Plasma oscillations of superconducting electrons in layered superconductors

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Weakly damped collective oscillations in layered superconductors are examined theoretically. A mode found at low temperatures is reminiscent of Josephson plasma oscillations at a tunnel junction. Near T_c these oscillations convert into a Carlson–Goldman mode with a line spectrum. Possibilities for detecting these oscillations experimentally are discussed.

Collective oscillations in superconductors have been discussed in many places in the literature. Phase oscillations with an acoustic spectrum have been found by Bogolyubov[1] and Anderson[2] for superconductors with neutral electrons. In conventional superconductors, however, the Coulomb interaction transforms such modes into plasma oscillations with a frequency much higher than the energy gap. These oscillations thus differ slightly from plasma oscillations in normal metals. Weakly damped collective oscillations in real superconductors (the Carlson–Goldman modes) have been seen experimentally[3] and explained theoretically.[4,5] This mode exists only near the transition temperature T_c, has a line spectrum, and is associated with oscillations in the phase of the order parameter in an electric field (see the review[6]).

In the present letter we examine normal oscillation modes in a layered metal in which there is a weak coupling between layers. The plasma oscillations of such a system in its normal state have been studied in numerous places (see, e.g., Ref. 7, where there is a discussion of the role played by these oscillations in the mechanism for high-T_c superconductivity). We consider the superconducting state of a layered metal. We show that plasma oscillations of superconducting electrons can exist in such a medium. As the temperature approaches T_c, these oscillations transform into Carlson–Goldman modes. Our calculation is based on kinetic equations for Green's functions[8–10] which were generalized in Ref. 11 to the case of layered superconductors. Although we use equations for conventional superconductors, we assume that the conclusions reached remain qualitatively correct for high-T_c superconductors.

A distinctive features of the equations for a layered superconductor is that the Wannier site representation is used for the direction perpendicular to the layers, while the Green's functions for directions in the plane of the layers depend on the coordinates and on the directions of the momentum. The equations simplify dramatically in the strong-coupling approximation, in which only transitions between neighboring layers are taken into account, and also under the condition $\epsilon_1 \ll 1/\tau$ or Δ, where ϵ_1 is equal to one-fourth of the width of the energy band for motion of the electrons across the layers, τ is the momentum scattering time, and Δ is the energy gap. In this case the equations become (here and below, $\hbar = e = k_B = 1$)

$$iv\nabla\mathscr{G}_{nm}+\epsilon_1\sum_{\iota\pm1}(\hat{A}_{nn+1}\mathscr{G}_{n+\iota m}-\mathscr{G}_{nm+1}\hat{A}_{m+\iota m})+i\sigma_z\frac{\partial}{\partial t}\mathscr{G}_{nm}+\frac{\partial}{\partial t'}\mathscr{G}_{nm}\sigma_z$$

$$-h_n\mathscr{G}_{nm}+\mathscr{G}_{nm}h_m=(i/2\tau)(\mathscr{G}_{nn}\mathscr{G}_{nm}-\mathscr{G}_{nm}\mathscr{G}_m), \tag{1}$$

where $h_n=i\sigma_y\Delta_n+\mu_n+\sigma_z vp_{Sn}$, $\mu_n=1/2(\partial\chi_n/\partial t)+\Phi_n$, Φ_n is a matrix element of the electric potential between the wave functions of layer n, χ_n is the phase of the order parameter, and p_{Sn} and v are the superconducting momentum and the Fermi velocity along the layers. The Green's function $\mathscr{G}$ is the matrix

$$G=\begin{pmatrix}\hat{g}^R & \hat{g}\\ 0 & \hat{g}^A\end{pmatrix},$$

each of whose components is in turn a matrix in spin indices. The Pauli matrices and

$$\hat{A}_{nm}=\cos\left(\frac{\chi_n-\chi_m}{2}\right)+i\sigma_z\sin\left(\frac{\chi_n-\chi_m}{2}\right)$$

act on the spin indices. In addition, a convolution in time is to be understood in (1). The term on the right side of (1) is an elastic-collision integral. Energy dissipation is ignored under the assumption that the characteristic frequencies are greater than the reciprocal of the energy relaxation time.

The current densities parallel and perpendicular to the layers and the charge density are calculated from

$$j_{\parallel}\sim\int d\theta\,\mathrm{Tr}\,\sigma_z\hat{g}_{nn}(t=t'),$$

$$j_{\perp}\sim\int d\theta\,v\,\mathrm{Tr}\,\sigma_z(\hat{A}_{n+1n}\hat{g}_{nn+1}-\hat{A}_{nn+1}\hat{g}_{n+1n})(t=t'), \tag{2}$$

$$\rho_n\sim\int d\theta\,\mathrm{Tr}\,\hat{g}_{nn}(t=t'), \tag{3}$$

where θ is the polar angle.

To seek the spectrum of normal modes we find the Green's functions in the linear approximation in p_s, μ and the phase difference between neighboring layers, $\varphi=\chi_{n+1}-\chi_n$. We take time and coordinate Fourier transforms, going over from the coordinates r along the layer to q, and going over from the discrete variable n to $|k|<\pi/d$, where d is the period of the layered structure. We then use (2) and (3) to calculate the linear response. Substituting the expressions for the current and charge into Maxwell's equations, we then find the normal modes of the system.

We restrict the discussion to the long-wave limit $kd\ll1$, $qv\ll\Delta$, and to the case of low frequencies, $\omega\ll\Delta$. We first consider low temperatures, $T<\Delta$. For the current densities we find

$$j_{\parallel}=\left(\frac{c^2}{4\pi\lambda_{\parallel}^2}-i\sigma_{\parallel}\omega\right)p_s,\quad j_{\perp}=\left(\frac{c^2}{4\pi\lambda_{\perp}^2}-i\sigma_{\perp}\omega\right)\varphi/2d, \tag{4}$$

where $\lambda_{\parallel,\perp}$ are the magnetic-field penetration depths for the corresponding directions. We find $\lambda_\perp \sim (v/\epsilon_1 d)\lambda_\parallel$, while for $\lambda_\parallel$ we find the standard expression. If we had not linearized Eqs. (1) in φ, then at small values of ω and k we would have found, instead of (4), the superconducting part of the current $j_\perp$ (the first term in parentheses):

$$j_{s\perp} = \frac{c^2}{8\pi\lambda_\perp^2 d} \sin\varphi \equiv j_c \sin\varphi, \tag{5}$$

which reflects the Josephson coupling of the layers.

The second terms in expressions (4) for the current, which describe the contribution of quasiparticles, are small at low temperatures (we have written only those terms which dominate the damping of the oscillations of interest here):

$$\sigma_{\parallel,\perp} = \sigma_{N,\perp} \begin{cases} (2\pi\Delta/\omega)^{1/2}/(1+2\omega\Delta\tau^2)\exp(-\Delta/T), & \Delta \gg \omega \gg T, \\ \dfrac{\Delta}{T}\ln(T/\omega)\exp(-\Delta/T), & \omega \ll T,\ 1/\Delta\tau^2, \end{cases} \tag{6}$$

where σ_N is the conductivity which would prevail in the normal state.

The quasiparticle contribution can be ignored in the expression for the charge density. As a result, we find

$$\rho = -(k_0^2/4\pi)\mu, \tag{7}$$

where k_0^{-1} is the Thomas–Fermi screening radius in the normal state.

We now express the electric field in terms of μ, φ, and p_s: $E_\parallel = i(q\mu + \omega p_s)$, $E_\perp = i(k\mu + \omega\varphi/2d)$. We substitute (5) and (7) into Maxwell's equations. Equating the determinant of the resulting system of equations to zero, we find a dispersion relation for the normal modes:

$$\omega^2 = \omega_0^2(1+\epsilon_\perp k^2/k_0^2)(1+k^2\lambda_\parallel^2+q^2\lambda_\perp^2)/(1+k^2\lambda_\parallel^2), \quad \omega_0 = c/(\lambda_\perp\sqrt{\epsilon_\perp}), \tag{8}$$

where $\epsilon_\perp$ is the dielectric costant. For brevity we have omitted an exponentially small damping from (8), and we have taken the limit $q \ll k_0$.

The assumption $\omega \ll \Delta$ which was used in deriving (8) simplified the calculations. The *necessary* condition for the existence of slightly damped plasma oscillations in (8) is the condition $\omega_0 < \Delta$, which holds at sufficiently large values of $\lambda_\perp$, i.e., when the coupling between the superconducting layers is sufficiently weak.

If a constant current $j < j_c$ flows perpendicular to the layers, then we need to replace $\lambda_\perp$ in Eq. (8) by $\lambda_\perp/[1-(j/j_c)^2]^{1/2}$ because of (6). In other words, the frequency of the plasma oscillations decreases.

We now consider temperatures near T_c, at which we have $\Delta < T$, and at which the contributions of the quasiparticles to the current and to the charge are not small. The expressions we find in this case are analogous to those for the isotropic case (see the review[6]):

$$j_{\parallel}\ \frac{c^2}{4\pi\lambda_{\parallel}^2}\ p_s - i\sigma_{\parallel}\ [q\mu+\omega p_s(1+J)],$$

$$j_{\perp} = \frac{c^2}{8\pi\lambda_{\perp}^2 d}\varphi - i\sigma_{\perp}\left[k\mu+\omega\frac{\varphi}{2d}(1+J)\right], \tag{9}$$

$$\rho = -(k_0^2/4\pi)\left[\left(\delta+i\frac{D_{\parallel}q^2+D_{\perp}k^2}{\omega}\right)\mu+i\left(D\| \ qp_s+D\perp \ k\frac{\varphi}{2d}\right)\right]. \tag{10}$$

For $\delta=\pi\Delta/4T \ll 1$ we have $D_{\parallel}=v^2\tau/2$ and $D_{\perp}=2e_1^2d^2\tau$; the quantity J is small and depends on the parameter $\Delta\tau$:

$$J=\frac{\Delta}{2T}\ln\left(\frac{\Delta}{\omega}\right) \quad \text{for } \Delta\tau \gg 1, \quad J=\frac{\Delta}{T}\ln\left(\frac{\Delta}{\omega}\right) \quad \text{for } \Delta\tau \ll 1.$$

The conductivities $\sigma_{\parallel}$ and $\sigma_{\perp}$ in the limit $\Delta\to 0$ are the same as those in the normal state. We might add that Eqs. (9) and (10) hold under the condition $\omega \gg D_{\parallel}^2{}_q + D_{\perp} k^2$.

Proceeding as at low temperatures, we calculate the spectrum of weakly damped normal modes. The nature of their dispersion relation turns out to depend on the relation between ω_0 and the dielectric relaxation frequency $\omega_r=4\pi\sigma_{\perp}/\epsilon_{\perp}$. At low temperatures we have $\omega_0 \gg \omega_r$, and the plasma oscillations in (8) exist. Near T_c, this mode persists as long as the condition $\omega_0 \gg \omega_r$ holds. At $\Delta < T$, the spectrum of plasma oscillations in the long-wave limit is

$$\omega^2=\omega_0^2 - i\omega\omega_r + c^2q^2/\epsilon_{\perp} + D_{\perp}k^2\omega^2/\delta\omega_r. \tag{11}$$

In addition to mode (11), there is a Carlson–Goldman mode in the superconductor, which propagates along the layers.

Since ω_0 is proportional to Δ near T_c, as we raise the temperature further the opposite condition, $\omega_0 \ll \omega_r$, begins to hold. In this case the spectrum becomes acoustic and is the same as a Carlson–Goldman mode for all directions of the wave vector:

$$\omega^2=\frac{D_{\parallel}q^2}{4\pi\sigma_{\parallel}\delta}\ 1-\frac{i\omega 4\pi\sigma_{\parallel}\lambda_{\parallel}^2}{c^2}J+\frac{c^2}{i\omega 4\pi\sigma_{\parallel}\lambda_{\parallel}^2}+\frac{D_{\perp}k^2}{4\pi\sigma_{\perp}\delta}\left(1-\frac{i\omega 4\pi\sigma_{\perp}\lambda_{\perp}^2}{c^2}J+\frac{c^2}{i\omega 4\pi\sigma_{\perp}\lambda_{\perp}^2}\right). \tag{12}$$

We see that the region of weak damping is bounded from below and from above and exists to the extent that the quantity $J \sim \Delta/T\ln(\Delta\omega)$ is small.

Could this plasma mode be observed experimentally? Low temperatures, at which the damping is very slight, would seen to be more convenient for such an observation. Since this mode is associated with oscillations of the electric field, it should affect the impedance of a superconductor. Let us consider a sample whose dimensions in the plane of the layers are smaller than $\lambda_{\perp}$. We put this sample in an electric field of frequency ω which is directed perpendicular to the layers. The spatial distribution of this field near the sample is determined by the dependence $k(\omega)$ which is found from (8) in the case $q=0$. With the damping in (6) taken into account, this dependence is

$$k^2=k_0^2\left[\frac{\omega^2}{\omega_0^2-i\omega\omega_r}-1\right]/\epsilon_\perp . \tag{13}$$

Once we have calculated the field distribution in the sample, we can find the impedance. For a sample of thickness w in the direction normal to the layers, the impedance per unit area in the plane of the layers is, under the condition $\delta\omega=\omega-\omega_0\ll\omega_0$,

$$Z=\frac{2\pi i\omega}{\epsilon_\perp(\delta\omega+i\omega_r)}\left(1-\frac{\tan\kappa}{\kappa}\right),\quad \kappa=k_0w[(\delta\omega+i\omega_r/2)/2\omega_0\epsilon_\perp]^{1/2}. \tag{14}$$

We see from (14) that under the condition $\mathrm{Im}\kappa<1$ (in this case, w is smaller than the oscillation damping length) the function Z is an oscillatory function of ω. Under the condition $\delta\omega\gg\omega_r$, a maximum $\mathrm{Re}Z_n=32w/[\pi(2n+1)^2\omega_r]$ is reached at frequencies $\delta\omega_n=(\omega_0/2)[\pi(2n+1)/k_0w]^2$, while $\mathrm{Im}Z$ changes sign and also magnitude, by an amount equal to $\mathrm{Re}Z$. The minimum value of $\mathrm{Re}Z$ is reached at frequencies $\delta\omega_m=2\omega_0(\pi m/k_0w)^2$, where it is $\mathrm{Re}Z=\pi w\omega_r/[2\epsilon_\perp(\delta\omega)^2]$.

Near ω_0 we also find distinctive features in the reflection coefficient for an electromagnetic wave incident at an angle α on a plane parallel to the layers. In this case two waves arise in the superconductor. Under the condition $\delta\omega\gg\omega_0(\sin\alpha)/k_0\lambda_\parallel$, the wave vector of one of these waves is given by (13), while that of the other satisfies

$$k^2\lambda^2=-(1-4\pi i\omega\sigma_\parallel\lambda^2/c^2)(\omega^2-i\omega\omega_r-\omega_0^2\cos\alpha)/(\omega^2-i\omega\omega_r-\omega_0^2). \tag{15}$$

Since ω_r and $\sigma_\parallel$ are exponentially small, both modes are weakly damped under the condition $\omega_0<\omega<\omega_0/\cos\alpha$. As a result, if the plate thickness w is such that the condition $\mathrm{Im}kw<1$ holds, then the reflection and transmission coefficients oscillate. For the transmission coefficient (in the case $w\gg1/k_0$) we find

$$T=\frac{4b\cos^2\alpha}{\cos^2\alpha|\cos(kw)|^2+b(1+\cos\alpha)^2|\sin(kw)|^2}, \tag{16}$$

where $b=|\omega\lambda k/c|^2\ll1$, and k is taken from (15). It can be seen from (16) that under the condition $\omega>\omega_0$ the dependence of k on ω and on the angle α leads to oscillations in T.

We note in conclusion that our calculation, which is based on the theory for conventional (low-temperature) superconductors, is not strictly applicable to high-T_c superconductors. Nevertheless, the expressions found for the current density and the charge, (4)–(7) and (9), (10), have a clear physical meaning, and we believe that these expressions can be used for a phenomenological description of high-T_c superconductors. We also believe that they should give a qualitatively a correct description of the normal modes in these superconductors.

As we mentioned earlier, weakly damped plasma oscillations can also exist in the most anisotropic superconductors, in which the condition $\omega_0\ll\Delta$ holds. According to (9), a necessary condition here is that $\lambda_\perp$ be greater than the wavelength of light corresponding to the size of the energy gap. This condition apparently holds in BSCCO, in which the role of $\lambda_\perp$ is played by λ_c. This condition should also hold in the Y/PrBCO superconductors and in superconducting dichalcogenides of transition metals which have been intercalated to the point of a pronounced anisotropy.

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