Heat-flux limitation factor in a plasma

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The theory of nonlocal heat transfer yields an expression for the heat-flux limitation factor. This factor is derived as a function of the ion charge in a plasma with a high degree of ionization.

In the Knudsen limit of rarefied gases, in which the mean free path is much longer than the length scale of the spatial variations in the particle distribution, heat transfer is governed by the equation for the heat flux density:[1]

$$q \simeq 0.65 n \kappa_B T v_T, \tag{1}$$

where n is the number density of particles, κ_B is the Boltzmann constant, T is the temperature, $v_T = (\kappa_B T/m)^{1/2}$ is the thermal velocity, and m is the mass of the particles. On the other hand, numerous experimental studies of the properties of hot rarefied plasmas produced by laser light have established the idea that, under conditions such that electron mean free path l_e is much longer than the length scale (L_e) of the spatial variations in the electron distribution, there is a substantial deviation from Knudsen law (1). The following expression for the density of the electron heat flux has won the greatest popularity:[2]

$$q_e = f n_e \kappa_B T_e v_{Te}, \tag{2}$$

where f is a heat-flux limitation factor, which is small in comparison with one. Various experimental studies have yielded different values of the factor f required for describing the experimental data: from 0.05 (Ref. 3) to 0.1 (Ref. 4).

Two physically distinct points of view have arisen regarding the reason for the appearance of a heat-flux limitation factor. One involves an ion acoustic turbulence, which arises when the electron flux density in a nonisothermal plasma is sufficiently high, under the condition $ZT_e \gg T_i$, where $T_{e(i)}$ is the electron (ion) temperature, and Z is the degree of ionization of the ions. The typical heat-transfer velocity can be determined by the ion acoustic velocity $v_s = (Z\kappa_B T_e/m_i)^{1/2}$, where m_i is the mass of an ion. In the latter case, the heat-flux limitation factor can be written[5]

$$f = 0.18(Z/A)^{1/2}, \tag{3}$$

where A is the mass number. Expression (3) holds for heat fluxes which are not too high, such that, according to Ref. 6, the turbulent Knudsen number is not very high: $K_N \ll 1$. At high values of the turbulent Knudsen number, $K_N \gg 1$, we have, according to Ref. 6,

$$f = 0.28(K_N Z/A)^{1/2}. \tag{4}$$

The other reason for the limitation on the heat flux is considerably simpler in a sense. Shvartz *et al.*[7] state that the systematic use of a Fokker–Planck collision integral for the charged particles leads to a limitation factor $f=0.1$. That is not, however, the only assertion. Luciani *et al.*,[8] for example, have proposed a nonlocal description of the heat flux in which the following expression is used for the heat flux density:

$$\mathbf{q}_e(\mathbf{r}) = -\int d\mathbf{r}' K(\mathbf{r}-\mathbf{r}') \frac{\partial T_e(\mathbf{r}')}{\partial \mathbf{r}'}. \tag{5}$$

The following expression was used for the Fourier transform of the nonlocal thermal conductivity:

$$K(\mathbf{k}) = \kappa_{SH}[1+(30k\lambda_e)^2]^{-1}. \tag{6}$$

Here we have the following expression for the Spitzer–Härm thermal conductivity:

$$\kappa_{SH} = C_{SH} n_e \kappa_B l_e v_{Te}, \tag{7}$$

where n_e is the electron density, and

$$l_e \equiv v_{Te}/\nu_{ei} = (3/4\sqrt{2\pi})(\kappa_B T_e)^2/e^4 Z n_e \Lambda.$$

In the limit $Z \gg 1$ we have $C_{SH} = 128/3\pi$, and the effective distance

$$\lambda_e = l_e (2Z/9\pi)^{1/2}, \tag{8}$$

used in Ref. 8, is proportional to the square root of the product of the electron-ion mean free path l_e and the electron-electron mean free path.

Among other possible factors which might be invoked to explain the onset of expression (6), we note that, in a description of heat transfer by the Hilbert–Chapman–Enskog method, corrections of higher-order approximations turn out to give a power series in the square of the product of the electron mean free path and the wave vector $\mathbf{k}$.[9] In this sense, expression (6) corresponds to the Padé approximation.[10]

However, subsequent numerical simulations which were aimed at a computer study of heat transfer in a plasma and which used the Landau Fokker–Planck collision integral for both electron-ion and electron-electron collisions yielded a conclusion regarding the nonlocal thermal conductivity which was not as definite as that reached in Ref. 8. It can be said that the various assertions which have been made in recent years regarding the nonlocal thermal conductivity reduce to the formula

$$K(k) = \kappa_{SH}[1+(\alpha k \lambda_e)^\beta]^{-1}. \tag{9}$$

In describing stimulated Brillouin scattering in a plasma, Rose and DuBois[11] used expression (9) with $\beta=1$ and $\alpha=50$, in accordance with the results of Ref. 12. Their results were criticized by Epperlein and Short,[13] according to whom the values $\beta=4/3$ and $\alpha=30$ (found in a numerical simulation[14]) should be used to describe stimulated Brillouin scattering. The parameter values $\beta=1.44$ and $\alpha=21$ were subsequently mentioned in Refs. 13 and 14. In this connection we should point out that in the various studies there is an obvious lack of coordination among the conclusions which different authors draw, or which given authors draw at different times, from numerical simulations in attempts to write universal law (9). There is accordingly a need for an analytic theory for solving the electron kinetic equation in the limit of long mean free paths. Such a theory was constructed in Ref. 15; it predicted the following values for the parameters in (9):

$$\beta=\beta_{\text{th}}=\frac{10}{7},\quad \alpha=\alpha_{\text{th}}\equiv 48\ \frac{2^{0.3}}{21^{0.3}\pi^{0.1}}\left[\Gamma\left(\frac{2}{7}\right)\Gamma\left(\frac{3}{7}\right)/\Gamma\left(\frac{1}{7}\right)\right]^{0.7}\approx 21.1.$$

The results of that study were used in Ref. 16. However, the relationship between a nonlocal transfer of the type in (9) with the standard formula for a limitation on heat transfer, (2) has yet to be established. This relationship can be established in the 1D case, in which we have, according to (5) and (9),

$$q_e(x)=-\int_{-\infty}^{\infty} dx'\,\frac{dT_e(x')}{dx'}\int_{-\infty}^{\infty}\frac{dk}{2\pi}\exp(ik(x-x'))\kappa_{SH}[1+(\alpha k\lambda_e)^{\beta}]^{-1}. \tag{10}$$

The following kernel corresponds to the case of an extremely sharp spatial variation in the temperature:

$$K(x=0)=\frac{\kappa_{SH}}{2\pi}\int_{-\infty}^{\infty} dk[1+(\alpha k\lambda_e)^{\beta}]^{-1}=\frac{\kappa_{SH}}{\lambda_e\alpha\beta}\ \operatorname{cosec}(\pi/\beta). \tag{11}$$

Using (7) and (8), and ignoring the coordinate dependence of the electron density n_e, we find the following approximate expression from (10):

$$\begin{aligned} q_e(x)&=-f\frac{3}{2}n_e\kappa_B\int_{-\infty}^{\infty} dx'\ \frac{dT_e(x')}{dx'}\ v_{Te}(x') \\ &=-fn_e\kappa_B[T_e(+\infty)v_{Te}(+\infty)-T_e(-\infty)v_{Te}(-\infty)], \end{aligned} \tag{12}$$

where the heat-flux limitation factor is

$$f=\frac{2}{3}\ 3\sqrt{\frac{\pi}{2}}\,\frac{C_{SH}}{\alpha\beta}\quad \operatorname{cosec}\left(\frac{\pi}{\beta}\right)Z^{-1/2}. \tag{13}$$

In particular, for the parameter values $\alpha=\alpha_{\text{th}}$, $\beta=\beta_{\text{th}}$, and $C_{SH}=128/3\pi$ corresponding to the analytic theory of Ref. 15, we have

$$f=1.4Z^{-1/2}. \tag{14}$$

According to Ref. 15, we have $Z\gg 1$. At the same time, it is obvious from (13) that the Padé approximation[10] leads to approximately the same value:

$$f=[32\sqrt{2}/15\sqrt{\pi}]Z^{-1/2}=0.57^{-1/2}. \tag{15}$$

We can thus regard the result concerning the dependence of the heat-flux limitation factor on the degree of ionization as being of universal validity within the range of applicability of the collisional treatment. We wish to stress that this dependence arises because (11) contains the ratio of the effective mean free path λ_e to the electron-ion mean free path l_e. Comparing (14), (15) with (3), (4), we see that there is a qualitative difference between the purely collisional theory [(14), (15)] and the turbulent theory in terms of the dependence on the characteristics of the plasma ions. In addition, a comparison of the consequences of these two theories which hold at $Z \gg 1$ leads to the assertion that collisional effects outweigh turbulent effects at small turbulent Knudsen numbers only at degrees of ionization so high as to be difficult to achieve in a real plasma:

$$Z \gg 7\sqrt{A}. \quad (16)$$

It can therefore be assumed that under these conditions the flux limitation factor in a plasma with $Z \gg 1$ is determined by ion acoustic turbulence. At large values of the turbulent Knudsen number, $K_N \gg 1$, on the other hand, a comparison of (4) and (14) leads to the following condition under which ion acoustic turbulence is not a governing factor:

$$Z > 50/K_N. \quad (17)$$

Finally, in connection with the possible use of (14), we should make what we regard as the strongest assertion: In the case of fast processes, for which the time scale is (on the one hand) short in comparison with, for example, the reciprocal growth rate $[\sqrt{\pi/8}\omega_{Li}^2/\omega_{Le}]^{-1}$, for the damping of short-wave ion acoustic waves, while (on the other) large in comparison with the electron-electron mean free time ν_{ee}^{-1}, an ion-acoustic turbulence simply is not realized, and the limitation on the heat flux is described by (14). The necessary condition $[\sqrt{\pi/8}\omega_{Li}^2/\omega_{Le}] \ll \nu_{ee}$ is satisfied for a plasma which is not too nearly ideal, under the condition $e^2/r_{De} \gg 10^{-4}\kappa_B T_e$, where r_{De} is the electron Debye length. The latter condition holds at $n_e(\text{cm}^{-3}) \gg 10^{20} T_e$ (keV).

In summary, it has been shown on the basis of a nonlocal analytic theory for collisional transfer in a rarefied plasma how a limitation factor on the heat flux arises, and the dependence of this factor on the degree of ionization of the plasma ions has been established.

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