Vortex core anomaly from the gapless fermions in the core

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The massless chiral fermions which are localized in the core of a quantized vortex in Fermi superfluids and superconductors produce an anomaly in the vortex core structure at low temperature. The result obtained by Kramer and Pesch[4] for axisymmetric vortex in a s-wave superconductor is generalized to the more complicated vortices.

1. INTRODUCTION

The gapless fermions interacting with the Bose fields of the order parameter lead to the anomalous behavior of superfluids and superconductors at low temperature, $T \ll T_c$. In the superfluid ^{3}He-A, where the gapless fermions are chiral, two classes of phenomena take place: 1) the chiral anomaly—nonconservation of the linear momentum of the coherent condensate motion due to the spectral flow, and 2) singularity in the gradient expansion of the order parameter field, which is equivalent to the zero-charge effect in particle physics, since it comes from the logarithmic polarization of fermionic vacuum (see the review article[1]).

In ordinary superconductors the massless chiral fermions appear in cores of quantized vortices.[2] They also lead to similar phenomena: 1) the spectral flow along the branches of chiral fermions in the vortex core gives rise to the momentum transfer from the superfluid component to the normal component,[3] and 2) the singularity in the order parameter field in the vortex core region—an anomalous increase in the slope of the order parameter at $T \ll T_c$—was analytically found in Ref. 4 and numerically confirmed in Ref. 5. The slope of the gap $\Delta(r)$ near the origin, $(d\Delta/dr)|_{r=0}$, increases as T_c/T at low T,[4] while the core size remains on the order of the coherence length ξ.[5] The latter singularity was calculated for the particular case of the axisymmetric vortex in the s-wave isotropic superconductor. Our goal was to determine how the singularity is modified in the case of more complicated vortices, including those in unconventional superfluids and superconductors, such as superfluid ^{3}He and high-T_c materials.

2. ANOMALOUS BRANCHES OF LOCALIZED FERMIONS

The quantized vortices in superfluids and superconductors contain the anomalous branches of the low-energy fermions which are localized in the vortex core.[2] The energy spectrum of these fermions in the semiclassical approximation is characterized by two quantum numbers: the momentum k_z along the vortex axis and the impact parameter $\tilde{y} = \hat{z} \cdot (\mathbf{q} \times \mathbf{r})/q = r \sin(\alpha - \phi)$, where $\mathbf{q}$ is the projection of $\mathbf{k}$ onto the x–y plane with $q^2 = k_F^2 - k_z^2$, α and ϕ are the angles in the x–y plane of $\mathbf{q}$ and $\mathbf{r}$, respectively.

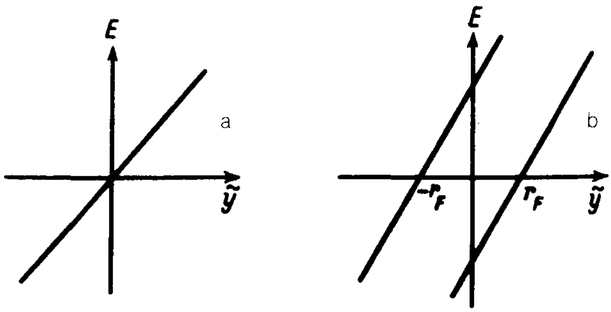

FIG. 1. Energy spectrum of fermionic zero modes localized in the vortices in terms of the impact parameter. a—One doubly degenerate branch of fermions in ordinary singly quantized vortices in an s-wave superconductor; b—two branches in doubly quantized vortex or in the ^{3}He-B vortex with broken parity, the Fermi surface appears at a finite impact parameter, $\tilde{y}=\pm r_F$.

In the conventional, singly quantized Abrikosov vortex in an ordinary s-wave superconductor, there are two identical branches corresponding to two spin projections with the spectrum

$$E(k_z,\tilde{y})=\tilde{y}q\omega_l(q). \tag{2.1}$$

In the quantum limit, which takes into account the quantization of the angular momentum, $q\tilde{y}=\hbar n_l$, the quantum $\omega_l(q)$ is the distance between the levels with neighboring n_l. Usually this interlevel distance is on the order of $\Delta^2(\infty)/E_F \ll \Delta(\infty)$ [$\Delta(\infty)$ is the gap far from the vortex, and E_F is the Fermi energy], and we consider the region $T \gg \omega_l$, where this quantization can be ignored. These branches are anomalous since, if they are considered as a continuous function of $\tilde{y}$, they cross zero energy level. The crossing occurs at $\tilde{y}=0$ for all k_z (Fig. 1a), which causes a one-dimensional Fermi surface (Fermi line) to form. At low temperature the sharp Fermi distribution of the chiral fermions in the vicinity of the Fermi line leads to the anomaly in the vortex core. This anomaly is strengthened by the unique circumstance that the position of the Fermi line does not depend on k_z.

The number of fermionic zero modes, N_{zm}, is completely defined by the topology of the vortex, i.e., by the winding number n of the vortex:[3] $N_{zm}=2n$. A similar relationship between the number of fermionic zero modes in the core of the string and the string winding number n is found in the relativistic field theories.[6,7] The difference is that in the core of the string the branches cross zero as functions of k_z, while in condensed matter the vortices cross zero as functions of $\tilde{y}$.

The fact that the anomalous branch in Eq. (2.1) crosses zero energy at zero impact parameter $\tilde{y}$ is the result of the symmetry of the vortex, which requires that $-E(-\tilde{y})$ should be the branch of the fermionic states in the vortex core. Therefore, if there is only one anomalous branch, then the crossing should occur at $\tilde{y}=0$, according to the equation $E(-\tilde{y})=-E(\tilde{y})$ for this unique branch. The situation is different in the case of vortices with winding number $n=2$, 3, etc., which contain several branches of zero modes. Several nonidentical branches can occur even in the case of singly quantized vortices, e.g., in the triplet-paired states, such as ^{3}He-A and ^{3}He-B, where the degeneracy over the spin states is lifted and the modes with different spin projections are not identical.

According to the relationship between the winding number and the number of branches,[3] in a doubly quantized vortex ($n=2$) in ordinary s-wave superconductors there should be two anomalous branches (each being degenerate over spin). It is evident from the symmetry consideration that there is no reason for two branches to cross zero at $\tilde{y}=0$. Instead the equation $E_1(-\tilde{y})=-E_2(\tilde{y})$ is satisfied if the branches cross zero at two different points $\tilde{y}=\pm r_F(q)$ (Fig. 1b), which are symmetric with respect to the origin:

$$E_1(k_z,\tilde{y})=[\tilde{y}-r_F(q)]q\omega_l(q), \quad E_2(k_z,\tilde{y})=[\tilde{y}+r_F(q)]q\omega_l(q). \tag{2.2}$$

The position of the Fermi surfaces, $\pm r_F(q)$, of zero modes therefore begins to depend on q, which should lead to the smoothing of the singularity in the vortex core. In the vortex with winding number $n=3$ two branches cross zero at mutually symmetric points $\tilde{y}=\pm r_F(q)$, while the third branch should be antisymmetric and should cross zero at $\tilde{y}=0$.

In the case of singly quantized vortices in the triplet-paired ^{3}He-B two anomalous branches (corresponding to two different spin components) have been calculated by Schopohl.[8] While for the most symmetric vortex, the o-vortex, the branches cross zero at $\tilde{y}=0$, for the vortex with a broken parity in the core, the v-vortex, the crossing occurs at finite $\tilde{y}=\pm r_F(q)$. The finite value of r_F takes place in the ^{3}He-A continuous vortices, where the parity is also broken.[9] This r_F depends on the angle in the x–y plane if the axial symmetry is broken in the vortex core.[8]

3. ORDER PARAMETER JUMP IN THE CONVENTIONAL AXISYMMETRIC VORTICES

Let us consider the influence of the zero modes on the gap function within the vortex core. The gap equation can be found from the BCS action which contains two terms:

$$S=\int d^3r dt \frac{|\Delta|^2}{g}+\mathrm{Tr}\ln(i\omega-\mathbf{H}), \tag{3.1}$$

where g is the interaction constant, and the second term, in which $\mathbf{H}$ is the Hamiltonian for fermions in the presence of the vortex, is the fermion contribution. The trace is over all the fermionic states ν, and also over the thermal frequencies. Variation over the gap function, $\delta S/\delta\Delta^*=0$, gives the self-consistent equation for the gap function:

$$\Delta(\mathbf{r})=g\sum_\nu u_\nu(\mathbf{r})v_\nu^*(\mathbf{r})\tanh\frac{E_\nu}{2T}, \tag{3.2}$$

where u_ν and v_ν are the Bogolyubov wave functions.

Let us first consider the ordinary vortices with the energy spectrum in Eq. (2.1). The low-energy anomalous branches (zero modes) lead to the singular contribution to the gap function:

$$\Delta_{\rm sing}(\mathbf{r}) = g \sum_{\mathbf{q}} u_{\mathbf{q}}(\mathbf{r}) v_{\mathbf{q}}^*(\mathbf{r}) \tanh \frac{rq\omega_l(q)\sin(\alpha-\phi)}{2T}, \tag{3.3}$$

which comes from the fact that in the intermediate asymptotic region, $T_c \gg T \gg \omega_l$, the sharp coordinate dependence of the Fermi function near the Fermi surface of the chiral fermions produces the abrupt behavior of the order parameter near the vortex axis. This behavior is characterized by the new scale

$$\xi_2 = k_F^{-1} \frac{T}{\omega_l} \sim \xi \frac{T}{T_c}. \tag{3.4}$$

At low temperatures $T \ll T_c$, when the Fermi function is narrow and close to the step function, this scale ξ_2 becomes smaller than ξ and begins to define the properties of the vortex core near the origin. In this low-temperature limit we can use the step function

$$\frac{rq\omega_l(t)\sin(\alpha-\phi)}{2T} \approx \Theta[r\sin(\alpha-\phi)] = \Theta(r)\Theta[\sin(\alpha-\phi)], \tag{3.5}$$

and we can ignore the coordinate dependence of the Bogolyubov functions which have the characteristic length scale on the order of $\xi \gg \xi_2$:

$$u_{\mathbf{q}}(0) v_{\mathbf{q}}^*(0) \equiv \lambda(q) e^{i\alpha} \sim e^{i\alpha} |\Delta(\infty)|. \tag{3.6}$$

The oscillations on the scale k_F^{-1} and the resulting Friedel oscillations of the gap function in the vortex core become important only at $T \sim \omega_l$.[4,5]

As a result, the Fermi-liquid distribution of quasiparticles which occupy the gapless branch results in the stepwise behavior of the order parameter with the infinite slope at the origin:

$$\Delta_{\rm sing}(\mathbf{r}) = g\Theta(r) \int_{-k_F}^{k_F} \frac{dk_x}{2\pi} \lambda(q) \int_0^{2\pi} \frac{d\alpha}{2\pi} e^{i\alpha} \Theta[\sin(\phi-\alpha)]$$

or

$$\Delta_{\rm sing}(\mathbf{r}) = \Theta(r) e^{i\phi} \, |\Delta_{\rm sing}(\infty)|, \tag{3.7}$$

where the contribution of the zero modes to the gap far from the vortex,

$$|\Delta_{\rm sing}(\infty) = g \frac{2}{\pi} \int_{-k_F}^{k_F} \frac{dk_z}{2\pi} \lambda(q), \tag{3.8}$$

is on the order of $g|\Delta(\infty)$. This contribution is small compared with the regular contribution only due to the small coupling constant g, which is on the order of $1/\ln(E_F/T_c)$. Therefore, the gap $\Delta(\mathbf{r})$ has a jump at the origin, from zero value to $\Delta_{\rm sin}(\infty)$. This jump is smoothed out over the distance on the order of $\xi_2 \ll \xi$, if one takes into account the finite T. Because of the regular contribution to the order parameter from the fermions with the gap, the order parameter will then gradually

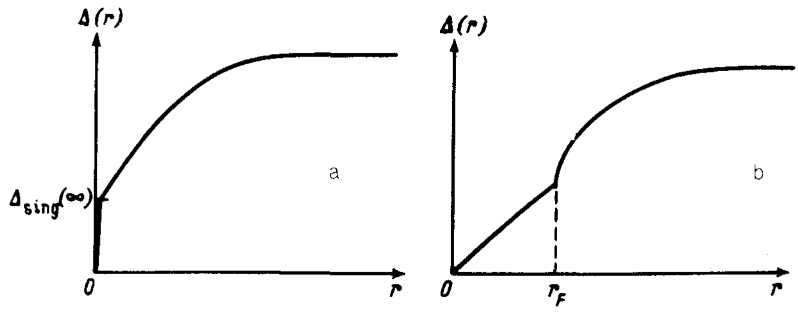

FIG. 2. Singularity in the order parameter within the vortex core due to zero modes. a—Stepwise discontinuity at the vortex axis in ordinary singly quantized vortices in an s-wave superconductor with zero modes in Fig. 1a; b—square-root singularity at a finite distance r_F in two-dimensional vortices with zero-mode behavior in Fig. 1b.

increase from $\Delta_{\rm sing}(\infty)$ to the value $\Delta(\infty)$ at a distance on the order of the usual coherence length ξ (see Fig. 2a). This double-scale behavior of the order parameter is in agreement with the numerical calculations.[5]

The radial derivative of the order parameter has the δ-function singularity which corresponds to the infinite slope at the origin:

$$\partial_r\Delta_{\rm sing}(\mathbf{r})=\delta(r)e^{i\phi}|\Delta_{\rm sing}(\infty)|. \tag{3.9}$$

4. ZERO MODES WITH A FERMI SURFACE AT THE FINITE IMPACT PARAMETER

Let us now consider how the result obtained above is modified when the Fermi surface of zero modes appears at a finite r_F in Eq. (2.2). In the axially symmetric vortex the distance of the Fermi surface from the origin, $r_F(q)$, does not depend on ϕ and is on the order of the coherence length in the ^{3}He-B vortices with broken parity.[8] It should be of the same order of magnitude in doubly quantized vortices in ordinary superconductors.

The contribution to the order parameter from the zero modes now contains two Fermi steps which are symmetrically shifted from the origin:

$$\Delta_{\rm sing}(\mathbf{r})=ge^{i\phi}\int_{-k_F}^{k_F}\frac{dk_z}{2\pi}\lambda(q)\int_0^{2\pi}\frac{d\alpha}{2\pi}\sin\alpha[\Theta(r\sin\alpha-r_F(q))$$
$$+\Theta(r\sin\alpha+r_F(q))]$$
$$=ge^{i\phi}\int_{-k_F}^{k_F}\frac{dk_z}{2\pi}\lambda(q)\sqrt{1-\frac{r_F^2(q)}{r^2}}\,\Theta[r-r_F(q)]\,. \tag{4.1}$$

We see that the singularity is essentially smoothed out because of the dependence of the Fermi surface on k_z (or q). The singularity can survive only for two-dimensional vortices, i.e., for the vortices in very thin films with size quantization, or in the layered materials, like high-T_c superconductors. In the first case, k_z is quantized, i.e., $q=k_F$ is fixed. As a result, the Fermi line of zero modes becomes the Fermi point. We have

$$\Delta_{\rm sing}(\mathbf{r})=e^{i\phi}|\Delta_{\rm sing}(\infty)|\sqrt{1-\frac{r_F^2}{r2}}\,\Theta(r-r_F)\ . \qquad (4.2)$$

This is the square-root singularity whose infinite slope is at the Fermi surface $r=r_F$, which appears against the background of the regular contribution (see Fig. 2b). At $r_F=0$, Eq. (4.2) transforms into Eq. (3.7).

If the axial symmetry is spontaneously broken, as in superfluid ^{3}He vortices, or externally broken in superconductors due to the presence of a crystal field, then the Fermi surface will depend on ϕ. The anisotropy of the Fermi surface leads to the smoothing out of the square-root singularity even in two-dimensional vortices.

5. CONCLUSION

The anomalous behavior of the order parameter in the vortex core in the temperature region $T_c \gg T \gg T_c^2/E_F$ is attributable to the Fermi surface (Fermi line) formed by the chiral fermions which are localized in the vortex core. The sharp distribution function of the fermions in the vicinity of the Fermi surface leads to a singularity in the order parameter. In the case of ordinary axisymmetric vortices in *s*-wave superconductors the Fermi surface is formed at the vortex axis, which results in the sharp stepwise distribution of the order parameter near the vortex axis. For the more complicated vortices, in which the Fermi surfaces of chiral fermions are formed far from the vortex axis, the singularity is integrated over the Fermi surface and is smoothed out. The only exception is represented by the two-dimensional systems (or the quasi-two-dimensional layered superconductors), in which the Fermi line of the chiral fermions becomes the Fermi point. In this case the square-root singularity in the order-parameter field is observed far from the vortex axis.

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