Supersymmetry and principle of action at a distance

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A combination of supersymmetry and the principle of action at a distance, which was used previously by Wheeler and Feynman for an alternative description of classical Maxwellian electrodynamics, is analyzed. It is shown that this combination opens up the possibility of working from differential elements of the spinor coordinates of the world lines of particles in superspace to construct vector and spinor fields which satisfy Maxwell's equations and Weyl's equations with currents.

In the Wheeler–Feynman theory,[1] the electromagnetic field a_μ arises as a secondary effect, constructed from the world coordinates of relativistic charged particles. When there are two particles moving along world lines with coordinates $x^\mu(t)$ and $y^\mu(\tau)$, the potential a^μ of the effective field acting on particle x^μ can be written as the relativistically invariant and reparametrization-invariant integral[1]

$$a^\mu(x)=e\int d\tau\dot{y}^\mu(\tau)\delta(s_0^2), \tag{1}$$

where $\delta(s_0^2)$ is the Dirac δ-function, whose argument is the square of the relativistic interval $s_0^\mu\equiv x^\mu-y^\mu(\tau)$ between the particles, and e is the electric charge of the particles. The effective field in (1) satisfies Maxwell's equations and the Lorentz gauge condition

$$\partial^\mu f_{\mu\nu}(x)=-4\pi j_\nu(x),\quad \epsilon_{\mu\nu\rho\sigma}\partial^\nu f^{\rho\sigma}(x)=0,\quad \partial^\mu a_\mu(x)=0, \tag{2}$$

while the current $j^\mu(x)$ is given by the standard expression

$$j^\mu(x)=e\int dy^\mu\delta^{(4)}(s_0). \tag{3}$$

We would like to analyze the Wheeler–Feynman approach along with the principle of supersymmetry.[2–4] To do this, it is natural to switch from the world coordinates of the observation point x^μ to the superspace coordinates $z^M=(x^\mu,\ \theta^\alpha,\bar{\theta}_{\dot\alpha})$, introducing several additional Grassmann spinor coordinates $(\theta^\alpha,\ \bar{\theta}_{\dot\alpha})$ in the Weyl representation. Correspondingly, we should transform from the world coordinates of the source particle to supercoordinates $\zeta^M=[y^\mu(\tau),\ \xi^\alpha(\tau),\ \bar{\xi}_{\dot\alpha}(\tau)]$. The supersymmetry transformations for the coordinates z^M and ζ^M are

$$\delta x^\mu=i\theta\sigma^\mu\bar{\epsilon}-i\epsilon\sigma^\mu\bar{\theta},\quad \delta\theta^\alpha=\epsilon^\alpha,\quad \delta\bar{\theta}_{\dot\alpha}=\bar{\epsilon}_{\dot\alpha},$$

$$\delta y^\mu=i\xi\sigma^\mu\bar{\epsilon}-i\epsilon\sigma^\mu\bar{\xi},\quad \delta\xi^\alpha=\epsilon^\alpha,\quad \delta\bar{\xi}_{\dot\alpha}=\bar{\epsilon}_{\dot\alpha}. \tag{4}$$

To find a supersymmetric generalization of representation (1), it would thus be sufficient to replace the velocities $\dot{y}^\mu(\tau)$ and the intervals s_0^μ by corresponding quantities which are invariant under supersymmetry transformations (4). The simplest generalization of this type is one using the replacement

$$\dot{y}^\mu \mapsto \omega^\mu_\tau = \dot{y}^\mu - i(\dot{\xi}\sigma^\mu\bar{\xi} - \xi\sigma^\mu\dot{\bar{\xi}}),$$

$$s_0^\mu \mapsto s^\mu = x^\mu - y^\mu - i(\theta\sigma^\mu\bar{\xi} - \xi\sigma^\mu\bar{\theta}). \tag{5}$$

In this letter, however, we will take another approach to generalize the velocities and intervals—an approach which sheds some light on the physical role played by the spinor coordinates of superspace. We implement this proposed supersymmetric generalization by making the substitutions

$$\dot{y}^\mu \mapsto \dot{\xi}^\alpha,\ \dot{\bar{\xi}}^{\dot{\alpha}},\quad e \mapsto \mu,$$

$$s_0^\mu \mapsto s_L^\mu = x_L^\mu - y_R^\mu - 2i\theta\sigma^\mu\bar{\xi},\quad s_R^\mu = (s_L^\mu)^* = x_R^\mu - y_L^\mu + 2i\xi\sigma^\mu\bar{\theta}, \tag{6}$$

where x_L^μ and y_L^μ are the coordinates in the left chiral basis,[3] given by

$$x_L^\mu \equiv x^\mu + i\theta\sigma^\mu\bar{\theta},\quad y_L^\mu \equiv y^\mu + i\xi\sigma^\mu\bar{\xi}, \tag{7}$$

and x_R^μ and y_R^μ are the complex conjugate coordinates in the right chiral basis,

$$x_R^\mu = (x_L^\mu)^* = x^\mu - i\theta\sigma^\mu\bar{\theta},\quad y_R^\mu = (y_L^\mu)^* = y^\mu - i\xi\sigma^\mu\bar{\xi}. \tag{8}$$

Substituting (6) into integral representation (1), we obtain chiral superfield spinor potentials:

$$A^\alpha = \mu\int d\tau\dot{\xi}^\alpha\delta(s_R^2),\quad \bar{A}^{\dot{\alpha}} = \mu\int d\tau\,\dot{\bar{\xi}}^{\dot{\alpha}}\delta(s_L^2), \tag{9}$$

where μ is a constant with the dimensionality of a length in the system ($h=c=1$). By virtue of the chiral nature of the superfields A^α and $\bar{A}_{\dot{\alpha}}$,

$$D_\alpha A_\beta \equiv \left(\frac{\partial}{\partial\theta^\alpha} + i(\sigma^\rho\bar{\theta})_\alpha\partial_\rho\right)A_\beta = 0,$$

$$\bar{D}_{\dot{\alpha}}\bar{A}_{\dot{\beta}} \equiv \left(-\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i(\theta\sigma^\rho)_{\dot{\alpha}}\partial_\rho\right)\bar{A}_{\dot{\beta}} = 0,$$

the superfield conditions fixing the $U(1)$ gauge hold automatically:

$$D^\alpha A_\alpha = 0,\quad \bar{D}_{\dot{\alpha}}\bar{A}^{\dot{\alpha}} = 0. \tag{10}$$

These conditions play the same role as is played by the Lorentz gauge condition in the Wheeler–Feynman theory. Conditions (10) leave some freedom in the superfield gauge transformations

$$A'_\mu = A_\mu + \partial_\mu\Lambda,\quad A'_\alpha = A_\alpha + D_\alpha\Lambda,\quad \bar{A}'_{\dot{\alpha}} = A_\alpha + \bar{D}_{\dot{\alpha}}\Lambda, \tag{11}$$

which are characterized by the real scalar superfield $\Lambda(x, \theta, \bar{\theta})$, which is bounded by the conditions

$$D^\alpha D_\alpha\Lambda = 0,\quad \bar{D}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}}\Lambda = 0. \tag{12}$$

The superpotentials A^{α} and $\bar{A}_{\dot{\alpha}}$ introduced above [see (9)] automatically satisfy the first two equations, $F_{\alpha\beta}=F_{\dot{\alpha}\dot{\beta}}=0$, of the set of connections imposed on the superfield field strengths in order to eliminate nonphysical component fields.[3] From the third equation,

$$F_{\alpha\dot{\beta}}\equiv D_{\alpha}\bar{A}_{\dot{\beta}}+\bar{D}_{\dot{\beta}}A_{\alpha}+2i\sigma^{\mu}_{\alpha\dot{\beta}}A_{\mu}=0, \tag{13}$$

we find an integral representation for the vector superfield potential $A_{\mu}(x,\theta,\bar{\theta})$:

$$A^{\mu}=-\mu\int d\tau\Big(\bar{\xi}\tilde{\sigma}_{\mu\rho}(\bar{\theta}-\bar{\xi})\partial^{\rho}\delta(s_L^2)+(\theta-\xi)\sigma_{\rho\mu}\dot{\xi}\partial^{\rho}\delta(s_R^2)\Big)$$
$$+\frac{\mu}{2}\int d\tau\Big(\dot{\bar{\xi}}(\bar{\theta}-\bar{\xi})\partial^{\mu}\delta(s_L^2)+(\theta-\xi)\dot{\xi}\partial^{\mu}\delta(s_R^2)\Big). \tag{14}$$

We know from the standard superfield theory of gauge fields[3] that the superfield field strengths remaining after the imposition of the connections which we just listed can be expressed in terms of the chiral superfields W and $\bar{W}$, which are bounded by the additional conditions $D_{\alpha}\bar{W}_{\dot{\beta}}=\bar{D}_{\dot{\alpha}}W_{\beta}=0$ and $DW-\bar{D}\bar{W}=0$, by means of the following relations:

$$F_{\mu\alpha}=i\sigma_{\mu\alpha\dot{\beta}}\bar{W}^{\dot{\beta}},\quad F_{\mu\dot{\alpha}}=iW^{\beta}\sigma_{\mu\beta\dot{\alpha}},\quad F_{\mu\nu}=-\frac{1}{2}(\bar{D}\tilde{\sigma}_{\mu\nu}\bar{W}-D\sigma_{\mu\nu}W). \tag{15}$$

Using these relations and integral representations (9) and (14), we easily find expressions for the superfields W and $\bar{W}$ in terms of the spinor superpotentials in (9):

$$W^{\alpha}=\frac{i}{4}F_{\mu\dot{\alpha}}\tilde{\sigma}^{\mu\dot{\alpha}\alpha}=\frac{1}{8}\bar{D}_{\dot{\beta}}\bar{D}^{\dot{\beta}}A^{\alpha}+\frac{i}{2}\partial^{\dot{\alpha}\alpha}\bar{A}_{\dot{\alpha}},$$
$$\bar{W}^{\dot{\alpha}}=\frac{i}{4}\tilde{\sigma}^{\mu\dot{\alpha}\alpha}F_{\mu\alpha}=-\frac{1}{8}D^{\beta}D_{\beta}\bar{A}^{\dot{\alpha}}+\frac{i}{2}\partial^{\dot{\alpha}\alpha}A_{\alpha}. \tag{16}$$

The superfields W and $\bar{W}$ describe a (1/2, 1) multiplet which contains the auxiliary field $D(x)$ and the $U(1)$-invariant field strength $v_{\mu\nu}=\partial_{[\mu}a_{\nu]}$ of the Abelian vector field $a_{\mu}(x)=iA\mu(x,\theta,\bar{\theta})|_{\theta=\bar{\theta}=0}$:

$$v_{\mu\nu}(x)=-\frac{1}{2}(\bar{D}\tilde{\sigma}_{\mu\nu}\bar{W}-D\sigma_{\mu\nu}W)|_{\theta=\bar{\theta}=0}$$
$$=i\mu\int d\tau[\dot{\bar{\xi}}\tilde{\sigma}_{[\mu|\rho}\bar{\xi}\partial_{\nu]}\partial^{\rho}\delta(\tilde{s_L}^2)+\xi\sigma_{[\rho|\mu}\dot{\bar{\xi}}\partial_{\nu]}\partial^{\rho}\delta(\tilde{s_R}^2)]. \tag{17}$$

This multiplet also contains spinor Weyl fields $\lambda_{\alpha}(x)$ and $\bar{\lambda}_{\dot{\alpha}}(x)$:

$$\lambda^{\alpha}(x)\equiv iW^{\alpha}|_{\theta=\bar{\theta}=0}=-\frac{1}{2}\mu\int d\tau\,\dot{\bar{\xi}}_{\dot{\alpha}}\partial^{\dot{\alpha}\alpha}\delta(\tilde{s_L}^2),$$
$$\bar{\lambda}^{\dot{\alpha}}(x)\equiv-i\bar{W}^{\alpha}|_{\theta=\bar{\theta}=0}=\frac{1}{2}\mu\int d\tau\dot{\xi}_{\alpha}\partial^{\dot{\alpha}\alpha}\delta(\tilde{s_R}^2), \tag{18}$$

where the intervals $\tilde{s_L^\mu} \equiv s_L^\mu|_{\theta=\bar{\theta}=0} = x^\mu - y_R^\mu$ and $\tilde{s_R^\mu} \equiv s_R^\mu|_{\theta=\bar{\theta}=0} = x^\mu - y_L^\mu$ are independent of θ and $\bar{\theta}$.

The vector field (17) and the spinor fields (18) satisfy Maxwell's and Weyl's equations with currents:

$$\partial_\mu v^{\mu\nu}(x) = J^{(2)\nu}(x), \quad \epsilon_{\mu\nu\rho\sigma}\partial^\nu v^{\rho\sigma}(x) = 0,$$
$$\partial_{\alpha\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}}(x) = J_\alpha^{(1)}(x), \quad \partial^{\dot{\alpha}\alpha}\lambda_\alpha(x) = \bar{J}^{(1)\dot{\alpha}}(x). \tag{19}$$

The vector and spinor currents $J_\nu^{(2)}$, $J_\alpha^{(1)}$, and $\bar{J}_{\dot{\alpha}}^{(1)}$ on the right sides of (19) are given by

$$J_\mu^{(2)}(x) = -2\pi\mu i \int d\tau[\,\dot{\bar{\xi}}\,\tilde{\sigma}_{\mu\rho}\bar{\xi}\partial^\rho\delta^{(4)}(\tilde{s_L}) - \xi\sigma_{\rho\mu}\dot{\xi}\partial^\rho\delta^{(4)}(\tilde{s_R})\,],$$

$$J_\alpha^{(1)}(x) = 2\pi\mu \int d\tau\dot{\xi}_\alpha\delta^{(4)}(\tilde{s_R}), \quad \bar{J}_{\dot{\alpha}}^{(1)}(x) = -2\pi\mu \int d\tau\,\dot{\bar{\xi}}_{\dot{\alpha}}\delta^{(4)}(\tilde{s_L}). \tag{20}$$

Equations (19) and (20) are found by differentiating representations (17) and (18) and by using a supersymmetric generalization of the Dirac equation[5]

$$\Box\delta(\tilde{s_L}^2) = -4\pi\delta^{(4)}(\tilde{s_L^\mu}),$$

$$\Box\delta(\tilde{s_R}^2) = -4\pi\delta^{(4)}(\tilde{s_R^\mu}).$$

Correspondingly, for the auxiliary field $D(x) = -(1/4)(DW + \bar{D}\bar{W})|_{\theta=\bar{\theta}=0}$ we find the equation

$$D(x) = J^{(0)}(x), \quad J^{(0)}(x) \equiv 2\pi\mu i \int d\tau[\dot{\xi}\xi\delta^{(4)}(\tilde{s_R}) - \dot{\bar{\xi}}\bar{\xi}\delta^{(4)}(\tilde{s_L})]. \tag{21}$$

Equations (19)–(21) can be written as a common superfield potential for W and $\bar{W}$ from (16):

$$D^\alpha W_\alpha + \bar{D}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}} = \mathcal{J}(x,\theta,\bar{\theta}) \tag{22}$$

with the superfield current

$$\mathcal{J}(x,\theta,\bar{\theta}) = 8\pi\mu \int d\tau[\dot{\xi}(\theta-\xi)\delta^{(4)}(s_R) - \dot{\bar{\xi}}(\bar{\theta}-\bar{\xi})\delta^{(4)}(s_L)]. \tag{23}$$

In component form, we have

$$\mathcal{J} \equiv -4J^{(0)} + 4\theta^\alpha J_\alpha^{(1)} - 4\bar{\theta}_{\dot{\alpha}}\bar{J}^{\dot{\alpha}(1)} - 8(\theta\sigma_\rho\bar{\theta})J^{(2)\rho}$$
$$-i\theta\theta\bar{\theta}_{\dot{\alpha}}\partial^{\dot{\alpha}\alpha}J_\alpha^{(1)} + -i\bar{\theta}\bar{\theta}\theta^\alpha\partial_{\alpha\dot{\alpha}}\bar{J}^{(1)\dot{\alpha}} + \theta\theta\bar{\theta}\bar{\theta}\,\Box\,J_0 \tag{24}$$

with the components determined by the integrals in (20) and (21).

It follows from the explicit expression for the electromagnetic current $J_\mu^{(2)}$ in (20), which contains the generators $\sigma_{\mu\nu}$ and $\bar{\sigma}_{\mu\nu}$ of the $SO(3, 1)$ Lorentz group in the Weyl spinor representation, that there is the possibility of a physical interpretation of the dimensional constant μ as the magnitude of an anomalous magnetic moment of an electrically neutral particle.[1)] The derivation of an effective electromagnetic field carried out above through the use of differential elements of spinor coordinates of the superparticles can thus be thought of as a mechanism which is the dual of the Wheeler–Feynman mechanism for describing classical Maxwellian electrodynamics. The existence of this duality can be seen clearly by comparing both representations (1) and (14) and expressions (3) and (20) for vector currents. This duality can be understood as an expression of a sort of symmetry of Maxwell's equations under the replacement of pairs $(e, dy^{\mu}) \leftrightarrow (\mu, d\xi^{\alpha}, d\bar{\xi}_{\dot{\alpha}})$ in the expressions for the effective electromagnetic field and the current. In other words, the anomalous magnetic moment μ plays the same role with respect to Grassmann spinor world coordinates as is played by the electric charge e in the ordinary space of world coordinates. A relationship between the Grassmann coordinates and the anomalous magnetic moment close to that which we are discussing here has been pointed out previously[6] in a generalization of the Kaluza–Klein mechanism to superspace.

Another new result associated with the supersymmetric generalization of the principle of action at a distance is the possibility of constructing spin-1/2 fields from spinor differential elements of the world lines of the superparticles.

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[1)]The problem of constructing a superfield action to describe the interactions of such neutral superparticles is currently being studied.

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