Dynamic quantization of current with a doubled electron charge

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A current quantization is predicted: the appearance of steps on the current-voltage characteristic at current values $I=2ef$ in a system of quantum dots in the absence of a Coulomb blockade.

It has been predicted that a particular quantum system (a low-capacitance tunnel junction) under Coulomb blockade conditions should generate regular jumps in the voltage across the junction at a frequency $f=I/e$ when a constant current I flows (under current-source conditions; e is the charge of an electron).[1] A qualitative explanation runs as follows: Under Coulomb blockade conditions, electrons tunnel through the junction one after another. Their repetition frequency I/e determines the jumps in the voltage. In our opinion, no direct experiments to observe oscillations of the voltage under current-source conditions have been carried out.

Steps on the current-voltage characteristic at current values $I=ef$ have been observed in several experiments.[2] Those experiments were carried out on a series of several tunnel junctions with a constant bias voltage which did not move the system away from Coulomb blockade conditions. The application of an alternating voltage with a frequency f and a finite amplitude above the Coulomb barrier to one of the junctions gives rise to steps on the current-voltage characteristic at current values $I=ef$.

In some other experiments on a series of junctions, the dynamic conductance dI/dU was measured by a lock-in method involving a finite-amplitude signal.[3] The structural features which appeared on the dI/dU curve corresponded, as a function of the current, to the values $I=ef$.[3]

These structural features (and, later, the plateaus on the current-voltage characteristic at the current $I=ef$ which were found in the experiments of Ref. 2 have been interpreted as resulting from a mixing of natural oscillations with an alternating external voltage at the frequency f.[3] In our opinion, those experiments cannot be taken as confirmation of natural oscillations under current-source conditions. There is another way to interpret the appearance of plateaus on the current-voltage characteristics as in Ref. 2, and this other interpretation is more successful. The steps on the current-voltage characteristic at $I=ef$ essentially signify a passage of one electron through the system over one cycle of the external voltage. A detailed dynamic picture of the appearance of steps on the current-voltage characteristics has not been drawn.

In this letter we wish to propose an experiment in which a dynamic quantization

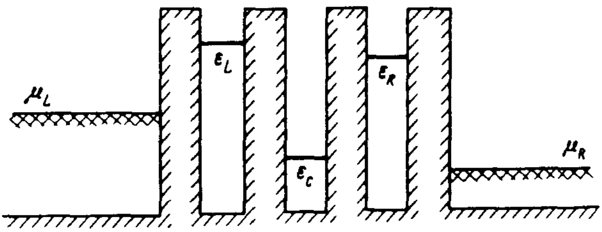

FIG. 1. Positions of the unshifted levels in the quantum wells.

of the current (the appearance of steps on the current-voltage characteristic at $I=ef$) arises in the absence of a Coulomb blockade.

We consider three weakly linked quantum dots (Fig. 1) connected to bulk metal electrodes. We assume that the dimensions of the dots are such that the energy levels in the outer wells lie above the state in the central well (by a distance greater than the width of the levels). The levels in the outer wells need not coincide. Each level is doubly spin-degenerate (we are assuming that the Coulomb interaction is unimportant). A static bias voltage V such that the conditions $\mu_L > \epsilon_c > \mu_R$ hold is applied to the banks (Fig. 1); we assume that V is greater than the level with ϵ_c.

We consider the following periodic process with a characteristic period $\tau = 1/f$. We can begin the discussion at any stage in the periodic process. We assume that the level in the central well is empty. During the first half-period a voltage is applied to the well on the left and causes a slow (adiabatic) lowering of the level in the left well and a crossing of a resonance with the level in the central well (Fig. 2a). The states in the left and central wells are each filled by a pair of electrons from the left bank over a time scale τ_{res} (when the levels are at resonance), by virtue of the condition $\mu_L \gg \epsilon_c$. The level in the left well then rises and empties. Two electrons are left in the central well. If the frequency $f = 1/\tau$ is such that the conditions

$$\tau_{\text{res}} \ll \tau \ll \tau_{\text{outres}} \tag{1}$$

hold (τ_{outres} is the time of the transition from the central well to the right bank when the level in the right well is not at resonance with the central bank), then the electrons are unable to tunnel into the right bank over the time of the flow out of the left bank. During the second half-period the alternating potential is applied to the right well and

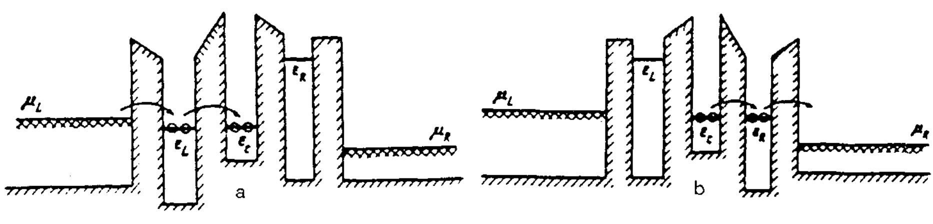

FIG. 2. Positions of the levels at one instant. a—During the first half of a period, when electrons from the left well tunnel into the central well; b—during the second half of the period, when electrons tunnel from the central well into the right well.

causes a lowering of the level in this well in such a way that this level crosses resonance with the level in the central well (Fig. 2b). Over the characteristic time $\tau_{\rm res}$, two electrons from the central well go into the right bank, since we have $\mu_R < \epsilon_c$. The central well is empty again, and the system is back in its original state.

The inequality on the left in (1) means that the motion of the levels occurs adiabatically slowly in comparison with the charge flow time under resonance conditions. The right inequality states that the motion of the levels is fast in comparison with the charge escape time when the levels are far from resonance.

These arguments mean that over one cycle of the alternating external voltage of frequency f a current $I=2ef$ flows through the system. For the level configuration in Fig. 1 ($\mu_L > \epsilon_c > \mu_R$), a background current of course flows through the system. This background current is much smaller than the quantized portions $I=2ef$. The right inequality in (1) automatically keeps this background current small.

We turn now to the necessary analytic calculations. The Hamiltonian of the system incorporating the basic aspects of this problem can be written

$$H = \sum_{\substack{k,\sigma \\ \alpha=L,R}} \epsilon_{k\sigma\alpha} a^+_{k\sigma\alpha} a_{k\sigma\alpha} + \sum_{\alpha} (\epsilon_c c^+_{c\sigma} c_{c\sigma} + \epsilon_L c^+_{L\sigma} c_{L\sigma} + \epsilon_R c^+_{R\sigma} c_{R\sigma})$$

$$+ \sum_{k\sigma} (T_{kL} c^+_{L\sigma} a_{k\sigma L} + T_{CL} c^+_{c\sigma} c_{L\sigma} + T_{CR} c^+_{c\sigma} c_{R\sigma} + T_{kR} c^+_{R\sigma} a_{k\sigma R} + \text{H.a.}), \tag{2}$$

where $a^+_{k\sigma\alpha}$ are the creation operators in the left bank ($\alpha=L$) and in the right bank ($\alpha=R$), the operators $c^+_{c,L,R\sigma}$ create electrons in the central bank and in the left and right isolated wells, and the $T_{kL,CR,CL,kR}$ are the corresponding matrix elements for jumps between wells and banks.

First calculating the background current (here it is convenient to take the approach of Ref. 4), we find

$$I_{\rm backgr} = \frac{e}{\hbar} \int \frac{d\omega}{2\pi} \frac{\gamma_L(\omega)\gamma_R(\omega)}{\gamma_L(\omega)+\gamma_R(\omega)} \, \text{Im}\{G^R_c(\omega)\}[f_L(\omega) - f_R(\omega)], \tag{3}$$

where $G^R_c(\omega)$ is the exact Green's function for the electrons in the central well. This Green's function can be written in the form

$$G^R_c(\omega) = \frac{1}{\omega - \tilde{\epsilon}_c(\omega) + i(\gamma_L(\omega) + \gamma_R(\omega))}, \tag{4}$$

where $\tilde{\epsilon}_c(\omega)$ is the exact energy of the level which originated from the level in the isolated central well, $f_L(\omega)$ and $f_R(\omega)$ are Fermi distributions in the banks, shifted by the voltage ($\mu_L - \mu_R = V$), and $\gamma_{L,R}(\omega)$ are the rates of tunneling into the left and right banks from the central well, given by

$$\gamma_{L,R}(\omega) = \frac{|T_{cL,R}|^2 \gamma_{0L,R}}{(\omega - \tilde{\epsilon}(\omega)_{L,R})^2 + \gamma^2_{0L,R}}. \tag{5}$$

Here $\tilde{\epsilon}_{L,R}(\omega)$ are the renormalized energy levels in the left and right banks, and

$$\gamma_{0L,R}=\sum_k |T_{kL,R}|^2\delta(\omega-\epsilon_{k,L,R}). \tag{6}$$

From (6) we see that, if the density of states at the Fermi level in the banks changes only slightly, then the constants $\gamma_{0L,R}$ can be assumed independent of the energy.

Far from resonance $[\Delta\epsilon=\tilde{\epsilon}_c-\tilde{\epsilon}_{L,R}\gg\gamma_{L,R}(\tilde{\epsilon}_c)]$ we have

$$\gamma_{L,R}(\omega)=\frac{|T_{cL,R}|^2\gamma_{0L,R}}{(\omega-\tilde{\epsilon}_{L,R}(\omega))^2}. \tag{7}$$

An estimate of $\gamma_{0L,R}$ based on (6) is (we are assuming $\gamma_{0L}\simeq\gamma_{0R}\simeq\gamma_0$)

$$\gamma_0\simeq T^2/W, \tag{8}$$

where W is the width of the band in the electrodes, and $T\simeq T_{kL,R}$ are the transmissions of the barriers. As a result, we find

$$\gamma_{L,R}(\tilde{\epsilon}_c)\simeq T^4/W(\Delta\epsilon)^2. \tag{9}$$

The band width W is determined by the integral representing jumps between atoms in the electrodes, so the condition $W\gg T$ holds. Using (3) and the estimates above, we estimate the background current to be

$$I_{\text{backgr}}\simeq 2e\gamma(\tilde{\epsilon}_c)/\hbar\simeq eT^4/W(\Delta\epsilon)^2. \tag{10}$$

The time over which the charge is established in the quantum well is determined by the parameters $\gamma_{L,R}$.[5] The inflow of charge into the central well when the levels are at resonance is determined by the time scale

$$1/\tau_{\text{res}}\simeq\gamma_L(\tilde{\epsilon}_c)\simeq T^2/W. \tag{11}$$

The time scale of the departure of the charge through the right level, when the latter is far from resonance with the level in the central well, is

$$1/\tau_{\text{outres}}\simeq\gamma_R(\tilde{\epsilon}_c)\simeq T^4/W(\Delta\epsilon_c)^2. \tag{12}$$

If the frequency f of the variation of the external potential satisfies the conditions

$$T^2/W\gg f\gg T^4/W(\Delta\epsilon_c)^2, \tag{13}$$

then the condition for a dynamic quantization of the current holds: a transfer of two charges through the system over one cycle of the potential. The right inequality in (13) automatically means that the background current is small in comparison with the quantized portions $I=2ef$. This inequality also determines the precision of the quantization. Since T is exponentially small in comparison with W, the interval in inequality (13) for the frequency is fairly wide. With $\Delta\epsilon\simeq W$ (away from resonance), the ratio of the left and right sides of inequality (13), whose value is $(T/W)^{-2}$, is an exponentially large quantity.

If the Coulomb interaction of the electrons in the wells is important, the spin degeneracy is lifted (each level can be occupied by only one electron), and quantized portions $I=ef$ flow through the system.

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