Collective order-parameter modes in superconducting UPt_3

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(Submitted 29 June 1993; resubmitted 8 July 1993)
Pis'ma Zh. Eksp. Teor. Fiz. **58**, No. 3, 220–224 (10 August 1993)

The spectrum of collective modes in the limit $q=0$ in superconducting UPt_3 is calculated in the model of p-wave pairing proposed by Machida and Ozaki [Phys. Rev. Lett. **66**, 3293 (1991)]. A Lagrangian formulation of the Ginzburg–Landau theory is used. In a zero field the spectrum consists of (1) four modes at temperatures $T<T_{c2}$ and (2) two doublets and one singlet at $T_{c1}>T>T_{c2}$.

The double superconducting transition in UPt_3 (see the review[1]) is convincing evidence that Cooper pairing is not a standard property of the heavy-fermion superconductors. We are thus led to ask about the symmetry of the superconducting state. Collective modes of the order parameter were proposed for this purpose in Ref. 2. In ordinary superconductors, with s-wave pairing, these modes are not observed (primarily because of the strong interaction with plasma modes). In the case of p-wave pairing as in ^3He, however, collective modes exist (although there is a dissipation, which seriously hinders an experimental study of these modes). These modes exist because the order parameter is transformed by a multidimensional representation of the symmetry group of the superfluid phase.[3] For a corresponding reason we might expect collective oscillations of the order parameter in unconventional superconductors. Hirschfeld *et al.*[2] used a matrix kinetic equation and showed that, in the case of the 2D E_1 representation of the hexagonal group, collective modes can be observed in the absorption of electromagnetic waves, despite the screening currents and the excitation of quasiparticles near points and lines at which the order parameter vanishes. The theoretical prediction of the magnitude of the effects associated with existence of collective modes depends on the characteristics of the model considered. We should thus bear in mind that there are different opinions about the nature of the superconductivity of the heavy-fermion alloys. For example, a model used in Refs. 4 and 5 for the superconducting transition in UPt_3 is based on the presence of defects (and the proximity of the crystal to a transition to an fcc structure). The more conventional approach of Refs. 6 and 7 is based on triplet pairing. In the present letter we wish to show that the model of Ref. 6 imposes some severe restrictions on the spectrum of collective modes, so that if the latter can in fact be observed it would become possible to draw some conclusions about the symmetry of the superconducting state in UPt_3.

In the model of Ref. 6 the order parameter is a complex three-dimensional vector $\eta=(\eta_1, \eta_2, \eta_3)$ which is related to the gap by the equation

$$\Delta(k)=\sum_{i=1}^{3}(i\sigma_2\sigma_i)l(k)\eta_i,$$

where σ_i are the Pauli matrices. The orbital part of the gap, $l(k)$, is odd under spatial reflection. It belongs to a 1D representation of the D_{6h}, A_{2u}, B_{1u}, or B_{2u} type. A very important point is that there is an antiferromagnetic order, which is determined by the moment M, which lies in the basal plane and is directed along an axis in such a way that the pairing between the superconducting and antiferromagnetic systems is determined by the energy

$$f_c=-\gamma M^2(2|\eta_1|^2-|\eta_2|^2-|\eta_3|^2). \tag{1}$$

The complete Ginzburg–Landau energy in the absence of an external field is given by

$$f_{\mathrm{GL}}=\alpha_0(T-T_0)\eta\eta^*+\frac{1}{2}\beta_1(\eta\eta^*)^2+\frac{1}{2}\beta_2|\eta^2|^2+f_c, \tag{2}$$

where T_0 is the temperature of the superconducting transition (the antiferromagnetic order is being ignored here). We assume $\gamma>0$ and $\beta_1>0$ in accordance with the stability condition. According to Ref. 6, this moment causes a splitting of the superconducting transition. Equations (1) and (2) are invariant under rotations around the x axis, the phase transformation $\eta\rightarrow e^{i\phi}\eta$, and time reversal, which corresponds to complex conjugation. The symmetry group is therefore

$$G_0=SO(2)_x\times U(1)\times T_R \tag{3}$$

(in addition to the discrete symmetries mentioned above). To a large extent, this circumstance determines the form of the minima of the Ginzburg–Landau energy given by Eqs. (1) and (2). Since the group G_0 is, from the topological standpoint, a torus (taken twice), the minimum may be, provided that there are no additional degeneracies, a point, a circle, or a torus again (see the general theory in Ref. 3).

The minima of functional (2) satisfy the necessary conditions for an extremum:

$$\frac{\partial F}{\partial\eta_i}=0,\quad \frac{\partial F}{\partial\eta_i^*}=0,\quad i=1,2,3. \tag{4}$$

In more detail, Eqs. (4) are

$$\Sigma\begin{bmatrix}\eta\\ \eta^*\end{bmatrix}=0, \tag{5}$$

where Σ_{ij} is given by

$$\begin{aligned}
\Sigma_{11}=\Sigma_{44} &= d_1 &&= A+\beta_1(\eta\cdot\eta^*)-3\gamma M^2, &&\\
\Sigma_{ii} &= d_2 &&= A+\beta_1(\eta\cdot\eta^*), && i=2,3,5,6,\\
\Sigma_{i+3,i} &= z &&= \beta_2(\eta\cdot\eta), && i=1,2,3,\\
\Sigma_{i,i+3} &= z^* &&= \beta_2(\eta\cdot\eta)^*, && i=1,2,3,
\end{aligned}$$

and A by

$$A=\alpha(T-T_0)+\gamma M^2. \tag{6}$$

From Eq. (5) we see that we have $\det\Sigma=0$. Hence

$$\det\ \Sigma=(d_1^2-|z|^2)(d_2^2-|z|^2)^2,$$

and the necessary conditions for an extremum are

$$d_1=\pm|z|,\quad d_2=\pm|z|. \tag{7}$$

Analysis of these equations shows that the minima of Ginzburg–Landau energy (2) are given by the general formulas

$$\eta_1=re^{i\phi},\quad \eta_2=iae^{i\phi}\cos\theta,\quad \eta_3=iae^{i\phi}\sin\theta, \tag{8}$$

where the parameters r and a satisfy the conditions

$$a^2=\begin{cases}0 & \text{if } r^2\leqslant\frac{3}{2}\gamma M^2/\beta_2,\\ r^2-\frac{3}{2}\gamma M^2/\beta_2 & \text{otherwise.}\end{cases} \tag{9}$$

There is thus a branch point for the structure of the minimum of the Ginzburg–Landau energy, given by Eqs. (1) and (2). The first transition from a normal phase to a superconducting phase occurs at the temperature

$$T_{c1}=T_0+\frac{2\gamma M^2}{\alpha}. \tag{10}$$

The second transition, also of second order, occurs at the temperature

$$T_{c2}=T_{c1}-\frac{3\gamma M^2\beta_1}{2\alpha\beta_2}, \tag{11}$$

which corresponds to this branch point $a=0$, $r^2=\frac{3}{2}\gamma M^2/\beta_2$. In the temperature interval $T_{c1}\geqslant T\geqslant T_{c2}$ the minimum is a circle; below T_{c2} it is a torus. The expression which we have derived for the temperature of the second transition, (11), and the form of the order parameter below T_{c2} are at odds with the results of Refs. 6 and 7. In particular, Ohmi and Machida[7] assert that the minimum is reached under the conditions $|\Im\eta|=|\Re\eta|$ and $\Im\eta\perp\Re\eta$. That assertion is correct only for very low temperatures, at which the antiferromagnetic order can be ignored.

The second transition thus generates a topological restructuring of the minimum of the Ginzburg–Landau energy, leading to substantial differences in the behavior of collective modes. Since we are interested primarily in the qualitative behavior of the spectrum in this letter, we can take the phenomenological approach which has been taken previously[8–10] to study superfluid ^{3}He, specifically, a time-dependent Ginzburg–Landau theory:

$$L=\Lambda\partial_t\delta\eta\partial_t\delta\eta^*-V(\delta\eta,\delta\eta^*).$$

Here Λ is an adjustable parameter, and V is an expansion of the Ginzburg–Landau energy at the points of the minimum in the order parameter, in which small terms of up to second order are retained. We ignore spatial gradients, considering the spectrum only for "$q=0$." We assume that the external field is turned off. An important point is that collective modes are oscillations near the equilibrium values of the order parameter determined by the minimum of the Ginzburg–Landau energy. This minimum is degenerate, and we should consider only oscillations in directions which are transversal with respect to it (in order-parameter space). We focus on the low-temperature case ($T \leqslant T_{c2}$). The transversal variations are specified by

$$\delta\eta_1=u_1+iau_2,\quad \delta\eta_2=ru_2+iu_3,\quad \delta\eta_3=u_4.$$

Solving the equations for small oscillations with respect to variations in η_{123} for the Lagrangian written above, we find the following expressions for the frequencies of collective modes:

$$\omega_{1,3}^2=\frac{k+l}{2}\pm\sqrt{\frac{(k-l)^2}{4}+4a^2r^2(\beta_1-\beta_2)^2}, \tag{12}$$

$$\omega_2^2=A-3\gamma M^2a^2+\beta_1(r^2+a^2)+\beta_2\left[8a^2r^2+\frac{(r^2-a^2)^2}{r^2+a^2}\right], \tag{13}$$

$$\omega_4^2=A+(\beta_1+\beta_2)r^2+a^2(\beta_1-\beta_2), \tag{14}$$

where A is given by (6),

$$k=A-3\gamma M^2+3r^2(\beta_1+\beta_2)+a^2(\beta_1-\beta_2),$$

$$l=A+r^2(\beta_1-\beta_2)+3a^2(\beta_1+\beta_2).$$

Going through similar calculations, we find the spectrum for temperatures $T_{c2}\leqslant T \leqslant T_{c1}$, at which the minimum is a circle:

$$\omega_1^2=A-3\gamma M^2+3(\beta_1+\beta_2)r^2, \tag{15}$$

$$\omega_{2,4}=A+(\beta_1+\beta_2)r^2, \tag{16}$$

$$\omega_{3,5}=A+(\beta_1+\beta_2)r^2. \tag{17}$$

In other words, we find two doublets and one singlet. It follows from Eqs. (12)–(14) that in the limit $T\rightarrow T_{c2}$, in the low-temperature region, the frequencies ω_1, ω_2 and ω_4, ω_3 become the corresponding frequencies given by Eqs. (15)–(17) for the high-temperature phase. Significantly, at temperatures $T\geqslant T_{c2}$, the frequencies $\omega_{2,4}$ of collective modes are determined by the antiferromagnetic order, since we have, by virtue of Eqs. (7),

$$\omega_2^2=\omega_4^2=3\gamma M^2,\quad \omega_1^2=2r^2(\beta_1+\beta_2),\quad \omega_{3,5}^2=\alpha(T-T_0)+\gamma M^2+r^2(\beta_1-\beta_2). \tag{18}$$

We see that the high- and low-temperature phases differ significantly in the behavior of the spectrum of collective modes; the existence of a degeneracy above T_{c2} is the most striking feature in this regard. In a magnetic field, this degeneracy should lead to a splitting of the spectrum similar to that which occurs in ^{3}He–B (see Ref. 10 and the bibliography there). The specific equations derived above for the frequencies are only "qualitatively" correct, because of the approximate nature of the formalism which was used (see the discussion of the Lagrangian approach in Ref. 10). However, the degeneracy mentioned above is of a more fundamental nature, because of the symmetry described by the G_0 group. At low temperatures, the order-parameter space is a torus, on which the symmetry group acts without fixed points. In directions transversal to the torus there are no other symmetries and, correspondingly, no degeneracy. Above T_{c2}, the order-parameter space is a circle, on which the subgroup $U(1)$ of the G_0 group acts--without fixed points--while $SO(2)_x$ does not act at all [or, in accordance with the standard terminology, the $U(1)$ symmetry is broken, while $SO(2)_x$ is not]. Consequently, for an arbitrary minimum point we can choose a five-dimensional space N which is transversal with respect to the circle minimum in such a way that the $SO(2)_x$ subgroup acts on N, leaving P fixed. As a result, N is split into two planes and one straight line [since the irreducible real representations of $SO(2)$ are one- and two-dimensional]. From the standpoint of the spectrum, this situation corresponds to two doublets and one singlet.

The singlet corresponds to a one-dimensional scalar representation of the symmetry group G_0, so it may not be observable, because of the interaction with charge currents, as in s-wave-pairing superconductors. Significantly, the ω_1^2 mode represents a possible way out of this situation; it does not contain antiferromagnetic contributions γM^2. The structure of the degeneracy is thus closely related to the antiferromagnetic order.

I wish to thank P. Wölfle for stimulating discussions and Karlsruhe University for hospitality. This study had financial support from the DAAD, and part of it was carried out while the author was working temporarily at Karlsruhe University. I am indebted to G. Volovik and M. Zhitomirskiĭ for calling my attention to errors in the original version of this paper. I am also indebted to M. Kagan for many useful discussions of the physics of heavy fermions.

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Translated by D. Parsons