Stable topological solitons (vortices) in 2D antiferromagnets in an external field

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A topologically localized soliton (vortex) in an external magnetic field has a nonzero momentum at a zero velocity. A vortex with a nonzero velocity is therefore stable with respect to collapse, in contrast with the standard situation.

The dynamics of the magnetization of a classical two-sublattice antiferromagnet can be described in the anisotropic n-field σ model[1,2] (see also Ref. 3). For an antiferromagnet with easy axis along the z axis in an external magnetic field $\mathbf{H}$, the Lagrangian can be written in terms of the antiferromagnetism unit vector $\mathbf{l}(\mathbf{l}^2=1)$:

$$L=\frac{E_0}{4\pi}\int\left\{\frac{1}{2c^2}\dot{\mathbf{l}}^2-\frac{1}{2}(\nabla\mathbf{l})^2+\frac{1}{r_0^2}(l_x^2+l_y^2)+\frac{g}{c^2}(\mathbf{H}\cdot[\dot{\mathbf{l}}\times\mathbf{l}])\right\}d^2x. \tag{1}$$

The first two terms here govern the dynamics of the antiferromagnet in the exchange approximation; $c=gM_0(\alpha\delta)^{1/2}/2$ is the velocity of spin waves; $E_0=4\pi\alpha a M_0^2$ is the minimum energy of a topological soliton (vortex; more on this below); α and δ are the inhomogeneous- and homogeneous-exchange constants; a is a lattice constant; M_0 is the magnetization of a sublattice; $\dot{\mathbf{l}}=\partial\mathbf{l}/\partial t$; $g=2|\mu_0|/\hbar$; and μ_0 is the Bohr magneton. The third term determines the anisotropy energy in the form $w_a=\beta/2(l_x^2+l_y^2)$, and $r_0=\sqrt{\alpha/\beta}$ is a length scale satisfying $r_0\gg a$. In a field $\mathbf{H}\neq 0$, the last term disrupts the formal Lorentz invariance with a selected velocity c which is characteristic of an antiferromagnet at $H=0$ (even if the "Dzyaloshinskiĭ" interaction[4] is ignored).

The resultant magnetization of the antiferromagnet, $\mathbf{M}$ [$(\mathbf{M}\cdot\mathbf{l})=0$], contains two terms, the dynamic term $\mathbf{M}_d$ and the static term $\mathbf{M}_s$:

$$\mathbf{M}=\mathbf{M}_d+\mathbf{M}_s=\frac{2}{g\delta}[\mathbf{l}\times\dot{\mathbf{l}}]+\frac{4}{\delta}[\mathbf{H}-\mathbf{l}\times(\mathbf{l}\cdot\mathbf{H})]. \tag{2}$$

In an isotropic magnetic material ($H=0$, $r_0\to\infty$) there are static localized ($\mathbf{l}\to\mathbf{e}_z$ as $x,y\to\infty$) topological solitons (Belavin–Polykov vortices) which exhibit scale invariance. Their structure is described by the following equations[5] (see also Ref. 3):

$$\tan\frac{\theta}{2}=\left(\frac{R}{r}\right)^{|\nu|},\quad \phi=\nu\chi+\phi_0,\quad l_z=\cos\theta,\quad l_x+il_y=\sin\theta e^{i\phi}. \tag{3}$$

Here r, χ are the polar coordinates in the plane of the magnetic material; the integer $\nu=\pm 1, \pm 2, \ldots$ is the topological charge of the vortex (the degree of the mapping of the plane of the magnetic material, xy, onto the sphere $\mathbf{l}^2=1$; see Ref. 3) and ϕ_0 and

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the vortex radius R are arbitrary parameters. The vortex energy $E = |\nu| E_0$ is independent of ϕ_0 and R in the exchange approximation, so the 2D problem is scale-invariant for an isotropic magnetic material. If the magnetic anisotropy is taken into account, a positive term proportional to R^2 appears in the energy. For $|\nu| = 1$, for example, we have[6,3]

$$E = E_0[1 + \lambda(R/r_0)^2], \quad \lambda = \ln(2.42 r_0 / R_0). \tag{4}$$

Consequently, the energy does not reach a minimum as a function of R at any value $R \neq 0$; this is the reason for the collapse of the vortex (this is the Derrick–Hobbart theorem regarding the instability of non-1D solitons[7,8,3]). A vortex may be stabilized by conservation of the z projection of the resultant magnetization $\int M_z d^2x$ (Ref. 3), but in this case a soliton with a small $R < r_0$ corresponds to precession of the vector $\mathbf{l}$, and we have $\phi = \omega t + \nu\chi + \phi_0$. The precession frequency is not small; it is comparable to the frequency of linear magnons: $\omega_0 = gH_0$, $H_0 = M_0(\beta\delta)^{1/2}/2$.

In the case under discussion here, there can be another type of dynamic stabilization of a vortex. The stabilization in this case occurs because, in a field $\mathbf{H}$ which is not parallel to the easy axis $\mathbf{e}_z$ of the antiferromagnet, the momentum of a immobile vortex (with a velocity $v = 0$) with a finite radius R and a topological charge $\nu = \pm 1$ is nonzero. We write the general expression for the momentum of the field of the vector $\mathbf{l}$ as follows:

$$\mathbf{P} = \int \mathbf{p} d^2x, \quad \mathbf{p} = -\sum_i (\partial L/\partial \dot{l}_i) \nabla l_i.$$

Because of the last term in Lagrangian L, terms without $\mathbf{l}$ appear in the momentum density. Calculating their contribution to the momentum for a vortex $\theta = \theta_0(r)$, $\phi = \nu\chi + \phi_0$, we easily find, with $v = 0$ and $\nu = \pm 1$,

$$\mathbf{P} \equiv \mathbf{P}_0 = P_0 \mathbf{n}, \quad P_0 = (gHE_0/4c^2) \int_0^\infty dr \left(r \frac{d\theta_0}{dr} + \sin\theta_0 \cos\theta_0 \right),$$

$$\mathbf{n} = \mathbf{e}_x \sin\Psi + \nu \mathbf{e}_y \cos\Psi, \quad \Psi = \phi_0 - \Phi, \quad H_x = H\cos\Phi, \quad H_y = H\sin\Phi. \tag{5}$$

The direction of $\mathbf{P}_0$ (the vector $\mathbf{n}$) depends on the orientation of the field in the plane of the magnetic material and also on the internal parameter ϕ_0 of the vortex. For $|\nu| \neq 1$, we have $P = 0$ with $v = 0$ for a solution of the type selected above. Below we restrict the discussion to small-radius vortices with $R \ll r_0$ and $|\nu| = 1$, but we assume $R \gg a$, as we must in order to find a correct macroscopic description. Under the condition $R \ll r_0$, we can use (3) for $\theta_0(r)$; we then easily find

$$P_0 = (E_0/2c^2) gHR. \tag{5'}$$

We can show that the condition $|\mathbf{P}| \propto R \neq 0$ with $v = 0$ leads to stability of a moving vortex with collapse; i.e., the vortex radius $R = R_0$ is fixed. We take a variational approach.[6] We choose a trial function in the form in (3) with $r \to r' = (x'^2 + y'^2)^{1/2}$, $\chi \to \chi' = \arctan(y'/x')$, $x' = x - v_x t$, $y' = y - v_y t$. We evaluate Lagrangian (1) as a function of the vortex velocity v and the parameters R and ϕ_0 of the trial function:

$L = L(\mathbf{v}, R, \phi_0)$. An extremum of L as a function of R and ϕ_0 determines the equilibrium parameters of the vortex (this extremum is not necessarily a minimum[3,6]). Using (3) and (4), we easily find

$$L(\mathbf{v},R,\phi_0) = E_0 v^2/2c^2 + (E_0/4c^2) g H R \mathbf{vn} - E_0 \; [1+\lambda(R/r_0)^2]. \tag{6}$$

For a given $v = |\mathbf{v}|$, the extremum in (6) corresponds to that value of ϕ_0 at which the condition $\mathbf{v}\cdot\mathbf{n} = v > 0$ holds [the physical meaning here is that the dynamic and static contributions to the magnetization of the vortex are parallel; see (2)]. This condition can be rewritten in the form $\eta + \nu(\phi_0 - \Phi) = \pi/2$, where η and Φ are the polar angles which specify the direction of the velocity $\mathbf{v}$ and the field $\mathbf{H}$; $\nu = \pm 1$; and ϕ_0 is the vortex parameter [see Ref. 5]. It is easy to verify that this condition can be satisfied for any orientation of $\mathbf{v}$ with respect to $\mathbf{H}$, by choosing the appropriate value of ϕ_0. Making use of the condition $\mathbf{v}\cdot\mathbf{n} = |\mathbf{v}|$, we easily find the following result for the equilibrium vortex radius R_0 in the leading-log approximation in R_0/r_0:

$$R_0 = r_0^2 \, vgH/gc^2\lambda', \quad \lambda' \simeq \ln(c^2/r_0 vgH). \tag{7}$$

We can now find the behavior of the "equilibrium" Lagrangian of the vortex, $L(v) \equiv L(\mathbf{v}, R_0, \phi_0)$, as a function of the vortex velocity $\mathbf{v}$, and we can determine the energy and momentum of the vortex:

$$\mathbf{P} = m_* \mathbf{v}, \quad E = E_0 + \frac{P^2}{2m_*}, \quad m_* = \frac{E_0}{c^2}\,[1 + A(gHr_0/c)^2] \simeq \frac{E_0}{c^2}, \quad A \simeq 1. \tag{8}$$

By virtue of the choice of the value of ϕ_0 which corresponds to an extremum of L, the energy does not depend on the orientation of the momentum with respect to the magnetic field. We have thus obtained the standard quadratic isotropic dispersion relation with an effective mass m_*, $m_* \simeq E_0/c^2$ at $H \ll H_0$. A calculation from (6) and (7) contributes terms of the type $A(gHr_0/c)^2 \simeq (H/H_0)^2$ to m_*; these terms are omitted from the final expression [here $H_0 = M_0(\beta\delta)^{1/2}/2$ is a typical value of the field]. In calculating the effective mass we need to allow for the circumstance that the actual solution for a moving vortex is of the form $\theta = \theta_0(r') + v\vartheta(x', y')$, $\phi = \nu\chi' + \phi_0 + v\psi(x', y')$, and we need to find the increments ϑ and ψ explicitly.[9] Going through this procedure, we find a correction to m_* which is on the same order, $(H/H_0)^2$ [omitted from (8)]. The actual calculation can be carried out as was done in Ref. 9 for a ferromagnet, but the analysis of that complicated problem goes beyond the scope of the present letter.

In the calculations we used the assumption $a \ll R_0 \ll r_0$, i.e., $(a/r_0)^2 \ll (v/c) \times (H/H_0) \ll 1$. Since H_0 is fairly large (H_0 is equal to the field of the spin-flip transition in the orientation $\mathbf{H} \parallel \mathbf{e}_z$), since it varies from a few to several tens of kiloersteds for various antiferromagnets, and since we have $(a/r_0)^2 \simeq \beta/\delta \leqslant 10^{-3}$–$10^{-4}$, this assumption holds over a wide range of vortex velocities for reasonable values of the field. We might add that similar effects ($\mathbf{P} \neq 0$ at $v = 0$ and a stability of moving vortices) result from certain types of "Dzyaloshinskiĭ" interactions, which can disrupt the Lorentz invariance of the dynamics of the vector $\mathbf{l}$.[4] In an antiferromagnet with an n-fold easy axis (n may be even or odd), vortices with $|\nu| = n/2$ or $|\nu| = n$ become stabilized. Correspondingly, we have $R_0 \propto (v/c) H_D/H_0$, where H_D is the corresponding "Dzyaloshinskiĭ" field.[4]

A stabilization of 3D topological solitons due to momentum conservation has been discussed by Volovik and Mineev[10] for the A phase of superfluid ^{3}He. A different behavior was found for this system: $R \propto 1/v$, $P \propto 1/v^3$, and $E \propto \sqrt{P}$. This behavior is characteristic of vortex rings in a liquid. Papanicolaou[11] has recently suggested that conservation of the momentum of a 3D soliton in a ferromagnet with a Hopf index can lead to its stabilization (for a soliton of this type we have $E \propto \alpha R + \beta R^3$ and $P \propto vR^2$). Writing $L = E - \mathbf{P} \cdot \mathbf{v} \propto \alpha R + \beta R^3 - vR^2/gM_0$, we find that an extremum of L can arise only at sufficiently high velocities, $v \geqslant gM_0 \sqrt{\alpha\beta}$. The conditions for the stabilization of 2D solitons in magnetic materials are thus less stringent than those for 3D solitons.

The density of a gas of vortices with a finite energy is finite at any temperature $T \neq 0$. The contribution of a gas of localized vortices to the response functions of 2D magnetic materials was discussed in Refs. 12 and 13, among other places, but the problem of a collapse of solitons arose. As we have shown in the present letter, for a wide range of antiferromagnets with disrupted Lorentz invariance, moving solitons are stable with respect to collapse. Consequently, it may be pertinent to take into account a gas of such solitons in analyzing the thermodynamics of quasi-2D antiferromagnets.

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