Superconformal string amplitudes for π-meson interactions

V. A. Kudryavtsev

B. P. Konstantinov St. Petersburg Institute of Nuclear Physics, Russian Academy of Sciences, 188350 Gatchina, Russia

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Operator vertices of a new type are constructed for superconformal tree amplitudes. Amplitudes with these vertices describe the interactions of an arbitrary number of π mesons with a realistic spectrum of resonant states. A leading Regge trajectory $\alpha_\rho=\frac{1}{2}+\alpha' t$ corresponds to this spectrum. The operator structure of the amplitudes reproduces the terms of dual quark diagrams in many ways.

Attempts made in the 1970s to describe hadron physics in terms of string models failed because the spectrum of systematic conformal and superconformal models for open strings[1] necessarily included a massless vector particle, while the spectrum of closed strings necessarily included a spin-2 massless particle, in obvious contradiction of the hadron spectrum observed experimentally. The interpretation of the massless tensor particle as a graviton, and of the massless vector particle as a gluon, made it possible to treat these classical string models as the foundation for a theory which unifies all the fundamental interactions, including gravitation, at a Plankian energy scale $E\sim(\alpha')^{-1/2}\sim 10^{19}$ GeV. On the other hand, the problem of constructing string amplitudes for interactions of hadrons at the energy scale characteristic of strong interactions, $E\sim 1$ GeV, remained an open question, despite the impressive rectilinearity and universality of both meson and baryon Regge trajectories. These and other arguments in favor of a string interpretation of hadrons remain valid, and they are spurring a search for systematic string models of hadrons.

In the present study we have found a solution of this problem, and we have constructed superconformal string amplitudes, at least for the interaction of mesons at the tree level. The spectrum of physical states of these partial amplitudes has no massless particles with spin 1 or 2, and it has a ρ–f leading Regge trajectory: $\alpha_\rho(t)=\frac{1}{2}+\alpha' t$.

It turns out that the key to the operator approach for the new string amplitudes is a generalization (formulated below) of the classical string vertices of Lovelace, Olive, *et al.*[2] The Lovelace and Olive vertices describe the interaction of N arbitrary string states in the Veneziano or Neveu–Schwarz–Ramond model. The corresponding N-string amplitudes are given in this formalism by integrals of the vacuum expectation value for a product of a vertex operator and the wave functions Φ_i $(i=1, 2, \ldots, N)$ of string states over the complex variables z_i $(i=1, \ldots, N)$ with a certain weight. The wave function $|\Phi_i\rangle$ for the ith string is written in its own ith basis of the creation operators for boson string modes, $a_{n\mu}^{(i)+}=a_{-n\mu}^{(i)}$, corresponding to the Fourier coefficients of an expansion of the coordinate of the ith string $X_\mu^{(i)}(z_i)$: $a_0^{(i)}=P^{(i)}$, $[a_{n\mu}^{(i)}, a_{n\nu}^{(i)}]=-g_{\mu\nu}\delta_{n,-m}$.

In the case of a Neveu–Schwarz string, the wave functions Φ_i involve anticommutating string modes $b_r^{(i)+}=b_{-r}^{(i)}$, i.e., field expansion coefficients:

$$H^{(i)}(z_i)=\Sigma b_r^{(i)}z_i^r, \quad \{b_{r\mu}^{(i)}, b_{s\nu}^{(i)}\}=-g_{\mu\nu}\delta_{r,-s}.$$

To construct the vertex operator we use infinite-dimensional matrices $(U_\epsilon^{(i)})_{nm}$ and $(V_\epsilon^{(i)})_{nm}$ for the modes $a_n^{(i)}(n, m=0, 1, 2 \ldots)$ and $(U_{1/2}^{(i)})_{rs}$ and $(V_{1/2}^{(i)})_{rs}$ for the operators $b_r^{(i)}$. These matrices are infinite-dimensional representations D_{nm}^J of linear-fraction transformations $z'=(az+b)/(cz+d)(ad-bc=1)$. For the transformations which send z_{i-1} into 0, z_i into ∞, and z_{i+1} into 1, we have $D_{nm}^J=(U_J^{(i)})_{nm}$; for those which send ∞ into z_{i-1}, 0 into z_i, and 1 into z_{i+1}, we have $D_{nm}^J=(V_J^{(i)})_{nm}$. The matrices D_{nm}^J are determined completely by the linear-fraction transformation itself and by the conformal spin J (correspondingly, $J=\epsilon$, $\epsilon\to 0$ or $J=1/2$):

$$\left(\frac{az+b}{cz+d}\right)^n |cz+d|^{-2J}c_n^J=\sum_m D_{nm}^J z^m c_m^J, \quad c_n^J=\sqrt{\frac{\Gamma(n+2J)}{\Gamma(n+1)}}. \tag{1}$$

The corresponding expression for the N-particle amplitude in the case of the Neveu–Schwarz string is

$$V_N=\int \prod_i dz_i F(z_1 \ldots z_N) \prod_{i=1}^N \langle 0_i| \exp\left\{\frac{1}{2}\sum_{i\pm j} a_n^{(i)}(U_\epsilon^{(i)})_{nm}(V_\epsilon^{(j)})_{mk}a_k^{(j)}\right.$$
$$\left.+\frac{1}{2}\sum_{i\neq j} b_r^{(i)}(U_{1/2}^{(i)})_{rs}(V_{1/2}^{(j)})_{sp}b_p^{(j)}\right\} \prod_i \Phi_i|0_i\rangle, \tag{2}$$

where $F(z_1 \ldots z_N)$ is a weight function which determines the measure of the integration along z_i, which is consistent with crossing symmetry and conformal symmetry.

The Lovelace–Olive vertices written above involve the field components $X^{(i)}(z_i)$ of the zeroth conformal string and components of the anticommutating field $H^{(i)}(z_i)$ of conformal spin 1/2. We now introduce, as a supplement to these "gluon" fields (X, H), some new anticommutating "quark" fields $\psi^{(i)}(z_i)$ of conformal dimensionality 1/2, which correspond to a purely fermionic string.[3] These fields $\psi_{\alpha\beta}^{(i)}$ are Dirac bispinors in terms of α $(\alpha=1, 2, 3, 4)$, while they are isospinors in terms of β $(\beta=1, 2)$. In other words, they carry a spin of 1/2 and an isospin of 1/2 in ordinary space. A superconformal algebra of generators G_r^f corresponding to these fields was also formulated in Ref. 3 ($G_r^f=G_r^{(0)}$ in the notation of Ref. 3). We now construct a new vertex operator (to supplement the earlier one), with the correct crossing-symmetry and factorization properties:

$$(\tilde{\psi}_r^{(1)}(U_{1/2}^{(1)}V_{1/2}^{(2)})_{rs}\psi_s^{(2)})(\tilde{\psi}^{(2)}U_{1/2}^{(2)}V_{1/2}^{(3)}\psi^{(3)})\ldots(\tilde{\psi}^{(N)}U_{1/2}^{(N)}V_{1/2}^{(1)}\psi^{(1)})$$

$$\equiv\prod_{i=1}^{N}\tilde{\psi}_r^{(i)}(U_{1/2}^{(i)}V_{1/2}^{(i+1)})_{rs}\psi_s^{(i+1)}, \qquad (3)$$

where

$$\tilde{\psi}=\psi\gamma_6\tau_2, \quad \{(\tilde{\psi}_r)_\alpha,(\psi_s)_\beta\}=\delta_{\alpha\beta}\delta_{r,-s}.$$

The operator (3) is seen to have the structure of the Hartree–Rosener quark dual diagram.[4] The incorporation of this operator in the N-string vertex leads to the generalization which we have been seeking for the N-string Lovelace–Olive amplitude:

$$V_N\int\prod_i dz_i F(z_1\ldots z_N)\langle 0|\prod_{i=1}^{N}(\tilde{\psi}^{(i)}U_{1/2}^{(i)}V_{1/2}^{(i+1)}\psi^{(i+r)})$$

$$\times\exp\left\{\frac{1}{2}\sum_{i\neq j}a^{(i)}U_\epsilon^{(i)}V_\epsilon^{(j)}a^{(j)}+\frac{1}{2}\sum_{i\neq j}b^{(i)}U_{1/2}^{(i)}V_{1/2}^{(j)}b^{(j)}\right\}\prod_{i=1}^{N}\Phi^{(i)}|0\rangle. \qquad (4)$$

The new amplitude in (4) [like (1)] has all the necessary factorization and duality properties.

It is now a straightforward matter to go over from the approach of N-string vertices to an operator representation for the amplitude of the interaction of N ground states:

$$A\sim\langle 0|V_1V_2\ldots V_N|0\rangle. \qquad (5)$$

As in the Neveu–Schwarz model for an open string, we choose the vertex operators in the form of a commutator with a Virasoro supergenerator:

$$V_i(z_i)=z_i^{-L_0}[G_{1/2},W_i]z_i^{L_0}. \qquad (6)$$

If, in accordance with (4), we take the operator W_i to be $[\tilde{\psi}^{(i)}(1)\gamma_5\tau^{(i)}\times\psi^{(i+1)}(1)]\exp ik_iX(1)$, and if we take $k_i^2=0$, then the requirement of a single conformal dimensionality for $V_i(z)$ reduces to the requirement of a conformal dimensionality of 1/4 for the fields $\psi^{(i)}$ instead of 1/2. We can meet this requirement by replacing the free "quark" field $\psi(z)$ by the composite field $\hat{\psi}(z)$:

$$\hat{\psi}^{(i)}(z_i)=\psi^{(i)}(z_i):\exp\frac{1}{\sqrt{2}}C^{(i)}(z_i):. \qquad (7)$$

Here the new scalar field $C^{(i)}(z_i)$ is analogous to the field of zeroth conformal dimensionality $X^{(i)}(z_i)$. The field $C^{(i)}(z_i)$, along with the new anticommutating field $f^{(i)}(z_i)$ of dimensionality 1/2, appears in a new pair of Neveu–Schwarz fields. This pair of fields, $C^{(i)}$, $f^{(i)}$, is analogous to the old pair $[X(z),H(z)]$. The superconformal Virasoro generator G_r in this new model is the sum of an ordinary Neveu–Schwarz operator G_r^{NS} for all free Neveu–Schwarz fields and an operator G_r^f for the free fields of the purely fermion sector:[3] $G_r=G_r^{NS}+G_r^f$.

The superconformal vertex which we are seeking for the emission of a π meson can now be written as in (6) with the following operator W_i:

$$W_i=g\colon \exp\left(-\frac{1}{\sqrt{2}}C^{(i)}(1)\right)\colon [\tilde{\psi}^{(i)}(1)\gamma_5\tau^{(i)}\psi^{(i+1)}(1)]\colon$$

$$\times\exp\left(\frac{1}{\sqrt{2}}C^{(i+1)}(1)\right)\colon \exp[ik_iX(1)],$$

$$C^{(i)}(z)=\sum_n C_n^{(i)}z^n\frac{1}{\sqrt{n}},\quad [C_n^{(i)},C_m^{(i)}]=\delta_{n,-m}. \tag{8}$$

The quantities $C^{(i)}(1)$ and $C^{(i+1)}(1)$ in (8) must have opposite signs if the zeroth modes are to cancel out for each $C^{(i)}$. [The situation here is analogous to momentum conservation $\Sigma k_i=0$ for $X(z)$.]

The amplitude for the interaction of a number of π mesons N is written, with the help of (6) and (8), in the standard form of an integral of the vacuum expectation value:

$$A_N=\int\prod_i\frac{dz_i}{z_i}\langle 0|V_1(z_1)V_2(z_2)\ldots V_N(z_N)|0\rangle$$

$$\times\frac{|z_1-z_N||z_1-z_{N-1}||z_{N-1}-z_N|}{dz_1dz_{N-1}dz_N}. \tag{9}$$

We wish to stress that, in the factorization for a given channel i with momentum $P(i)=\Sigma_{n=1}^i k_n$, the set of operators which determine the wave function of the state includes only components of the four fields $\psi^{(1)}$, $C^{(1)}$, $\psi^{(i+1)}$, $C^{(i+1)}$. Accordingly, amplitude (9) [like amplitude (2)] has the correct factorization properties. The spectrum of physical states of this amplitude with the vertices in (6) and (8) has no tachyons or massless particles with spin 1 or 2. The only massless particle here is a pseudoscalar isovector π meson. The spectrum is also free of states with isospin above 1, since only two fields ψ, with an isospin of 1/2, appear in each channel. (The structure of the quark dual diagram of Ref. 4 is being reproduced.) The leading trajectory in this model is $\alpha_\rho(t)=1/2+\alpha' t$. A calculation carried out for a four-point amplitude leads to the familiar Lovelace–Shapiro formula[5]

$$A_4=-g^2\frac{\Gamma(1-\alpha_s)\Gamma(1-\alpha_t)}{\Gamma(1-\alpha_s-\alpha_t)}\mathrm{Tr}(\tau^{(1)}\tau^{(2)}\tau^{(3)}\tau^{(4)}). \tag{10}$$

Amplitude (9), with the vertices given by (6) and (8), is thus a superconformal multiparticle generalization of Lovelace–Shapiro amplitude (10) with a leading trajectory $\alpha_t=1/2+\alpha' t$. The introduction of an additional nonzero fifth component of the momentum, to make the mass of the π meson nonzero, leads to π-meson scattering lengths which are completely satisfactory from the experimental standpoint.[5]

The loop corrections, the physical spectrum, and other properties of this model will be discussed in further publications.

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