Statistics of charge fluctuations in quantum transport in an alternating field

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The distribution of charge fluctuations during current flow through a region with an alternating field which causes an inelastic scattering is analyzed. The distribution is formed by several independent Bernoulli random processes in which the frequencies of attempts are found as a function of the potential difference and the field frequency. The independent outcomes in one attempt correspond to a coherent passage of several electrons. The probabilities for the outcomes are expressed in terms of multiparticle scattering amplitudes.

Introduction. In quantum conductors at low temperatures, shot noise, which stems from the discrete nature of electric charge, is weakened to values below the classical level.[1–3] The reason lies in the Fermi statistics of electrons, which determines temporal correlations of the current. A complete statistical description of the noise is given by the distribution of the probabilities for charge to pass over a given time. For purely elastic scattering, the quantum noise at $T=0$ is described by Bernoulli statistics, which leads to a binomial distribution.[4]

This simple picture gives way to a more complex and more interesting one when the scattering is inelastic, since in this case the Fermi correlations between states with different energies become important.[5] Let us examine charge fluctuations in a case in which current is flowing through a region with a variable field, which alternates periodically in time. This case can be realized experimentally at a point contact, a tunnel junction, or an ordinary contact in a microwave field and also in a system which itself generates an rf field, as in the time-varying Josephson effect.

We will determine the statistics of the charge for an arbitrary relation between the field frequency Ω and the external voltage V. To take full account of the interference of states with energies differing by a multiple of $\hbar\Omega$, we introduce several auxiliary channels, corresponding to windows of width $\hbar\Omega$ along the energy scale. It then becomes possible to use a method developed for multichannel elastic scattering.[4] The statistics which we find has some interesting features as a function of V and Ω. At an integer value of $eV/\hbar\Omega$ the statistics corresponds to a generalized binomial distribution and arises by virtue of a random Bernoulli process with a test frequency Ω and with outcome probabilities which are expressed in terms of multiparticle scattering amplitudes. For arbitrary V, the statistics is a mixture of two "pure" Bernoulli components, corresponding to the integers N and $N+1$ which lie nearest $eV/\hbar\Omega$: $N<eV/\hbar\Omega<N+1$. As an example we discuss an interesting scattering law for which all the multiparticle amplitudes and the statistics can be found exactly.

In most cases of practical interest the junction can be regarded as quasi-1D, so we consider a 1D system in which charges pass through an oscillating barrier: a region with alternating fields, an electric one $\Phi(x,t)$ and a magnetic one $A(x,t)$. We assume that the field is strictly confined to the interval $-d<x<d$ and varies periodically in time, $\Phi(x,t+2\pi/\Omega)=\Phi(x,t)$, with a similar relation for $A(x,t)$. The solution for the quantum-mechanical problem

$$i\hbar\frac{\partial}{\partial t}\psi(x,t)=\left\{\frac{1}{2}\left[-i\frac{\partial}{\partial x}-\frac{e}{c}A(x,t)\right]^2+e\Phi(x,t)\right\}\psi(x,t)$$

leads to the scattering states

$$\psi_{L,k}(x,t)=e^{-iEt+ikx}+\begin{cases}\sum\limits_n B_{L,n}e^{-iE_nt-ik_nx}, & x<-d,\\ \sum\limits_n A_{L,n}e^{-iE_nt+ik_nx}, & x>d,\end{cases}$$

$$\psi_{R,k}(x,t)=e^{-iEt-ikx}+\begin{cases}\sum\limits_n A_{R,n}e^{-iE_nt-ik_nx}, & x<-d,\\ \sum\limits_n B_{R,n}e^{-iE_nt+ik_nx}, & x>d,\end{cases}\tag{1}$$

where $A_{L(R),n}$ and $B_{L(R),n}$ are functions of the initial energy E, and k_n and E_n are given by the relations $\hbar^2k_n^2/2m=E_n=E+n\hbar\Omega$ and $E=\hbar^2k^2/2m$. The transmission amplitude $A_{L(R),n}$ and the reflection amplitude $B_{L(R),n}$ correspond to an energy change of $n\hbar\Omega$; i.e., they contain both elastic $(n=0)$ and inelastic $(n\neq 0)$ channels.

Ignoring relaxation in the banks of the junction, we assume that the left and right states have an equilibrium Fermi filling, $n_{L(R)}(E)=1/(e^{(E-\mu_{L(R)})/T}+1)$. As usual, the potential difference is set by $eV=\mu_R-\mu_L$. The spin degeneracy g leads to g noninteracting scattering channels.

Relationship with the statistics of multichannel scattering. We are interested in the distribution of the charge which has passed through the barrier over a measurement time $t\gg\Omega^{-1}$. We will calculate it after we have found the relationship with the problem (already solved) of the statistics of charge in a system containing an arbitrary number M of channels between which the scattering is purely elastic.[4] In this case, a characteristic distribution function of the charge is given by

$$\chi(\lambda)=\exp\left[gt\int_{-\infty}^{\infty}\ln\chi_E(\lambda)\frac{dE}{2\pi\hbar}\right],$$
$$\chi_E(\lambda)=\det\,(1-n_E+n_E\mathbf{A}^+\bar{\mathbf{A}}),\tag{2}$$

where $\mathbf{A}_{jk}$ is an $M\times M$ scattering matrix, $\bar{\mathbf{A}}_{jk}=e^{i(\lambda_j-\lambda_k)}\mathbf{A}_{jk}$, $(\mathbf{n}_E)_{jk}=n_j(E)\delta_{jk}$, and $n_j(E)$ is the energy distribution of the particles in channel j. The argument of χ is the vector $\boldsymbol{\lambda}=(\lambda_1,...,\lambda_M)$, formed from auxiliary variables. As usual, the characteristic function χ is expanded in a Fourier series:

$$\chi_E(\tilde{\lambda})=\sum_{i_1,...,i_k;\, j_1,...,j_k} e^{i(\lambda_{j_1}+...+\lambda_{j_k}-\lambda_{i_1}-...-\lambda_{i_k})}P_{i_1,...i_k|\, j_1,...,j_k}, \tag{3}$$

where $P_{i_1,...i_k|j_1,...,j_k}$ is the probability for the transition of k charges from channels $i_1,..., i_k$ into channels $j_1,..., j_k$. Since the particles are indistinguishable, the order of the channels $i_1,...,i_k$ and $j_1,..., j_k$ is unimportant, and permutations are not considered in sum (3).

We will use result (2), (3) in the following way. We partition the energy scale into identical windows:

$$\mu_L+(m-1)\hbar\Omega<E<\mu_L+m\hbar\Omega. \tag{4}$$

We speak of the states with energies in these windows as individual scattering channels. In channel m we switch to a shifted energy $\tilde{E}=E-\mu_L-(m-1)\hbar\Omega$, $0<\tilde{E}<\hbar\Omega$. Since the energy changes by a multiple of $\hbar\Omega$ in the course of the scattering in the original problem, (1), the scattering for the shifted energy $\tilde{E}$ is purely elastic. We can thus use (2), (3), replacing $\int_{-\infty}^{\infty}dE$ by $\int_0^{\hbar\Omega}d\tilde{E}$. Instead of the two channels L and R of the original problem we now have an infinite number of scattering channels $|\alpha,m\rangle$, $\alpha=L,R$, and m is an arbitrary integer. The matrix $\mathbf{A}$ is expressed in terms of the Fourier harmonics (1) of the transmission and reflection amplitudes:

$$\langle\alpha_2\, m_2|\mathbf{A}|\alpha_1 m_1\rangle=\begin{cases}A_{\alpha_1,m_2-m_1}(E), & \alpha_1=\alpha_2\\ B_{\alpha_1,m_2-m_1}(E), & \alpha_1\neq\alpha_2\end{cases}, \tag{5}$$

where $E=\tilde{E}+\mu_L+m_1\hbar\Omega$. (We omit the energy $\tilde{E}$ from the name of the channel since it is conserved.) Since we are interested in the transition from channels $\alpha=L$ to $\alpha=R$, regardless of the change in m, we choose $\boldsymbol{\lambda}$ to be

$$\lambda_{\alpha,m}\begin{cases}\lambda, & \alpha=L\\ 0, & \alpha=R\end{cases}. \tag{6}$$

Now, after we have determined scattering channels (4), written scattering matrix (5), and chosen the vector $\boldsymbol{\lambda}$, the problem is one of determining the determinant $\chi_E(\boldsymbol{\lambda})$ in (2). The basic difficulty here is that the increase in the number of channels causes all the matrices to become infinite. A calculation of the determinant is thus not a trivial problem, in contrast with Ref. 4.

Dependence of the distribution on V, Ω, and t at $T = 0$. We can take a step forward in the calculation of $\chi(\lambda)$ by ignoring the energy dependence of matrix elements (5). This approach leads to substantial simplifications, as we will see. The meaning of this approximation is that we assume T, $\hbar\Omega$, and eV to be small in comparison with $\hbar/\tau_f$, where $\tau_f\simeq\hbar\partial\ln A_{\alpha,m}/\partial E$ is a time scale of the crossing of the barrier.

We will come back to the calculation of the determinant in a bit; first we derive a general expression for $\chi(\lambda)$ under the assumption that the determinant is known. The most interesting case is absolute zero, for which the integration over the energy in (2) leads to a very simple relationship between $\chi(\lambda)$ and $\chi_{\tilde{E}}(\lambda)$. By virtue of our assumption, the $\tilde{E}$ dependence of $\chi_{\tilde{E}}(\lambda)$ is due entirely to $\mathbf{n}_E$, so it becomes a steplike function at $T=0$ (because of the discontinuities in $\mathbf{n}_E$ at $E=\mu_L,\mu_R$).

We introduce the infinite matrices $\mathbf{S}_N(\lambda)=\mathbf{1}-\theta_N+\theta_N\mathbf{A}^+\tilde{\mathbf{A}}$. Here N is an arbitrary integer, and the matrix θ_N is given by

$$\langle\alpha_2\, m_2|\theta_N|\alpha_1\, m_1\rangle=\begin{cases}\delta_{\alpha_2\alpha_1}\delta_{m_2 m_1}, & \alpha_1=L, \quad m_1\leqslant 0,\\ 0, & \alpha_1=L, \quad m_1>0,\\ \delta_{\alpha_2\alpha_1}\delta_{m_2 m_1}, & \alpha_1=R, \quad m_1\leqslant 0,\\ 0, & \alpha_1=R, \quad m_1>N.\end{cases} \tag{7}$$

More simply, we have $\theta_N=\mathbf{n}_E$ under the conditions $\mu_R-\mu_L=\hbar\Omega$, $T=0$. The matrices $\mathbf{S}_N(\lambda)$ give us possible values of the matrix $\mathbf{1}-\mathbf{n}_E+\mathbf{n}_E\mathbf{A}^+\tilde{\mathbf{A}}$ as a function of E at $T=0$. There is another way to describe the structure of the matrices $\mathbf{S}_N(\lambda)$— by using the columns of the matrix $\mathbf{A}^+\tilde{\mathbf{A}}$: We select from $\mathbf{A}+\tilde{A}$ exclusively those columns which have indices (L,m), $m\leqslant 0$, and (R,m'), $m'\leqslant N$, and we replace the other columns $(m>0,\ m'>N)$ by the corresponding columns of the unit matrix $\mathbf{1}$.

We now express $\chi(\lambda)$ in terms of $\chi_N(\lambda)=\det\mathbf{S}_N(\lambda)$. To do this we find the window in (4) in which μ_R fell; i.e., we choose N to satisfy $\mu_L+(N-1)\hbar\Omega < \mu_R < \mu_L+N\hbar\Omega$. In accordance with our comment regarding the steplike function $\chi_{\tilde{E}}(\lambda)$ we find

$$\mathbf{1}-\mathbf{n}_E+\mathbf{n}_E\,\mathbf{A}^+\tilde{\mathbf{A}}=\begin{cases}\mathbf{S}_N(\lambda), & 0<\tilde{E}<\hbar\Omega_V\\ \mathbf{S}_{N-1}(\lambda), & \hbar\Omega_V<\tilde{E}<\hbar\Omega\end{cases}, \tag{8}$$

where $\Omega_V=(\mu_R-\mu_L)/\hbar-(N-1)\Omega$. Replacing $\mu_R-\mu_L$ by eV and recalling that the integration over $\tilde{E}$ in (2) should be over the range $0<\tilde{E}<\hbar\Omega$, we finally find

$$\chi(\lambda)=[\chi_N(\lambda)^{g\Omega_V t/2\pi}\,[\chi_{N-1}(\lambda)]^{g(\Omega-\Omega_V)t/2\pi}, \quad \Omega_V=eV/\hbar-(N-1)\Omega. \tag{9}$$

The factorization of $\chi(\lambda)$ into two terms and the exponential t dependence of each of the two mean that the distribution is obtained as the result of two independent random Bernoulli processes, for which the probabilities of the outcomes in a single attempt are given by a Fourier decomposition of $\chi_{N-1}(\lambda)$ and $\chi_N(\lambda)$, and the frequencies of attempts are $g\Omega_V/2\pi$ and $g(\Omega-\Omega_V)/2\pi$, respectively.

Relation (9) expresses the distribution of probabilities in terms of "elementary" distributions P_N^k, for each N, given by a Fourier expansion

$$\chi_N(\lambda)=\sum_{k=-\infty}^{\infty} P_N^k e^{i\lambda k}. \tag{10}$$

The probabilities P_N^k are constants which can in principle be expressed in terms of the Fourier harmonics (1) of the scattering amplitudes. As can be seen from (9), the mean value of any quantity F is written in terms of the mean values over elementary distributions (10). A particularly simple relationship results for the cumulants (irreducible means):

$$\langle\langle F\rangle\rangle=\frac{g\Omega_V t}{2\pi}\langle\langle F\rangle\rangle_N+\frac{g(\Omega-\Omega_V)t}{2\pi}\langle\langle F\rangle\rangle_{N-1}. \tag{11}$$

Since the P_N^k are simply constants, expression (11) gives us an explicit expression for any mean value in terms of V, Ω, and t.

Expression (11) can serve to explain the results found in calculations of current fluctuations.[5] It has been shown that the noise intensity $S_\Omega=\langle\langle I_\omega I_{-\omega}\rangle\rangle$ is, for $\omega=0$, a piecewise-linear function of V with cusplike singularities at all $V_N=N\hbar\Omega/e$. In light of (11), this result takes on a totally clear meaning, since each V_N is a threshold whose crossing is accompanied by a change in the structure of the probability distribution. At $V=V_N$ the distribution is "pure," i.e., simply P_N^k, while at $V_N<V<V_{N+1}$ it is a mixture of P_N^k and P_{N+1}^k in the proportions $(V_{N+1}-V):(V-V_N)$. The distributions on the different sides of V_N are mixtures of different distributions: P_N^k, P_{N-1}^k for $V<V_N$ and P_N^k, P_{N+1}^k for $V>V_N$. The derivative $\partial S_0/\partial V$ must therefore have a discontinuity at $V=V_N$, and this discontinuity can be expressed in terms of the second moment of the distributions P_N^k, P_{N-1}^k, and P_{N+1}^k in the following way:

$$\frac{ge^3}{\pi\hbar}(2\langle\langle k^2\rangle\rangle_N-\langle\langle k^2\rangle\rangle_{N-1}-\langle\langle k^2\rangle\rangle_{N+1}). \tag{12}$$

[It is simple to verify that we have $S_0\langle\langle q^2(t)\rangle\rangle/\pi t$, where $q(t)$ is the charge which has passed over a time t.]

Calculation of probabilities. As an example we consider the case in which amplitudes (1) have only two harmonics:

$$A_{L,0}=A,\quad A_{L,-1}=a,\quad B_{L,0}=B,\quad B_{L,-1}=b,$$

$$A_{R,m}=\bar{A}_{L,-m},\quad B_{R,m}=-\bar{B}_{L,-m}. \tag{13}$$

Unitarity requires $|A|^2+|a|^2+|B|^2+|b|^2=1$, $A\bar{a}+B\bar{b}=0$. A distinctive feature of the scattering amplitudes selected here is that the energy cannot change by more than a fixed value—$\hbar\Omega$ in the case at hand—during the scattering. For all amplitudes which have this property a calculation of the determinant of the infinite matrix $\mathbf{S}_N$ reduces to taking the square root of the determinant of a finite matrix. It is easy to verify that we have

$$\det \mathbf{S}_N \det \mathbf{S}_N^+=\det\,(\mathbf{1}+\theta_N\,\mathbf{A}^+\,\tilde{\mathbf{A}}(\mathbf{1}-\theta_N)+(\mathbf{1}-\theta_N)\tilde{\mathbf{A}}^+\mathbf{A}\theta_N), \tag{14}$$

since $\theta_N^2=\theta_N$ and $(\mathbf{1}-\theta_N)^2=\mathbf{1}-\theta_N$. Since we have $\theta_N(\mathbf{1}-\theta_N)=0$, we note that $\theta_N\,\mathbf{A}^+\tilde{\mathbf{A}}(\mathbf{1}-\theta_N)$ and $(\mathbf{1}-\theta_N)\tilde{\mathbf{A}}^+\mathbf{A}\theta_N$ have only a finite number of nonzero matrix elements. For this reason, the matrix in the determinant on the right side of (14) differs from $\mathbf{1}$ in only a finite number of rows and columns. In principle, evaluating such a determinant is an elementary matter.

Instead of using the method described above, we will calculate $\chi(\lambda)$ by a method whose physical meaning is more transparent. We begin with the following comment:

The probabilities in (3) are determined exclusively by *filled* channels. These probabilities thus cannot change upon changes in the amplitudes for scattering from empty channels. We are thus justified in performing any transformation of the matrix **A** which involves only those columns with indices (L, m), $m>0$, and (R, m), $m>N$, and which leaves this matrix unitary. We choose new amplitudes A' in the following way. We set $N'=1+\max(0, N)$. In the channel $|R, N'\rangle$ we retain only the two amplitudes

$$\langle R, N'|\mathbf{A}'|R, N'\rangle = u\bar{A}, \quad \langle L, N'|\mathbf{A}'|R, N'\rangle = -u\bar{B}, \quad u=1/\sqrt{|A|^2+|B|^2}, \tag{15}$$

while in the higher-energy channels we set $\langle \alpha_2 m_2|\mathbf{A}'|\alpha_1 m_1\rangle = \delta_{\alpha_2\alpha_1}\delta_{m_2 m_1}$, $m_1 > N'$. In other words, we simply insert the columns of a unitary matrix. We obtain

$$
\mathbf{A}' = \begin{array}{c}
\begin{array}{ccccc} \overbrace{\hphantom{-\bar{B}\;\; A}}^{m=N'-3} & \overbrace{\hphantom{-\bar{B}\;\; A}}^{m=N'-2} & \overbrace{\hphantom{-\bar{B}\;\; A}}^{m=N'-1} & \overbrace{\hphantom{-\bar{B}\;\; A}}^{m=N'} & \overbrace{\hphantom{-\bar{B}\;\; A}}^{m=N'+1} \end{array} \\
\left[\begin{array}{cccccccccccc}
\ddots & & \ddots & & & & & & & & & \\
 & A & -\bar{B} & a & 0 & & & & & & & \\
\ddots & B & \bar{A} & b & 0 & & & & & \mathrm{O} & & \\
 & 0 & -\bar{b} & A & -\bar{B} & a & 0 & & & & & \\
 & 0 & \bar{a} & B & \bar{A} & b & 0 & & & & & \\
 & & & 0 & -\bar{b} & A & -\bar{B} & a & 0 & & & \\
 & & & 0 & \bar{a} & B & \bar{A} & b & 0 & & & \\
 & & & & & 0 & -\bar{b} & A & -u\bar{B} & 0 & 0 & \\
 & & & & & 0 & \bar{a} & B & u\bar{A} & 0 & 0 & \\
 & & \mathrm{O} & & & & & 0 & 0 & 1 & 0 & \ddots \\
 & & & & & & & 0 & 0 & 0 & 1 & \\
 & & & & & & & & & \ddots & & \ddots
\end{array}\right]
\end{array} \tag{16}
$$

It is not difficult to verify that the replacement of **A** by **A'** leaves the matrix unitary. The result of this transformation, which is important for our purposes, is that the scattering from channels $m \leqslant N'$ into empty channels $m > N'$ has disappeared. For this reason, the channels $m > N'$ can be eliminated from the problem altogether. We thus obtain a new formulation of the problem, in which the number of empty channels is *finite*.

The last step is to go over to holes. In this case the empty channels become filled channels and the filled ones empty. By virtue of this switch from **A** to **A'** the number of filled (by holes) channels is finite, equal to $|N|+2$. It thus becomes possible to express the multiparticle probabilities in Eq. (3) in the standard way as alternating products of one-particle hole amplitudes.[4]

Let us assume that there are two hole channels in the case $N=0$. The probabilities for the elementary processes are thus the squares of two-particle amplitudes given by the minors of the 2×2 matrix **A'**. The columns of the minors should be taken to be those corresponding to two empty channels $m=N'$ in matrix (16). We find the five nonzero amplitudes: $-au\bar{B}$, $au\bar{A}$, $-bu\bar{B}$, $bu\bar{A}$, and u^{-1}. The charge flux is -1 for the first amplitude, $+1$ for the fourth, and zero for the others. We then find the probabilities to be

$$P_0^0=u^2(|a|^2|A|^2+|b|^2|B|^2)+|A|^2|B|^2,$$

$$P_0^{-1}=u^2|a|^2|B|^2,$$

$$P_0^1=u^2|b|^2|A|^2.$$

For $N=-1$ there are three hole channels, so we need to take three-particle amplitudes, constructing 3×3 minors for the last three columns of $\mathbf{A}'$ with $m\leqslant N'$. We find only two amplitudes, $-u\bar{B}$ and $u\bar{A}$, whose squares give us probabilities:

$$P_{-1}^0=u^2|B|^2,\quad P_{-1}^1=u^2|A|^2.$$

For an arbitrary N we find the probabilities P_N^k from recurrence relations. The latter can be derived by expanding the corresponding $(|N|+2)\times(|N|+2)$ minors into first and second columns. For $N<0$ we find

$$P_N^k(|a|^2+|A|^2P_{N+1}^{k-1}+(|b|^2+|B|^2)P_{N+1}^k-|a|^2|A|^2(P_{N+2}^{k-2}-2P_{N+2}^{k-1}+P_{N+2}^k).$$

It is now a simple matter to find the generating function

$$P^-(x,z)=zu^2\frac{|A|^2x+|B|^2-|A|^2|a|^2z(x-1)^2}{1-z[(|A|^2+|a|^2)x+|B|^2+|b|^2]+|A|^2|a|^2z^2(x-1)^2}. \tag{17}$$

An expansion of this function in a power series in z and x yields the probabilities

$$P^-(x,z)=\sum_{N<0}\sum_{k=0}^{|N|}P_N^k x^k z^{|N|}. \tag{18}$$

The generating functions $\chi_N(\lambda)$ are related to P^- by

$$P^-(e^{i\lambda},z)=\sum_{N<0}z^{|N|}\chi_N(\lambda). \tag{19}$$

We can deal with $N>0$ in a corresponding way. In this case it is convenient to introduce

$$P^+(x,z)=\sum_{N\geqslant -1}\sum_{k=-1}^{|N|}P_N^{-k}y^k z^N. \tag{20}$$

Constructing recurrence relations, we find

$$P^+(y,z)=\frac{u^2}{yz}\,\frac{|A|^2+|B|^2y-|A|^2|B|^2z(y-1)^2}{1-z[(|A|^2+|a|^2)y+|B|^2+|b|^2]+|A|^2|a|^2z^2(y-1)^2}. \tag{21}$$

As in the case of P^-, the functions $\chi_N(\lambda)$ are found through an expansion in z; in the P^+ case, this expansion begins with z^{-1}.

Although our switch to the hole problem does simplify the calculations greatly, this switch is not completely trivial. In deriving the matrix $\mathbf{A}'$ by pruning $\mathbf{A}$ and then orthogonalizing, we obtain hole amplitudes of the type $-u\bar{B}$, which are not present among the matrix elements of $\mathbf{A}$ or $\mathbf{A}^{+}$.

An important feature of the problem, which is illustrated by the explicit expressions for the probabilities in (17) and (21), is the "elementary nature" of the P_N^k distributions. Distributions with larger N do not reduce to distributions with smaller N, in contrast with, say, $P_N^k = p^k(1-p)^{N-k}C_N^k$, for which the characteristic functions factorize: $\chi_N(\lambda) = [\chi_1(\lambda)]^N$, $\chi_1(\lambda) = 1-p+pe^{i\lambda}$. The reason why our $\chi_N(\lambda)$ cannot be factorized is that the different outcomes are coherent multiparticle scattering processes. The probabilities for these processes, given by the squares of multiparticle amplitudes, do not reduce to one-particle probabilities.

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[1] G. B. Lesovik, JETP Lett. **49**, 513 (1989).
[2] B. Yurke and G. P. Kochanski, Phys. Rev. **41**, 8184 (1989).
[3] M. Büttiker, Phys. Rev. Lett. **65**, 2901 (1990); M. Büttiker, in *Granular Nanoelectronics* (ed. D. K. Ferry) (Plenum Press, New York, 1991).
[4] L. S. Levitov and G. B. Lesovik, JETP Lett. (in press).
[5] G. B. Lesovik and L. S. Levitov, "Noise in an AC biased junction. Non-stationary Aharonov-Bohm effect" (preprint).

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