Equations (9) and (10) [and also (1) and (6)] actually remain valid when the magnetic moments form a regular lattice in the plane of the barrier, but we would have to replace c by $1/\Omega$, where Ω is the area of the unit cell.

Although the energy of the state in (9) is comparable in order of magnitude to the energy of polarized state (1) (for $|t|\simeq 1$), this state would be more difficult to observe in a tunneling experiment. The reason is that the ratio of the second term on the right side of Eq. (7) to the corresponding term in Eq. (5) describing an $S/I(F)/S$ contact is on the order of JS/V. This circumstance may mean that on the curves of the differential conductivity the peak corresponding to state (9) is much smaller than that corresponding to state (1). We wish to stress, however, that a correct comparison with experimental results would be possible only on the basis of a systematic time-dependent theory. That problem goes beyond the scope of the present letter.

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