Freezing in of a magnetic field in a kinetic description of a collisionless plasma

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The reason for the freezing in of a magnetic field and the reasons for the disruption of this effect in a collisionless plasma are analyzed. The concept of a "freezing-in preserver" is introduced. In a tokamak, the role of this preserver may be played by untrapped electrons, which are not at resonance with turbulence and which thus execute an integrable motion. As a result, the poloidal magnetic field is frozen in the toroidal field. Under these conditions the diffusion of particles and heat may be governed by turbulence, in agreement with experiment.

1. The "freezing in" of magnetic fields is a central concept in magnetic plasma confinement. In the differential formulation, freezing in means that the evolution of the magnetic field **B** is described by the equation

$$\mathbf{B}_t=\nabla\times[\mathbf{v}\times\mathbf{B}], \tag{1}$$

where **v** is the plasma velocity. In the equivalent integral formulation, the flux of **B** carried across any closed contour at the velocity **v** is conserved. Unfortunately, just what "the plasma velocity **v**" means in reality is not clear, since the particles are moving at individual thermal-drift velocities, which are generally not small in comparison with **v**, and we have yet to mention the thermal velocity of the particles along the magnetic field. If we take **v** to mean the average velocity of the particles, then terms which violate (1) appear in the pressure tensor. Experimentally, a plasma behaves as if this were not the case. The present letter is an effort to analyze this question. Another formulation of the question would run as follows: Which effects disrupt the freezing in as in (1) in a collisional plasma? We begin by clarifying some fundamental physical reasons for the appearance of the freezing-in law.

2. Equation (1) can be derived by taking the curl of the hydrodynamic equation of motion of cold electrons and setting the electron mass equal to zero. In this connection it is often stated that the incorporation of a finite mass disrupts the freezing in and leads to a leakage of plasma out of confinement systems.[1] Even when a nonzero mass is taken into account, the equation of motion for each component of the plasma can be written as the conservation of the generalized vorticity $\mathbf{\Omega}$:

$$\mathbf{\Omega}_t=\nabla\times[\mathbf{v}\times\mathbf{\Omega}], \tag{2}$$

where $\mathbf{\Omega}=\nabla\times\mathbf{P}$, and $\mathbf{P}=m\mathbf{v}+e\mathbf{A}/c$ is the generalized momentum of the particle. When the mechanical part is predominant, we obtain a result which was established in the last century, namely, Kelvin's theorem regarding circulation in an ideal liquid. When the electromagnetic part is predominant, we obtain the freezing in of a magnetic field. Conservation of the generalized vorticity was pointed out in the well-known review by Braginskiĭ[2] and also in a paper by Linden–Bell,[3] but this fact is not generally known. As a result, there have been many misunderstandings. Although incorporating a small electron mass implies a small deviation of the magnetic field from the freezing-in condition, these changes cannot build up, even over a long time, because of conservation of the generalized vorticity. When the mass is taken into account, the electrons are "glued" not to the magnetic field lines but to the Ω lines. The integral of motion does not disappear, but its definition changes slightly.

Conservation of the generalized vorticity is not a matter of chance. It is embodied in the canonical form of Hamilton's equation for a fluid particle, as was pointed out in Refs. 4 and 5. The hydrodynamic equations of motion for a pressure which depends on only the density can be derived from the Hamiltonian

$$H=P[\rho(\mathbf{q})]+e\phi(\mathbf{q})+\frac{(\mathbf{p}-e\mathbf{A}/c)^2}{2m}, \tag{3}$$

where $P[\rho(\mathbf{q})]$ is the normalized pressure, $\phi(\mathbf{q})$ and $\mathbf{A}(\mathbf{q})$ are the electrostatic and vector potentials,

$$\dot{\mathbf{P}}(\mathbf{q})=-\delta H/\delta\mathbf{q}, \quad \text{and} \quad \dot{\mathbf{q}}=\delta H/\delta\mathbf{p}. \tag{4}$$

We then find from (4) conservation of a relative integral Poincaré invariant:

$$I=\oint \mathbf{p}\cdot d\mathbf{q}, \tag{5}$$

where the integration contour is carried by the phase flux.[6] Since the generalized momentum is a function of the coordinates and the time in hydrodynamics, the contour integral can be transformed into the flux $\nabla\times\mathbf{p}$ across the surface enclosed by this contour. This is the integral formulation of freezing in: $\mathbf{\Omega}\equiv\nabla\times\mathbf{p}$. This integral is a consequence of the form of the Poisson brackets of (4); it does not depend on the form of the Hamiltonian. Such integrals, called "Casimir integrals," cannot be disrupted by adding anything to the Hamiltonian. The only possibility in our case is to incorporate thermal motion of the particles, i.e., the kinetics.

3. Before taking up the kinetics, we wish to introduce the important concept of a "preserver of freezing in."

Let us assume, for example, that the plasma electrons consist of a cold component ($T=0$) and a hot one. It is then simple to see that the generalized vorticity of the cold component is conserved regardless of the behavior of the hot particles. The cold component is thus a preserver of the freezing in. This assertion remains correct, however, only up to the time of the first breaking, which can easily occur in a cold flow. In a collisionless plasma, circulation along the contour is conserved even after the breaking, but it changes sign because the contour turns inside out. Here is an overall sketch of the violation of freezing in: In the absence of breaking, the magnetic part of the vorticity converts into the mechanical part reversibly. The breaking introduces a note of irreversibility and leads to a disruption of the freezing in if there is no group of particles (a preserver) in which there is no breaking. We have been disucssing breaking in hydrodynamics, but a similar irreversibility is introduced by a kinetic breaking.

4. Let us attempt to apply the concept of a preserver to a tokamak plasma. A distinctive feature of a tokamak is that the magnetic field lines lie on a system of nested toroidal surfaces, which experience a 3D perturbation because of turbulence. We would naturally expect that the turbulence would not lead to a mixing over the entire phase volume. There then exists a group of particles (probably fast untrapped particles) which execute an integrable motion and do not break; these particles are not at resonance with the perturbations. This group of particles is our preserver. As a result, the diffusion of magnetic field is much slower than the diffusion of particles and heat; *i.e.*, we have

$$\mathbf{B}_t=\nabla\times[\mathbf{v}\times\mathbf{B}], \quad (6)$$

where $\mathbf{v}$ is no longer the plasma velocity. We might say that the poloidal and toroidal magnetic fields are frozen in each other but not in the plasma. We would then have an explanation for an important experimental fact: The conductivity of the plasma in a tokamak is classical, while the other transport processes are anomalous. There is a completely clear explanation here: In a plasma, the conductivities due to the various groups of particles combine as parallel resistances do; if some particular group is not dissipated in the turbulence, it dominates the conductivity. The other transport coefficients are dominated by those groups of particles which are dissipated to the greater extent. Those coefficients are governed by the turbulence.

It was pointed out a long time ago[7] that many experimental facts can be explained by assuming that the electron–electron collision rate is anomalously high. The effective result of this assumption is close to the picture drawn above.

Arguments regarding the existence of a preserver look nearly universal for tokamaks, so we should cite an example for which these arguments are not correct. Sawtooth oscillations stem from the onset of local singularities of the current-sheet type.[8] High gradients can cause all the electrons which pass through this zone to experience nonadiabatic perturbations, and there may be no freezing in of the magnetic field in this zone.

The freezing-in preserver means that the turbulent fluctuations in the magnetic field, $\delta\mathbf{B}$, have an "iso-freezing-in" form: $\delta\mathbf{B}=\nabla\times[\delta\mathbf{r}\times\mathbf{B}]$. This assertion should not be confused with the result of Ref. 9, where the disruption of magnetic surfaces was studied for small and otherwise arbitrary fluctuations $\delta\mathbf{B}$. The iso-freezing-in of perturbations definitely must be taken into account in models for electron heat transport, as set forth in the review by Isichenko,[10] for example.

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