Longitudinal conductivity of a Boltzmann electron gas in n-InSb in the quantum limit in the magnetic field

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(Submitted 8 July 1993)
Pis'ma Zh. Eksp. Teor. Fiz. **58**, No. 4, 280–285 (25 August 1993)

The longitudinal conductivity (i.e., that along the magnetic field) of n-InSb due to an interaction of electrons with ionized impurities has been measured in the quantum limit in the magnetic field under conditions corresponding to a Boltzmann statistics of the electrons. For samples with an electron density $n=10^{15}$ cm^{-3} the results are close to the theoretical predictions of Agyres and Adams [Phys. Rev. **104**, 900 (1956)]. Their theory predicts a decrease in the resistance when a strong field is applied. As the electron density decreases, the difference between the experimental and theoretical results increases, and the magnetoresistance goes from negative to positive.

The picture of the kinetics of electrons as they are scattered by ionized impurities in the quantum limit in the magnetic field H, in which case only the lowest Landau subband is filled, has been reexamined in recent years. The transverse and longitudinal conductivities in the case of Fermi statistics for the electrons was studied in Refs. 1–3 and the transverse conductivity in the case of Boltzmann statistics was studied in Ref. 4. In this letter we report an experimental study of the longitudinal conductivity of the Boltzmann gas of electrons in n-InSb. The problem of the longitudinal conductivity in the quantum limit was solved theoretically by Argyres and Adams.[2] It has been suggested that the resistivity in the quantum limit in the magnetic field is lower than in the absence of a magnetic field, because small-angle scattering is "turned off," and that it depends only weakly on the magnetic field, tending toward a constant at infinity.

It was shown in Refs. 1–3 that localization effects can strongly influence the longitudinal conductivity in the quantum limit because the electron motion is approximately one-dimensional. Measurements on n-InSb and n-InAs under conditions of Fermi statistics for the electrons[6,7] support this suggestion. However, it follows from the results of Ref. 8 that electromagnetic fluctuations should suppress localization effects in the quantum limit in the magnetic field in the case of Fermi statistics if

$$kT>(\hbar/\tau)(\hbar V_F\epsilon_0/e^2)^{1/3}. \qquad (1)$$

Here k is the Boltzmann constant, T is the temperature, τ is the relaxation time of the momentum along the field, V_F is the electron Fermi velocity, ϵ_0 is the dielectric constant of the lattice, and e is the electron charge. This condition remains valid for a Boltzmann gas if we replace V_F by V_T—the velocity of an electron with an energy kT. Using the theoretical relation[5]

TABLE I.

Sample	1	2	3	4	5	6	7	8	9	10	11	12	13
n, 10^{13} cm^{-3}	100	34	20.5	14.4	9.5	8.3	5.3	3.8	3.0	3.0	1.0	0.86	0.5
ρ_0, m$\Omega \cdot$cm	20	33	62.7	68.3	8.94	130	185	310	525	–	–	2200	3480
N, 10^{14} cm^{-3}	19	7.0	7.56	5.3	4.7	5.1	4.3	4.85	6.3	–	–	5.7	5.5
U/k, K	23	18	21	20	20	22	23	26	32	–	–	41	48

$$\hbar/\tau \sim N V_T [e^2/(\epsilon_0 kT)]^2, \quad (2)$$

we find in place of (1)

$$kT > (e^2 N^{1/3}/\epsilon_0)^{9/7} / \mathscr{E}_B^{2/7}. \quad (3)$$

Here $\mathscr{E}_B = me^4/(2\epsilon_0^2\hbar^2)$ is the Bohr energy, and N is the impurity concentration. If condition (3) holds, localization effects are thus suppressed, and the magnetoresistance should be described by the theory of Argyres and Adams;[5] in the opposite case, the magnetoresistance may be larger.

The longitudinal resistivity has been studied experimentally under the conditions of Boltzmann statistics for electrons in Tl (Ref. 9) and n-InSb (Ref. 10). A negative magnetoresistance was observed in those studies, but the experimental field dependence did not agree with the theoretical one. Under the conditions of Ref. 9 the condition for the applicability of the Born approximation, $e^2/(\hbar V_T \epsilon_0) \ll 1$, did not hold. In addition, kT was smaller than the spatial fluctuations of the impurity potential U, so the results of Ref. 5 did not apply. In Ref. 10, measurements were carried out on n-InSb in fields up to 3.2 T, just at the beginning of the quantum limit. The results of measurements on various samples with approximately equal properties (the impurity concentration N and the electron density n) turned out to be very different, apparently because of either an inhomogeneity of the samples or deficiencies in the measurement method.

We have accordingly carried out systematic measurements of the longitudinal conductivity of n-InSb samples over a wide range of the electron density n by various methods: a four-point method at a frequency of 30 Hz and a two-point method at both low (30 Hz) and high (up to 1000 MHz) frequencies.

Test samples and experimental procedure. The measurements were carried out on n-InSb samples with electron densities n ranging from 5×10^{12} to 10^{15} cm^{-3} (Table I). Measurements of the longitudinal conductivity are complicated by the pronounced anisotropy along and across the magnetic field: Slight inhomogeneities in the sample or irregularities of the lateral surface can seriously distort the results. We accordingly used several methods in an effort to improve the reliability. Many of the measurements were carried out by the four-point method at a frequency of 30 Hz. Samples 1 cm long, about 1 mm wide, and about 2 mm thick with lugs for the potential contacts were cut on an electrospark discharge machine and then etched in SR-4A etchant. The voltages across the potential contacts usually depended on the polarity of the magnetic field, and they differed for the pairs of contacts on different sides of the sample. However,

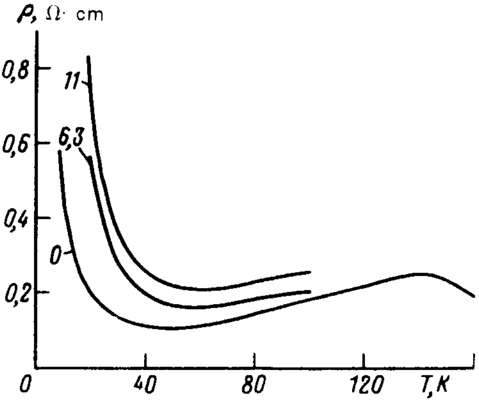

FIG. 1. Temperature dependence of the resistivity of sample 6 in a magnetic field and without a magnetic field. The curves are labeled with the magnetic field strength in teslas.

the average values for the two polarities of the magnetic field from each pair of contacts were usually approximately the same. These average values were used as the results. In addition, some measurements were carried out by the two-point method. A rectangular parallelepiped with dimensions of $2\times3\times4$ mm was cut from a material with a density of 10^{15} cm^{-3}. Tin films were deposited on the smaller faces. The current and potential leads were soldered to these films. The results of the measurements on this sample were the same as the results of measurements by the four-point method on a sample of the same material. For samples 10 and 11, whose ends were treated with indium, the two-point measurements were carried out at a high frequency (up to 1000 MHz) in order to eliminate the contact resistance by virtue of the capacitive coupling. In addition, the capacitive coupling made it possible to eliminate the effect of certain defects. Since the results of the high-frequency measurements were used as a control, we will not describe the measurement procedure here.

The electron density was found from results on the Hall resistance in strong fields. The concentration of ionized impurities, N, was determined for all samples except sample 1 from the Brooks–Herring formula for Boltzmann statistics (Table I). That for sample 1 was found from measurements of the resistance at $H=0$, $T=4.2$ K from the Brooks–Herring formula for Fermi statistics, since the Fermi energy in a zero magnetic field was high for this sample: $\mathscr{E}_F=30$ K. In a magnetic field $H>2$ T, however, the relation $\mathscr{E}_F\ll30$ K held for all samples.

Most of the measurements were carried out at temperatures above 20 K, at which essentially all the electrons have been removed from shallow donors into the conduction band, as could be seen from the Hall effect. The Hall constants varied with the temperature by less than 20% between 70 and 20 K.

Experimental results. The temperature dependence of the resistivity along the magnetic field is qualitatively similar to that without a field (Fig. 1). Below 50 K, the resistivity is governed by scattering by ionized impurities, and falls off with increasing temperature. Above 60 K, scattering by phonons becomes important, and the resistivity begins to increase. The decrease in ρ above $T=140$ K is due to an interband

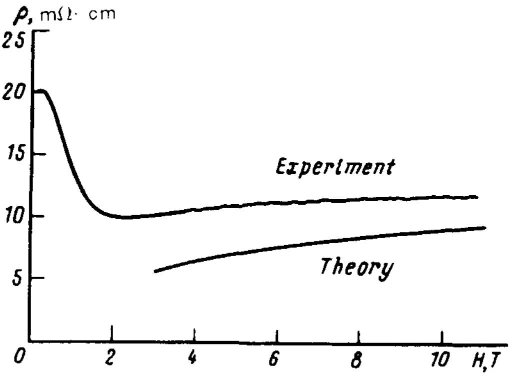

FIG. 2. Longitudinal resistivity of sample 1 versus the magnetic field at $T=30$ K. *1*—Experiment; *2*—Theory.

excitation of current carriers. In the temperature interval 20–40 K the resistivity in strong fields varies roughly as T^{η}, where $\eta \simeq 1.5$–2.

The field dependence was recorded at $T=30$ K in most cases. The behavior of the resistivity of samples 1 and 2 as a function of the magnetic field is qualitatively the same as that predicted by the Argyres–Adams theory:[5] As we move toward the quantum limit, the longitudinal resistivity decreases (Fig. 2) and then begins to increase slowly. According to the theory of Ref. 5, the resistivity tends in the limit $H \to \infty$ toward a constant value ρ_{∞} which is lower than ρ_0, but the value ρ_{∞} is approached from below rather slowly. For sample 1 we calculated a theoretical behavior of the resistivity as a function of the magnetic field in the quantum limit, in which the conductivity is governed essentially exclusively by electrons from the lower Landau subband (curve *2* in Fig. 2). This behavior does not differ greatly from the experimental behavior.

As the electron density n decreases, the ratio ρ/ρ_0 should decrease in the quantum limit according to the theory of Ref. 5. Actually, we find the opposite behavior: The ratio ρ/ρ_0 increases and becomes greater than one (Fig. 3). Figure 4 shows experimental and theoretical results on the behavior of the longitudinal resistivity, normalized to the resistivity in the absence of a magnetic field, as a function of the electron density at fixed values of the temperature and the magnetic field. These curves are qualitatively different. At low electron densities the ratios ρ/ρ_0 differ quantitatively by several orders of magnitude. The results of the high-frequency measurements are similar to the results found at 30 Hz, although the field dependence at the high frequency is slightly weaker than that at 30 Hz (Fig. 3).

Discussion of results. The monotonic behavior of ρ/ρ_0 as a function of n is evidence that our results are not due to some random macroscopic inhomogeneities in the samples or measurement errors which lead to a distortion of the current lines and which introduce an admixture of the transverse conductivity. Further evidence that there is no substantial admixture of the transverse conductivity comes from the circumstance that the temperature dependence of the conductivity is qualitatively the same as in the absence of a magnetic field and the circumstance that the resistivity varies only weakly with the field in the quantum limit.

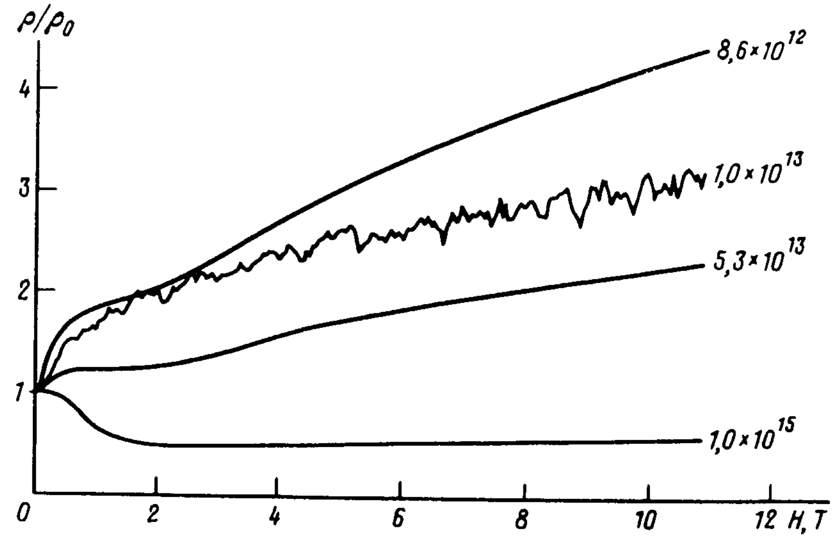

FIG. 3. Longitudinal resistivity versus the magnetic field at 30 K for several samples. The curve with noise is the result of measurements at a frequency of 600 MHz; the other curves correspond to 30 Hz. The curves are labeled with the electron density n in units of cm^{-3}.

Under our conditions, the impurity fluctuation potential U is rather high. Table I shows values of the quantity[11]

$$U/k \equiv (\bar{U}^2)^{1/2}/k = (2\pi e^4 N r_D/\epsilon_0)^{1/2}/k.$$

We see that this quantity exceeds 30 K for some of the samples. However, since the electrons with energies above 3 kT contribute nearly 50% of the conductivity, we could not expect the incorporation of this potential to change the calculated results by a factor of several orders of magnitude, especially since there is no correlation between the discrepancy between theory and experiment and the magnitude of this potential.

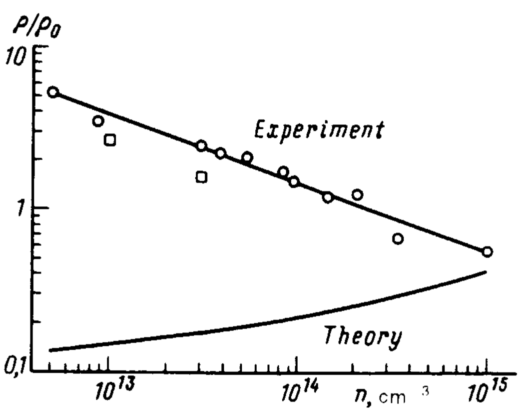

FIG. 4. Resisitivity in a magnetic field $H=6.3$ T at $T=30$ K, normalized to the resistivity in the absence of a magnetic field, versus the electron density n. Points: Experiment. Curve *2*: Theory. ○—Results of low-frequency measurements; □—high-frequency measurements.

For samples 1 and 7, the potentials are nearly equal, and the deviation of their resistivities from the theoretical values are greatly different (Fig. 4).

Our results cannot be explained on the basis of localization effects, since for n-InSb with an impurity concentration $N=5\times10^{14}$ cm^{-3} inequality (3) reduces to $kT>7.6$ K. This result means that in our samples localization effects should be suppressed at $T=30$ K. Inequality (4) holds worst for sample 1, which has the highest impurity concentration, $N=1.9\times10^{15}$ cm^{-3}, and for which we observe the least discrepancy between the theory of Ref. 5 and experiment. Furthermore in a field of $2T$ the localization factor[1] $(2k_T\lambda_H)^{-2} \simeq 1.3$ (here k_T is the wave vector of an electron with an energy kT along the magnetic field, and λ_H is the magnetic length), which determines the extent to which localization effects reduce the conductivity, is not large enough to account for a discrepancy of several orders of magnitude.

For the samples with a fairly high electron density, $n=10^{15}$ cm^{-3}, the experimental results and the theoretical results of Ref. 5 thus agree qualitatively, and they are not greatly different quantitatively. However, for some reason we do not understand, the relative conductivity ρ/ρ_0 increases with decreasing n, while it should decrease according to the theory of Ref. 5. For samples with small electron densities ($\leqslant10^{14}$ cm^{-3}) the results differ by several orders of magnitude.

This work was supported financially by the Russian Basic Research Foundation (Project 93-02-2794). This work was also supported in part by a grant from the Soros Foundation, awarded by the American Physical Society. We wish to thank V. F. Gantmakher for useful discussions.

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Translated by D. Parsons