Superconductivity with lines of GAP nodes: density of states in the vortex

G. E. Volovik
Landau Institute for Theoretical Physics, 117334, Moscow, Russia

(Submitted 23 August 1993)
Pis'ma Zh. Eksp. Teor. Fiz. **58**, No. 6, 457–461 (25 September 1993)

The density of states (DOS) produced by the vortices in the superconductors with the lines of zeros in the electronic energy spectrum is calculated with application to high-temperature superconductors. The DOS of the isolated vortex is $\propto N_F\xi\cdot\min\{R,\lambda\}$, where N_F is the DOS of the normal metal, ξ is the coherence length, λ is the penetration depth, and R is the distance between the vortices. Only a small part of the DOS results from the fermions which are localized in the vortex core. The symmetry of the isolated vortex line in superconductors in which the gap corresponds to the Γ_3 representation is discussed.

1. INTRODUCTION

Recent experiments with angle-resolved photoemission[1] revealed the existence of lines of the gap nodes in the high-temperature superconductor $Bi_2Sr_2CaCu_2O_8$. The position of the gap nodes allows us to identify the symmetry class of superconductivity. The pairing occurs into the spin-singlet state described by the one-dimensional Γ_3 representation of the (approximate) tetragonal symmetry group D_4 of the CuO_2 planes. This superconducting state corresponds to the symmetry class $D_4(D_2)\times T$ in the classification scheme of Ref. 2, where T is the time inversion symmetry. The gap function of such unconventional superconductivity has the general form

$$\Delta(\mathbf{k},\mathbf{r})=[(\mathbf{k}\cdot\hat{a})^2-(\mathbf{k}\cdot\hat{b})^2]f(\mathbf{k})\Psi(\mathbf{r}), \qquad (1.1)$$

where the real function $f(\mathbf{k})$ has the symmetry D_4 of the normal metal; $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ are unit vectors along the Cu–O bond directions (x and y), where z runs along the fourfold symmetry axis. The complex scalar $\Psi(\mathbf{r})$ is the order parameter, which depends on the coordinate $\mathbf{r}$ of the center of mass of the Cooper pair. Although the complete form of the gap function can be found only from the microscopic theory, the important property of its symmetry—the position of the gap nodes—does not depend on the particular features of the system. The energy of the Bogolubov excitations,

$$E(\mathbf{k})=\sqrt{\varepsilon^2(\mathbf{k})+|\Delta(\mathbf{k})|^2}, \qquad (1.2)$$

is zero on the four lines $k_n(k_z)$ $(n=1,2,3,4)$ defined by the equations $\mathbf{k}\cdot\hat{\mathbf{a}}=\pm\mathbf{k}\cdot\hat{\mathbf{b}}$, $\varepsilon(\mathbf{k})=0$.

The presence of the lines of nodes influences the low-temperature properties of a superconductor. In particular, it produces the power-law temperature dependence of the penetration depth, observed in $YBa_2Cu_3O_7$—a superconductor with CuO_2 planes.[3]

Here we consider the effect of nodes on the electronic properties of the vortex lines in $Bi_2Sr_2CaCu_2O_8$ and calculate the DOS in a mixed state of the superconductor in a magnetic field.

2. THE DOS OF THE FERMIONS LOCALIZED IN THE VORTEX CORE

In conventional superconductors (without gap nodes) the DOS, $N(0) \propto N_F \xi^2$, comes from the branch of localized fermions with low energy.[4] According to the general topological property, such anomalous branch, which crosses zero as a function of the impact parameter, should exist also for vortices in the case of unconventional pairing.[5] Let us therefore start with the DOS which comes from the localized fermions that occupy the anomalous branch.

The Bogolubov Hamiltonian for the fermions with a given spin projection is a 2×2 matrix

$$\mathbf{H} = \hat{\tau}_3 \varepsilon(\mathbf{k}) + \hat{\tau}_1 \operatorname{Re} \Delta(\mathbf{k},\mathbf{r}) - \hat{\tau}_2 \operatorname{Im} \Delta(\mathbf{k},\mathbf{r}). \tag{2.1}$$

As will be shown below, the main contribution to the DOS comes from the vicinity of the gap nodes and from outside the vortex core. If the azimuthal angle α of $\mathbf{k}$ is close to $\alpha_n = \pi/4(1+2n)$, the gap function outside the core of the vortex with winding number 1 has the general form

$$\Delta(\mathbf{k},\mathbf{r}) \approx \vec{\gamma}_n(k_z) \cdot [\mathbf{k} - \mathbf{k}_n(k_z)] e^{i\phi}, \tag{2.2}$$

where $\vec{\gamma}_n(k_z) \| \hat{z} \times \mathbf{k}_n$, and ϕ is the azimuthal angle of $\mathbf{r}$. The energy spectrum of electrons in the normal metal is

$$\varepsilon(\mathbf{k}) \approx -i\mathbf{v}_F(\mathbf{k}_n) \cdot \vec{\nabla} = (-i) v(k_z)(\cos \alpha_n \nabla_x + \sin \alpha_n \nabla_y). \tag{2.3}$$

Here only the gap slope at the node, $\gamma(k_z) = |\vec{\gamma}_n(k_x)|$, and the transverse component of the Fermi velocity at the nodes, $v(k_z) = |\mathbf{v}_{F\perp}(\mathbf{k}_n)|$, are defined by the particular features of the system and can be considered as phenomenological parameters.

Let us introduce a new coordinate and momentum variables

$$\tilde{x} = \mathbf{r} \cdot \hat{k}_{n\perp}, \quad \tilde{y} = \mathbf{r} \cdot \hat{\gamma}_n, \quad \tilde{k}_x = (\mathbf{k} - \mathbf{k}_n) \cdot \hat{k}_{n\perp}, \quad \tilde{k}_y = (\mathbf{k} - \mathbf{k}_n) \cdot \hat{\gamma}_n,$$

where $\hat{k}_{n\perp} = \mathbf{k}_{n\perp} / |\mathbf{k}_{n\perp}|$ are the unit vectors in the direction of the gap nodes in the a–b plane, $\hat{\gamma}_n = \vec{\gamma}_n / |\gamma|$ are the unit vectors in the perpendicular directions, and $\tilde{y}$ is the impact parameter. For a small impact parameter we then have

$$e^{i\phi} \approx e^{i\alpha} (\operatorname{sign})(\tilde{x}) - i \frac{\tilde{y}}{|\tilde{x}|}), \tag{2.4}$$

and the Hamiltonian is

$$\mathbf{H} = \mathbf{H}^{(0)} + \mathbf{H}^{(1)},$$

$$\mathbf{H}^{(0)} = -i\hat{\tau}_3 v(k_z) \nabla_{\tilde{x}} + \tilde{k}_y \gamma(k_z)(\cos \alpha_n \hat{\tau}_1 + \sin \alpha_n \hat{\tau}_2) \operatorname{sign}(\tilde{x}), \tag{2.5}$$

$$\mathbf{H}^{(1)} = \tilde{k}_y \gamma(k_z)(\cos \alpha_n \hat{\tau}_2 - \sin \alpha_n \hat{\tau}_1) \frac{\tilde{y}}{|\tilde{x}|}.$$

The Hamiltonian $\mathbf{H}^{(0)}$ has a zero eigenvalue with the localized eigenfunction

$$\Psi^{(0)}(\tilde{x})=\sqrt{|\tilde{k}_y|\frac{\gamma(k_z)}{2v(k_z)}}[1-\mathrm{sign}(\tilde{k}_y)(\cos\alpha_n\hat{\tau}_2-\sin\alpha_n\hat{\tau}_1)]\exp\left\{-\frac{\gamma(k_z)}{v(k_z)}\left|\tilde{x}\tilde{k}_y\right|\right\}, \quad (2.6)$$

and from the first order in perturbation $\mathbf{H}^{(1)}$ we obtain the spectrum of the anomalous branch of localized fermions:

$$E(\tilde{y},\tilde{k}_y,k_z)=2\tilde{y}\tilde{k}_y^2\frac{\gamma^2(k_z)}{v(k_z)}\int_\xi^\infty\frac{d\tau}{\tau}\exp\left\{-\frac{2\gamma(k_z)}{v(k_z)}\left|\tilde{k}_y\right|\tau\right\}\approx 2\tilde{y}\tilde{k}_y^2\frac{\gamma^2(k_z)}{v(k_z)}\ln\frac{k_F}{|\tilde{k}_y|}. \quad (2.7)$$

Here we used an estimate $\gamma(k_z)/v(k_z)\sim(\xi k_F)^{-1}$. The spectrum crosses zero at $\tilde{y}=0$ and it also touches zero at $\tilde{k}_y=0$. It should be kept in mind, however, that Eq. (2.7) is valid if its effective range of integration does not exceed the intervotex distance R or the penetration depth λ. This gives the condition for the angles at which the spectrum (2.7) holds:

$$k_F \gg |\tilde{k}_y| \gg \frac{v(k_z)}{\gamma(k_z)\min\{R,\lambda\}}\sim k_F\frac{\xi}{\min\{R,\lambda\}}. \quad (2.8)$$

Below the limit $v\{k_z)/[\gamma(k_z\}\min\{R,\lambda\}]$ the states are delocalized.

The anomalous branch of localized fermions gives the following contribution to the DOS:

$$N_{\mathrm{loc}}(0)=\int\frac{d\tilde{k}_y\,d\tilde{y}}{2\pi}\int\frac{dk_z}{2\pi}\delta[E(\tilde{y},\tilde{k}_y,k_z)]$$

$$=\int\frac{dk_z}{2\pi}\frac{v(k_z)}{\gamma^2(k_z)}\int\frac{d\tilde{k}_y}{2\pi\tilde{k}_y^2\ln(k_F/|\tilde{k}_y|)}. \quad (2.9)$$

The integral over $\tilde{k}_y$ diverges at small $\tilde{k}_y$, and the constraints in Eq. (2.8) give the following estimate for the contribution of the localized states:

$$N_{\mathrm{loc}}(0)\approx\frac{\min\{R,\lambda\}}{\ln(\min(R,\lambda\}/\xi)}\int\frac{dk_z}{2\pi\gamma(k_z)}\sim N_F\xi\cdot\min\{R,\lambda\}/\ln\frac{\min\{R,\lambda\}}{\xi}. \quad (2.10)$$

3. THE DOS OF DELOCALIZED FERMIONS

The divergence of the density of localized states at small angles, i.e., at the edge of the continuum, means that the main contribution to the DOS comes from the delocalized states. For these states we can use the semiclassical approach in which the local energy is Doppler shifted by the local superfluid velocity $\mathbf{v}_s$:

$$N_{\mathrm{deloc}}(0)=2\int\frac{d^3k}{(2\pi)^3}\int d^2r\,\delta[E(\mathbf{k},\mathbf{r})+m_e\mathbf{v}_F\cdot\mathbf{v}_s], \quad (3.1)$$

where m_e is the electron mass. The main contribution to the DOS comes from the vicinity of the gap nodes in the momentum space and from the region far outside the vortex core in the real space:

$$N_{\mathrm{deloc}}(0)=\frac{1}{4\pi^3}\sum_n \int d\tilde{k}_y dk_z \frac{d\varepsilon}{v(k_z)} d^2\tau\delta[\sqrt{\varepsilon^2+k_y^2\gamma^2(k_z)}+m_e\mathbf{v}_F(\mathbf{k}_n)\cdot\mathbf{v}_s]$$
$$=\int \frac{dk_z}{2\pi^2\gamma(k_z)} \sum_n \int d^2\tau|m_e\mathbf{v}_s\cdot\tilde{k}_{n\perp}|. \tag{3.2}$$

For the isolated vortex the superfluid velocity at the distance $\xi\ll\tau\ll\min\{R,\lambda\}$ is $\mathbf{v}_s=(\hbar/2m_e)(\hat{\phi}/\tau)$. As a result, the space integral is divergent at large distances:

$$N_{\mathrm{deloc}}(0)=2\int \frac{dk_z}{\pi^2\gamma(k_z)}\int_\xi^{\min\{R,\lambda\}} d\tau\sim N_F\xi\cdot\min\{R,\lambda\}. \tag{3.3}$$

This term does not contain a logarithm in the denominator and thus exceeds the density of localized states in Eq. (2.10).

For the vortex lattice in the magnetic field region $H_{c2}\gg H\gg H_{c1}$ the intervortex distance is $R\sim\xi\sqrt{H_{c2}/H}<\lambda$. The DOS averaged over the vortices therefore is

$$N(0)=KN_F\sqrt{\frac{H}{H_{c2}}}, \tag{3.4}$$

where the factor K is on the order of unity and, according to Eq. (3.2), is defined by the vortex lattice structure in the coordinate space and by the slope of the gap near the gap node in the momentum space.

4. SYMMETRY OF THE VORTEX LINE IN A $D_4(D_2)\times T$ SUPERCONDUCTOR

Finally, let us consider the symmetry of the isolated vortex in a superconductor of class $D_4(D_2)\times T$. According to Ref. 6, the elements of the maximal symmetry group of the vortex line can be found from the asymptote of the gap function far from the vortex core:

$$\Delta(\mathbf{k},\mathbf{r})=(k_x^2-k_y^2)f(\mathbf{k})e^{i\phi}. \tag{4.1}$$

Eight elements of the symmetry of this function form the group $D_4(E)$:

$$D_4(E)$$
$$=(E,\ C_z^{\pi}e^{i\pi},\ C_z^{\pi/2}e^{i\pi/2},\ C_z^{-\pi/2}e^{-i\pi/2},\ C_x^{\pi}T,\ C_y^{\pi}e^{i\pi}T,\ C_{x+y}^{\pi}e^{i\pi/2}T,\ C_{x-y}^{\pi}e^{-i\pi/2}T), \tag{4.2}$$

where C_i^{α} is the rotation about the i axis through an angle α. This group is isomorphic to the D_4 group; for the physical quantities such as $|\Delta(\mathbf{k},\mathbf{r})|^2$, which are gauge invariant and invariant under time inversion, this group coincides with the initial D_4 group of the CuO_2 planes.

In conventional superconductors with a D_4 group the symmetry of the vortex with asymptote $f(\mathbf{k})e^{i\phi}$ is similar but not the same:

$D_4'(E)$

$$=(E,\ C_z^{\pi}e^{i\pi},\ C_z^{\pi/2}e^{-i\pi/2},\ C_z^{-\pi/2}e^{i\pi/2},\ C_x^{\pi}T,\ C_y^{\pi}e^{i\pi}T,\ C_{x+y}^{\pi}e^{-i\pi/2}T,\ C_{x-y}^{\pi}e^{i\pi/2}T), \quad (4.3)$$

and Eq. (4.2) characterizes the symmetry of the vortex with an opposite winding number, i.e., with the asymptote $f(\mathbf{k})e^{-i\phi}$. This leads to two consequences.

1) The core of the vortex in the $D_4(D_2)\times T$ state should contain all the possible terms consistent with Eq. (4.2) and, in particular, it should contain the amplitude of conventional (s-wave) pairing with inverse circulation of the superfluid velocity, such as

$$\Delta_s(\mathbf{k},\mathbf{r})=f(\mathbf{k})\,|\Psi_s(\mathbf{r})|\,e^{-i\phi}. \quad (4.4)$$

Because of this correction, the total gap function has no lines of gap nodes within the core. This is not surprising since, in contrast with the point zeros, the lines of zeros are topologically unstable.[7] This circumstance, however, does not change the result for the DOS, since it comes mostly from the region outside the core.

2) There is an incompatibility between the symmetry and the topology, when the vortex line punctures the interface between the conventional and unconventional superconductors. The topology requires that the winding number of the vortex be the same on each side of the interface. On the other hand, if the symmetry $D_4(E)$ is conserved, the vortices on the two sides of the interface should have opposite winding numbers. The topology, however, is more important and therefore the symmetry should be broken. In particular, the rotation $C_z^{\pi/2}$, which is combined with different phase factors in Eqs. (4.2) and (4.3), is no longer an element of symmetry. The rotational symmetry about the fourfold axis is therefore broken near the interface.

5. CONCLUSION

While in the mixed state of conventional superconductors the electronic DOS, $N(0)\propto N_F H/H_{c2}$, comes from the low-energy states localized in the vortex cores, in the superconductors with lines of gap nodes the DOS, $N(0)\propto N_F\sqrt{H/H_{c2}}$, comes mostly from the continuous spectrum which is concentrated in the vicinity of the gap nodes and outside the vortex core.

I thank M. E. Zhitomirsky for discussions.

[1] Z. -X. Shen, D. S. Dessau, B. O. Wells *et al.*, Phys. Rev. Lett. **70**, 1553 (1993).
[2] G. E. Volovik and L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **38**, 1412 (1985) [Sov. Phys. JETP **61**, 843 (1985)].
[3] W. N. Hardy, D. A. Bonn, D. C. Morgan *et al.*, Phys. Rev. Lett. **70**, 2806 (1993).
[4] C. Caroli, P. G. de Gennes, and J. Matricon, Phys. Lett. **9**, 307 (1964).
[5] G. E. Volovik, JETP Lett. **57**, 244 (1993).
[6] M. M. Salomaa and G. E. Volovik, Rev. Mod. Phys. **59**, 533 (1987).
[7] P. G. Grinevich and G. E. Volovik, J. Low Temp. Phys. **72**, 371 (1988).

Published in English in the original Russian journal. Reproduced here with the stylistic changes by the Translations Editor.