

On a time dependence of physical constants

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Analysis of the wavelengths of lines of molecular hydrogen observed in the spectrum of the quasar PKS 0528-250 yields an estimate of an upper limit on a hypothetical change in the ratio of the electron mass to the proton mass, $\mu = m_e/m_p$, at a cosmological time scale. This upper limit on the rate of change of μ is $|\dot{\mu}/\mu| \leq 3 \times 10^{-13} \text{ yr}^{-1}$. This is two orders of magnitude lower than the value $\dot{\mu}/\mu = (2.3 \pm 0.3) \times 10^{-11} \text{ yr}^{-1}$ found from laboratory measurements [Yu. S. Domnin *et al.*, JETP Lett. **43**, 212 (1986)].

1. Domnin *et al.*¹ have reported the experimental observation of a change in the ratio of the frequencies of quantum standards in the optical and rf ranges over a time of 4.7 yr. They linked this change with a possible time variation of fundamental physical constants, in particular, a variation in the ratio of the electron mass to the proton mass, $\mu = m_e/m_p$. Their results correspond to a rate of change of μ equal to $\dot{\mu}/\mu = (2.3 \pm 0.3) \times 10^{-11} \text{ yr}^{-1}$. It follows that over a time comparable to the age of universe, $t_0 \approx 1.5 \times 10^{10} \text{ yr}$, μ should have increased by several tens of percent (if μ is assumed to be a linear function of cosmological time). Such an increase in μ could be observed easily through an analysis of astrophysical observations of extragalactic objects whose spectra was formed $\approx 10^{10} \text{ yr}$ ago. Particularly convenient for this purpose are the spectra of quasars, which contain absorption lines of molecular hydrogen, and the ratio of the wavelengths of molecular electronic–vibrational lines is known^{2,3} to depend on μ . At present, the lines of H_2 can be measured most reliably in the spectrum of the quasar PKS 0528-250,^{4,5} in which absorption lines of Lyman and Werner bands of H_2 have been identified, with a redshift $z=2.811$. According to the standard cosmological model,⁶ such a redshift corresponds to the epoch $t=t_0[1-(1+z)^{-3/2}] \approx 13 \times 10^9 \text{ yr}$ ago.

Our purposes in the present study were to find independent astrophysical estimates of the value of $\dot{\mu}/\mu$ through an analysis of H_2 absorption lines in the spectrum of the quasar PKS 0528-250 and to compare these estimates with laboratory measurements. Although high-precision laboratory measurements are much more accurate than astronomical estimates, the long time interval over which distant cosmological objects have been observed makes it possible to find estimates of the rate of change of μ which are manifold smaller than those amenable to direct laboratory checks.

2. Let us consider electronic-vibrational-rotational transitions of molecular hydrogen. The energy of the quantum states of a molecule can, as we know, be written as a Born–Oppenheimer expansion⁷ in even powers of the parameter $\chi = (m_e/M)^{1/4}$:

$$E = E_{\text{el}} + \chi^2 E_{\text{vib}} + \chi^4 E_{\text{rot}} + \dots, \quad (1)$$

TABLE I. The coefficients $k_1/10^{-3}$ for lines of the H₂ Lyman series.

Line	$R(0)$	$R(1)$	$P(1)$	$R(2)$	$P(2)$	$R(3)$	$P(3)$
$L\ 0-0$	7.79 ₁	8.25 ₁	9.53 ₁	9.59 ₂	11.70 ₂	11.79 ₂	14.70 ₂
$L\ 1-0$	0.72 ₁	1.22 ₁	2.38 ₁	2.62 ₂	4.54 ₂	4.89 ₂	7.54 ₂
$L\ 2-0$	-5.79 ₁	-5.24 ₂	-4.18 ₁	-3.81 ₂	-2.05 ₂	-1.50 ₃	0.94 ₂
$L\ 3-0$	-11.77 ₂	-11.20 ₂	-10.21 ₂	-9.73 ₃	-8.10 ₂	-7.39 ₅	-5.13 ₃
$L\ 4-0$	-17.27 ₃	-16.67 ₄	-15.76 ₂	-15.18 ₇	-13.67 ₃	-12.8 ₁	-10.72 ₅
$L\ 5-0$	-22.31 ₅	-21.69 ₈	-20.84 ₄	-20.2 ₁	-18.77 ₅	-17.8 ₂	-15.84 ₉
$L\ 6-0$	-26.9 ₁	-26.3 ₁	-25.49 ₇	-24.7 ₂	-23.4 ₁	-22.3 ₃	-20.5 ₂
$L\ 7-0$	-31.1 ₂	-30.4 ₂	-29.7 ₁	-28.9 ₄	-27.7 ₂	-26.4 ₅	-24.8 ₃
$L\ 8-0$	-34.9 ₃	-34.3 ₄	-33.6 ₂	-32.7 ₆	-31.6 ₃	-30.2 ₈	-28.7 ₄
$L\ 9-0$	-38.5 ₄	-37.8 ₆	-37.2 ₃	-36.2 ₉	-35.2 ₄	-34 ₁	-32.3 ₆
$L\ 10-0$	-41.8 ₇	-41.1 ₉	-40.5 ₅	-40 ₁	-38.5 ₇	-37 ₂	-35.7 ₉

where M is the total mass of the nuclei of the molecule. The frequency ratio of any two lines of the H₂ molecule observed at epoch z is then

$$\left(\frac{\nu_1}{\nu_2}\right)_z = \frac{a_1 + b_1 \sqrt{\mu_z/\mu} + d_1(\mu_z/\mu) + \dots}{a_2 + b_2 \sqrt{\mu_z/\mu} + d_2(\mu_z/\mu) + \dots}, \quad (2)$$

where a_i , b_i , and d_i are numerical constants ($a_i \gg b_i \gg d_i$, $i=1, 2$); $\mu = m_e/m_p$ is the current value of the ratio of the electron mass to the proton mass; and $\mu_z = (m_e/m_p)_z$ is the corresponding ratio at epoch z .

To find the relative change in the frequencies of H₂ lines as a function of the variations in the parameter μ , it is convenient to use a Danham expansion for the energy of vibrational-rotational levels:

$$E = \sum Y_{nm} \left(v + \frac{1}{2}\right)^n [J(J+1)]^m. \quad (3)$$

The Danham constants Y_{nm} here depend on μ ; this circumstance has been utilized in calculations of the energies of isotopically substituted molecules.² We express this dependence in terms of the parameter $\xi = (\mu_z - \mu)/\mu$, which characterizes the relative change in μ :

$$Y_{nm}(\xi) = Y_{nm}(0) (1 + \xi)^{-n/2 - m}. \quad (4)$$

We find the sensitivity of the i th molecular line to a change in the ratio m_e/m_p in terms of the sensitivity coefficient:

$$k_i = (d\nu_i/d\xi)/\nu_i. \quad (5)$$

The ratio of the frequency ratio of two electronic-vibrational-rotational lines at epoch z to the ratio of the frequencies of the same lines at the current epoch can then be written

$$\left(\frac{\nu_1}{\nu_2}\right)_z \bigg/ \left(\frac{\nu_1}{\nu_2}\right) = 1 + \xi K_{12} + O(\xi^2), \quad (6)$$

TABLE II.

No.	$\lambda_{\text{obs}}, \text{\AA}$	$\lambda_{\text{lab}}, \text{\AA}$	z_i	Line	$\xi, \%$	$2\sigma_\xi, \%$
1	4225.06	1108.70	2.81082	L 0-0 R(1)	-0.01	0.12
2	4230.50	1110.12	2.81085	L 0-0 P(1)	-0.04	0.11
3	4162.77	1092.27	2.81112	L 1-0 R(0)	-0.29	0.19
4	4164.48	1092.77	2.81094	L 1-0 R(1)	-0.14	0.15
5	4169.99	1094.19	2.81103	L 1-0 P(1)	-0.21	0.15
6	4105.15	1077.24	2.81080	L 2-0 R(0)	-0.05	0.18
7	4107.32	1077.76	2.81098	L 2-0 R(1)	-0.17	0.19
8	4121.28	1081.33	2.81131	L 2-0 P(2)	-0.45	0.31
9	4057.10	1064.71	2.81052	L 3-0 P(1)	-0.01	0.35
10	3998.92	1049.46	1.81045	L 4-0 R(0)	-0.28	0.36
11	4001.53	1059.00	2.81098	L 4-0 R(1)	-0.02	0.23
12	4005.43	1051.14	2.81056	L 4-0 P(1)	-0.19	0.30
13	4007.03	1051.55	2.81059	L 4-0 R(2)	-0.17	0.29
14	4014.04	1053.34	2.81077	L 4-0 P(2)	-0.10	0.20
15	3862.37	1013.50	2.81092	L 7-0 R(1)	0.03	0.14
16	3778.37	991.47	2.81088	L 9-0 R(0)	0.01	0.10
17	3780.76	992.07	2.81098	L 9-0 R(1)	0.08	0.11
18	3753.26	984.94	2.81065	L 10-0 P(2)	-0.17	0.10

where $K_{12} = k_1 - k_2$.

Table I shows the sensitivity coefficients k_i which we have calculated for transitions from low-lying rotational levels ($v''=0, J''=0, 1, 2, 3$) of the $X^1 \Sigma_g^+$ ground state to vibrational-rotational levels of the $B^1 \Sigma_u^+$ state ($v' = 0, 1, \dots, 10$; R and P branches) of the H_2 molecule. In this calculation we borrowed molecular constants from Ref. 6. We also allowed for a weak ξ dependence of the electronic term T_e of the $B^1 \Sigma_u^+$ upper excited state of the H_2 molecule, in the form $T_e = p + q\xi$. Here p and q were calculated from the empirical data of Ref. 8 for H_2 and D_2 . These quantities were found to be $91\,700.0$ and 5.6 cm^{-1} , respectively. The subscripts on each value of k_i in Table I give the error within which the last significant digit was determined [for example, $k_{p(3)}(L0-0) = (14.70 \pm 0.02) \times 10^{-3}$]. These errors were calculated from the errors in the measurement of molecular constants cited in Ref. 8.

As expected, the changes in the frequency of the corresponding transition increase with increasing index of the upper vibrational level, v' . For the various lines of the H_2 Lyman band the coefficients k_i have different signs; i.e., as μ varies, the wavelengths of some of the lines increase, while those of others decrease, depending on the sign of ξK_{12} (the same type of change in wavelength is observed between the lines of H_2 and D_2).

3. We turn now to the astrophysical data. Table II lists measured wavelengths of those absorption features which Foltz *et al.*⁵ identified with unblended lines of molecular hydrogen. In this table the uncertainty in the actual position of a molecular line can reach $\pm (0.2-0.4) \text{\AA}$, since the resolution of the spectrum of PKS 0528-250 was still not high enough to completely eliminate possible hidden blends. As the laboratory

wavelengths we used the calculated values of λ_{lab} calculated in Ref. 5 for a synthetic H_2 spectrum [at a ray concentration $N(\text{H}_2) = 10^{18} \text{cm}^{-2}$, an excitation temperature $T_{\text{ex}} = 100 \text{ K}$, and a Doppler parameter $b = 5 \text{ km/s}$], which was convolved with the instrumental function of the spectrograph. The mean square errors in the laboratory wavelengths can reach $\pm 0.1 \text{ \AA}$ in this case. The fourth column of Table II shows the redshifts of the H_2 lines: $z_i = \lambda_{\text{obs}}/\lambda_{\text{lab}} - 1$. Working from these data, we can estimate the parameter ξ from the relation

$$(1+z_i)/(1+z_0) = 1 + \xi(k_o - k_i). \quad (7)$$

The subscript i here corresponds to the index of the line in Table II; it runs over the values from 1 to 18. The subscript 0 refers to a reference line. This reference line can be any of the same 18 lines ($i \neq 0$). Through a linear regression analysis we find estimates of the parameter ξ and its error σ_ξ . Since the choice of reference line is arbitrary here, we carried out estimates of this sort systematically, using each H_2 line as the reference line. The values calculated for ξ and their errors are listed in the last column of Table II. The distribution of errors $2\sigma_\xi$ turns out to be Gaussian with a mean value of 0.20% and a standard deviation of 0.09%. For the final value of the parameter ξ we can thus choose the results of any of the regression analyses whose errors lie in the interval $(0.20 \pm 0.09)\%$. It turns out that there are 12 such versions. From this group we selected the result with the reference line $L1-0 R(0)$, which gives us an estimate of an upper bound on the value of ξ : $|\xi| \leq 0.5\%$ with a probability of 0.95. The estimate $|\xi| \leq 0.02\%$ found in Ref. 5 on the basis of only one pair of lines, $L 2-0 P(2)$ and $L 10-0 P(2)$, without a detailed analysis of the $\lambda(\xi)$ dependence, appears to us to be unjustifiably high.

Consequently, if the assumption that the ratio of the electron mass to the proton mass varies linearly with cosmological time is valid, then our results correspond to $|\dot{\mu}/\mu| \leq 3 \times 10^{-13} \text{ yr}^{-1}$. This figure is two orders of magnitude smaller than the value found for $\dot{\mu}/\mu$ in the laboratory experiment of Ref. 1.

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