Stochastic plasma heating by laser light

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(Submitted 25 June 1993; resubmitted 20 August 1993)
Pis'ma Zh. Eksp. Teor. Fiz. **58**, No. 7, 520–523 (10 October 1993)

A new method for fast plasma heating is proposed. It involves the onset of a stochastic instability in the motion of electrons in three or more high-frequency ($\omega \gg \omega_p$), large-amplitude electromagnetic waves. The elementary resonant mechanism for the wave–particle interaction is normal (Compton) or anomalous scattering. The heating rate is considerably greater than for other heating methods at a given power of the electromagnetic radiation.

Plasma heating by electromagnetic waves (in particular, in controlled-fusion devices) can be carried out in a "linear" regime in the case of weak fields. In this linear regime, the energy acquired by the particles from the wave is exchanged through collisions with other plasma particles fast enough that the deviation of the velocity distribution of these particles from a Maxwellian distribution remains small.[1] In the case of strong electromagnetic waves, for which the perturbation of the particle distribution function is large, unstable natural plasma waves can be excited, and the plasma may become turbulent (numerous beam–plasma and parametric instabilities occur). The scattering of plasma particles by the turbulent fluctuations of the electric field leads to a "turbulent" plasma heating in this case.[2,3]

We show below that there exists a mechanism of "direct" heating of plasma electrons by the field of strong electromagnetic waves. This direct mechanism leads to a plasma heating which is considerably more rapid than turbulent heating. This mechanism, which we call "stochastic," can be summarized as follows: The plasma electrons in the field of several ($N \geqslant 3$) electromagnetic waves may be at a Čerenkov resonance with the field of a combinational wave (a beat wave). When this nonlinear resonance becomes broad enough that its separatrix touches that of a nonlinear resonance at a different combinational wave, a local instability of the electron motion occurs, and the electrons experience a stochastic heating. The onset of nonlinear combinational oscillations in this case is the same as in the free-electron lasers.

This mechanism is similar to the known mechanism for microwave heating via electron cyclotron resonance (ECR heating; see Refs. 4 and 5, for example). In this case, integrals of motion characterizing the dynamics of the particles are disrupted in an ECR with one wave.

There is yet another possibility for "direct" heating of plasma particles: a heating through the effect of a random (noisy) electromagnetic field on a plasma. Below we compare the heating mechanism proposed here with some other mechanisms. We show that in many cases of practical interest the stochastic heating is more effective than the "noise" heating.

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Let us consider a charged particle which is moving in the field of several $(N+1)$ electromagnetic waves:

$$\mathbf{E}=\mathrm{Re}\sum_{n=0}^{N}\mathbf{E}_n,$$

$$\mathbf{E}_n=\mathscr{E}_n e^{i\Psi_n},\quad \mathbf{H}=\mathrm{Re}\sum_{n=0}^{N}\mathbf{H}_n,\quad \mathbf{H}_n=c[\mathbf{k}_n\mathbf{E}_n]/\omega_n,$$

where $\Psi_n\equiv\mathbf{k}_n\mathbf{r}-\omega_n t$. It is convenient to introduce the following variables: $\mathbf{P}_1=\mathbf{P}/mc$, $\mathrm{E}_{n,1}=eE_n/mc\omega_n$, $\mathbf{k}_{n,1}=\mathbf{k}_n c/\omega_n$, $\tau=\omega_0 t$, $\mathbf{r}_1=\mathbf{r}\omega_0/c$, $\omega_{n,1}=\omega_n/\omega_0$, and $\mathbf{v}_1=\mathbf{v}/c$. In terms of these variables (we omit the index 1) the equation of motion of a particle becomes

$$\dot{\mathbf{P}}=\frac{d\mathbf{P}}{d\tau}=\mathrm{Re}\left[\sum_{n=0}^{N}\mathbf{E}_n(\omega_n-\mathbf{k}_n\cdot\dot{\mathbf{r}})+\sum_{n=0}^{N}\mathbf{k}_n(\dot{\mathbf{r}}\cdot\mathbf{E}_n)\right], \tag{1}$$

where $\dot{x}\equiv dx/d\tau$. We assume that the dependent variables in (1) are the sums of fast and slow quantities. We consider the case in which all the waves are transverse, and their wave vectors are essentially collinear $(\mathbf{k}_m\cdot\mathscr{E}_n\ll 1)$. We then find the following system of equations for determining the slow quantities:

$$d\gamma^2/d\tau=\sum(\mathscr{E}_m\cdot\mathscr{E}_n)(\omega_m\mp\omega_n)\cos\theta_{mn},$$

$$d\theta_{mn}/d\tau=(\mathbf{k}_m\mp\mathbf{k}_n)\cdot\mathbf{v}-(\omega_m\mp\omega_n), \tag{2}$$

where γ is the energy of the particles, and θ_{mn} is the phase of the combinational wave. The summation in the first equation is over those pairs of waves for which the sum of phases $(\Psi_m\mp\Psi_n)$ varies slowly. The two signs here correspond to the cases of normal scattering and anomalous scattering (in the terminology of Ref. 6).

Let us consider the simplest possibility for stochastic plasma heating. Three waves act on the plasma. The first $(\omega_1,\mathbf{k}_1,\mathscr{E}_1)$ and the second $(\omega_2,\mathbf{k}_2,\mathscr{E}_2)$ are moving opposite the zeroth wave $(1,\mathbf{k}_0,\mathscr{E}_0)$. In other words, we are considering the case of normal scattering. Two combinational waves can arise under these conditions, with the phase velocities

$$v_{\mathrm{ph}i}=\Delta\omega_{0i}/(k_0+k_i),\quad i=\{1,2\},\quad \Delta\omega_{0i}\equiv 1-\omega_i.$$

The dispersion diagram in Fig. 1 illustrates the appearance of the combinational waves. The condition for the onset of the stochastic instability is the condition for an overlap of nonlinear resonances:

$$(v_{\mathrm{ph}2}-v_{\mathrm{ph}1})\leqslant\frac{\mathscr{E}_0}{\gamma_0^2\sqrt{k_0v_0}}\left[\sqrt{\mathscr{E}_1\Delta\omega_{01}}+\sqrt{\mathscr{E}_2\Delta\omega_{02}}\right], \tag{3}$$

where γ_0 and v_0 are the initial values of the energy and the velocity. If condition (3) holds, then we are left with only two terms on the right side of the first equation in (2), and the phases of the waves can be assumed to vary in a random way. After taking an

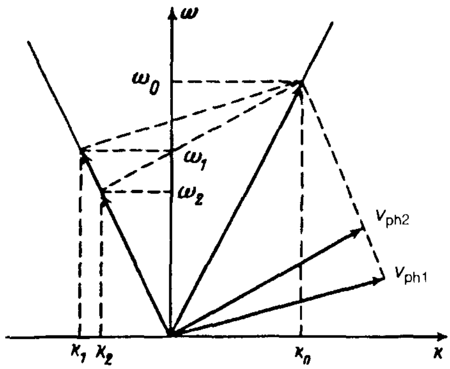

FIG. 1.

average over the random phases and over the random positions of the particles, we then find the following expression for the mean square change in the energy of the particles:

$$\langle(\Delta\gamma)^2\rangle \simeq \frac{\mathscr{E}^4(\Delta\omega)^2}{4\gamma_0^2}\tau, \tag{4}$$

where

$$\langle L\rangle \equiv \frac{1}{2\pi}\int_0^{2\pi} d(\mathbf{kr}) \lim_{T\to\infty} \frac{1}{T}\int_{-T}^{T} L dt.$$

In deriving (4) we assumed $\Delta\omega_{01}\simeq\Delta\omega_{02}\equiv\Delta\omega$, $\mathscr{E}_0\simeq\mathscr{E}_1\simeq\mathscr{E}_2\equiv\mathscr{E}$, and a time $\tau\gg\tau_k$, where the right side is the time scale of the decoupling of correlations in the motion, which is usually a few periods.

If all the plasma particles are to be involved in the stochastic heating, the inequality $\Delta v > v_{\text{ph1}}$ must hold. In the nonrelativistic approximation we find $\mathscr{E} > 0.25\sqrt{\Delta\omega v}$ from this condition; i.e., extremely moderate laser fields would be sufficient to trap and heat all the plasma electrons.

Let us compare the efficiency of the heating of particles by coherent fields with that of the heating of particles by a noisy field. For this purpose we consider the motion of a particle in a noisy field. From the equation for the energy,

$$\frac{d\gamma}{d\tau} = (\mathbf{v}\mathscr{E}_n),$$

under the same approximations as above, we easily find

$$\langle(\Delta\gamma)^2\rangle_n = v^2\mathscr{E}_n^2\tau.$$

Let us assume that the energy in the noise field is the same as that in the coherent radiation; i.e., we assume $\mathscr{E}_n^2\Delta\omega_n = \mathscr{E}^2\Delta\omega$. For the noise field, the spectral width is $\Delta\omega_n > \omega$, while that for the coherent field is $\Delta\omega = \omega/Q$, where Q is the quality factor of the optical cavity ($Q\sim 10^6$–10^7). The ratio of the increments in the energy of the particles in these fields (this ratio determines the heating efficiency) is then

$$K \equiv \frac{\langle (\Delta\gamma)^2 \rangle}{\langle (\Delta\gamma)^2 \rangle_n} > \frac{\mathscr{E}^2 (\Delta\omega)^2 Q}{4\gamma_0^2 v_0^2}. \tag{5}$$

In most cases of interest we would have $k \gg 1$.

Let us compare this mechanism with other mechanisms. There are two basic scenarios for plasma heating by intense electromagnetic waves. In the first, the wave incident on the plasma excites turbulent fluctuations in the electric and magnetic fields as the result of parametric processes (decay processes, stimulated Raman scattering, etc.[1,2,3]). These fluctuations lead to the plasma heating. The onset of turbulence takes a time which may be longer than the duration of the stochastic heating. In addition, the energy of the incident wave is distributed over a broad spectrum of natural waves in the course of the onset of turbulence. In this case, according to (5), the heating efficiency is lower, and the heating time longer, than in the case of stochastic heating.

The second scenario involves direct collisions between particles. The collision rate ν is proportional to the plasma density; with $n = 10^{22}$ cm^{-3} and $T = 7$ keV we would have $\nu = 10^{12}$ s^{-1}. If the wave frequency is $\omega = 5 \times 10^{15}$ s^{-1} and the wave amplitude $\mathscr{E} = 0.1$, then electrons are heated to 7 keV over a time $\Delta t_H = 2 \times 10^{-14}$ s, i.e., over a time much shorter than the collision time. There is thus a region of parameter values of the laser light and the plasma which is of interest for applications, in which the heating mechanism proposed here is more efficient than existing mechanisms.

Let us examine the possibility of using the stochastic method for controlled fusion. In this application it is necessary to heat the ions. It follows from (4) that the ion heating rate is low, $\tau_H \propto (m_i)^4$. The ions are heated more rapidly by collisions with hot electrons. The following scheme for ion heating is possible here: A laser field ($\mathscr{E} = 0.1$, $\omega = 5 \times 10^{15}$ s^{-1}) heats plasma electrons with $n = 10^{22}$ cm^{-3} to $T = 7$ keV over a time $t < 10^{-13}$ s. Over a time $t \sim 10^{-9}$ s, these heated electrons transfer energy to ions. This time is too short for a target of radius $r = 0.1$ cm to fly apart.

A rapid heating of electrons will prevent most instabilities, since their growth rates Γ are small: $\Gamma \Delta t_H < 1$. In this case, the absorbed laser energy will be converted more efficiently into heat—into a heating of the ions.

[1] K. N. Stepanov, Fiz. Plazmy **9**, 45 (1983) [Sov. J. Plasma Phys. **45**, 25 (1983).

[2] V. N. Tsytovich, *Theory of Turbulent Plasma* (Plenum, New York, 1974).

[3] V. P. Silin, *Parametric Effects of Intense Radiation on Plasmas* (Nauka, Moscow, 1973).

[4] A. J. Lichtenberg and M. A. Lieberman, *Regular and Stochastic Motion* (Springer-Verlag, New York, 1983).

[5] B. H. Quon, R. A. Dandl, W. Divergilio *et al.*, Phys. Fluids **28**, 1503 (1985).

[6] I. M. Frank, Usp. Fiz. Nauk **129**, 685 (1979) [Sov. Phys. Usp. **22**, 975 (1979)].

Translated by D. Parsons