Hysteresis of the current–voltage characteristic of a tunnel junction near the threshold $eV=\Delta_1+\Delta_2$

Yu. N. Ovchinnikov
L. D. Landau Institute of Theoretical Physics, Russian Academy of Sciences, 117334 Moscow, Russia

A. Yu. Ovchinnikov
Institute of Solid State Physics, Russian Academy of Sciences, 142432 Chernogolovka, Moscow Region, Russia

(Submitted 2 September 1993)
Pis'ma Zh. Eksp. Teor. Fiz. **58**, No. 7, 552–556 (10 October 1993)

Particles of the normal phase at the surface of an insulator separating two superconductors give rise to a descending region on the current–voltage characteristic above the threshold, at $eV>\Delta_1+\Delta_2$.

In the BCS model, a jump occurs on the current–voltage characteristic of a tunnel junction at a voltage eV equal to the sum of the gaps in the spectrum of one-particle excitations of the two superconductors:[1,2]

$$eV=\Delta_1+\Delta_2. \tag{1}$$

The magnitude of this jump depends on the temperature, but it remains nonzero up to the transition temperature.

At absolute zero, and for an ideal superconductor, the current through the junction is given by[2]

$$eR_{Nj(0)}=-\begin{cases}0, & eV<\Delta_1+\Delta_2,\\ \dfrac{\pi\sqrt{\Delta_1\Delta_2}}{2}\left(1+\dfrac{3(\Delta_1\Delta_2)(eV-\Delta_1-\Delta_2)}{8\Delta_1\Delta_2}\right), & 0<eV-\Delta_1-\Delta_2\ll\Delta_{1,2},\end{cases} \tag{2}$$

where R_N is the resistance of the junction in the normal state.

The various depairing mechanisms which are always operating in a superconductor smear the structural feature in the density of states. As a result, the step on the current–voltage characteristic is also smeared. For superconductors with paramagnetic impurities, the width of the transition region is on the order of $\Delta(\tau_s\Delta)^{-2/3}$, where τ_s is the mean free time with respect to spin flip. The differential resistance is always nonnegative.

We show below that droplets of a normal metal on the surface of an insulator (Fig. 1) always lead to a smearing of the structural feature in the density of states. Nevertheless, such defects lead to a contribution with a negative slope to the current–voltage characteristic.

In second order in the transmission of the barrier at absolute zero, the one-particle current j across the barrier in a inhomogeneous superconductor can be written[3]

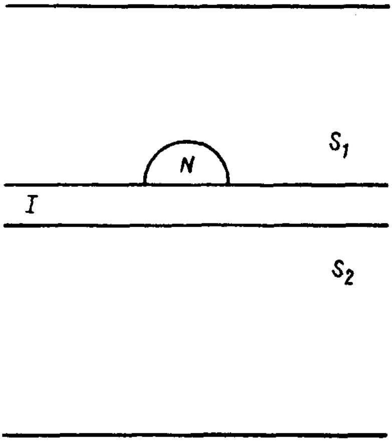

FIG. 1. Droplet of a normal metal (N) on the surface of an insulator (I) separating two superconductors (S_1 and S_2).

$$SeR_{Nj}=-\int_{E_1}^{eV-E_2} d\epsilon \int_S \mathrm{Re}\alpha_1(\epsilon)\mathrm{Re}\alpha_2(eV-\epsilon)dS, \tag{3}$$

where S is the area of the junction, and $E_{1,2}$ are the gaps in the spectrum of one-particle excitations. For superconductors in which the electron mean free path l is much smaller than the correlation length, the retarded Green's functions α and β satisfy the system of equations[4]

$$-D\frac{\partial}{\partial r}\left(\alpha\frac{\partial\beta}{\partial r}-\beta\frac{\partial\alpha}{\partial r}\right)+2i(\alpha\Delta-\beta\epsilon)=0, \quad \alpha^2-\beta^2=1, \tag{4}$$

and boundary conditions at the superconductor–normal-metal interface,[5]

$$\beta_+=\beta_-,$$

$$\left(\nu D\frac{\partial\beta}{\partial n}\right)_+=\left(\nu D\frac{\partial\beta}{\partial n}\right)_- . \tag{5}$$

Here $\nu=mp_0/2\pi^2$ is the density of states at the Fermi surface, and $D=vl_{\mathrm{tr}}/3$ is a diffusion coefficient.

To calculate the current in (3), we need the function α for energies close to Δ. In this region we have $|\alpha|$, $|\beta|\gg 1$, and Eqs. (4) can be reduced to a single simple equation:

$$-D\frac{\partial}{\partial r}\left(\frac{1}{\beta}\frac{\partial\beta}{\partial r}\right)+2i\Delta\left(\beta+\frac{1}{2\beta}\right)-2i\epsilon\beta=0. \tag{6}$$

We solve Eq. (6) in two cases: A layer of thickness d of a normal metal is on the surface of an insulator; a hemispherical droplet of radius R of a normal metal is on an insulator.

For a uniform coating of an insulator by a normal-metal layer of thickness d, we find the following expression from Eq. (6) and from boundary condition (5):

$$\beta=\begin{cases} C_0-\dfrac{i\epsilon C_0^2}{D_N}z^2-\dfrac{2C_0^4\epsilon^2}{3D_N^2}z^4+..., & 0\leqslant z\leqslant d,\\ \dfrac{1}{\sqrt{2}}\sqrt{\dfrac{\Delta}{\epsilon-\Delta}}+A\exp\left[-\left(\dfrac{-i2\sqrt{2}}{D_S}\sqrt{\Delta(\epsilon-\Delta)}\right)^{1/2}z\right], & d\leqslant z,\end{cases} \tag{7}$$

where

$$C_0=\frac{1}{\sqrt{2}}\sqrt{\frac{\Delta}{\epsilon-\Delta}}+\frac{i\Delta^2}{\epsilon-\Delta}\frac{\nu_N}{\nu_S\sqrt{D_S}}\frac{d}{[-i\cdot 2\sqrt{2\Delta(\epsilon-\Delta)}]^{1/2}},$$

$$A=\frac{i\Delta^2 d\nu_N}{\nu_S\sqrt{D_S(\epsilon-\Delta)(-i\cdot 2\sqrt{2\Delta(\epsilon-\Delta)})^{1/2}}}. \tag{8}$$

Equations (7) and (8) hold in the region

$$|\epsilon-\Delta|\gg\frac{\Delta^3 d^4}{D_N^2},\quad |\epsilon-\Delta|\gg\Delta\left(\frac{\nu_N}{\nu_S}\right)^{4/3}\left(\frac{d^2\Delta}{D_S}\right)^{2/3}. \tag{9}$$

The case of a normal-metal drop on an insulator can be analyzed in a corresponding way. The solution of Eq. (6) in this case is

$$\beta=\begin{cases} -C_0\dfrac{i\epsilon C_0^2}{3D_N}r^2-\dfrac{4\epsilon^2C_0^3}{45D_N^2}r^4+..., & r\leqslant R,\\ \dfrac{1}{\sqrt{2}}\sqrt{\dfrac{\Delta}{\epsilon-\Delta}}+\dfrac{A}{\sqrt{r}}K_{1/2}\left[-i\left(\dfrac{i\cdot 2\sqrt{2}}{D_S}\sqrt{\Delta(\epsilon-\Delta)}\right)^{1/2}r\right], & rR,\end{cases} \tag{10}$$

where $K_{1/2}(x)$ is the Bessel function, and C_0 and A are given by

$$A=i\sqrt{\frac{2}{\pi}}\frac{\Delta^2}{\epsilon-\Delta}\frac{\nu_N}{3\nu_S D_S}\left[-i\left(\frac{i\cdot 2\sqrt{2}}{D_S}\sqrt{\Delta(\epsilon-\Delta)}\right)^{1/2}\right]^{1/2}R^3,$$

$$C_0=\frac{1}{\sqrt{2}}\sqrt{\frac{\Delta}{\epsilon-\Delta}}+\frac{i\Delta^2R^2}{3D_N(\epsilon-\Delta)}\left(\frac{1}{2}+\frac{\nu_N D_N}{\nu_S D_S}\right). \tag{11}$$

Equations (10) and (11) hold at energies

$$|\epsilon-\Delta|\gg\frac{\Delta^3R^4}{18D_N^2},\quad |\epsilon-\Delta|\gg\frac{R^4\Delta 3}{10D_S^2}\left(\frac{\nu_N}{\nu_S}\right). \tag{12}$$

We set

$$\delta E = \max\left\{\frac{\Delta^3 d^4}{D_N^2},\quad \Delta\left(\frac{\nu_N}{\nu_S}\right)^{4/3}\left(\frac{d^2\Delta}{D_S}\right)^{2/3}\right\} \tag{13}$$

for the case of a uniform coating of an insulator by a normal-metal layer of thickness d. In the voltage region

$$\delta E \ll eV - E_1 - E_2 \ll E_{1,2} \tag{14}$$

expression (3) for the current density can then be rewritten as

$$j = j_{(0)} + j_{(1)} + j_{(2)}, \tag{15}$$

where $j_{(0)}$ is given by (2), and the currents $j_{(1),(2)}$ are related to layers of normal metal in respectively the first and second superconductors and are given by

$$eR_N j_{(1)} = -\frac{1}{2}\mathrm{Re}\int_{\supset} d\epsilon \frac{\sqrt{\Delta_2}}{\sqrt{2}\sqrt{eV-E_1-E_2-\epsilon}} \frac{i\Delta_1^2}{\epsilon} \frac{\nu_N^{(1)}}{\nu_S^{(1)}\sqrt{D_S^{(1)}}}$$

$$\times \frac{d_1}{(-i\cdot 2\sqrt{2\Delta_1\epsilon})^{1/2}} = \frac{\sqrt{\Delta_2}}{(eV-\Delta_1-\Delta_2)^{3/4}} \frac{\nu_N^{(1)}\Delta_1^{7/4}}{\nu_S^{(1)}\sqrt{D_S^{(1)}}} \frac{d_1 B(-1/4,1/2)}{2^{7/4}}. \tag{16}$$

Here $B(-1/4,\ 1/2) = -[4\sqrt{\pi}\Gamma(3/4)]/\Gamma(1/4)$ is the Euler B-function. The integral in (16) is evaluated along a contour which starts from the point $eV-E_1-E_2-i0$, circumvents 0, and ends at the point $eV-E_1-E_2+i0$.

The current $j_{(2)}$ is given by (16) with interchanged indices $(1 \rightleftharpoons 2)$.

For the case of a thin interlayer of a normal metal, with $d_{1,2} \ll \xi$, the currents $j_{(1),(2)}$ are small in comparison with $j_{(0)}$, but the differential resistance is negative over the broad region defined by

$$\Delta\left(\frac{\nu_N}{\nu_S}\right)^{4/3}\left(\frac{d^2\Delta}{D_S}\right)^{2/3} \ll eV - \Delta_1 - \Delta_2 < \left[-\frac{2^{1/4}\Delta_1\Delta_2 B(-1/4,1/2)}{\pi(\Delta_1+\Delta_2)}\right.$$

$$\left.\times\left(\frac{d_1\nu_N^{(1)}\Delta_1^{5/4}}{\nu_S^{(2)}\sqrt{D_S^{(1)}}} + \frac{d_2\nu_N^{(2)}\Delta_2^{5/4}}{\nu_S^{(2)}\sqrt{D_S^{(2)}}}\right)\right]^{4/7}. \tag{17}$$

The region of a negative differential resistance is fairly wide; in terms of the parameter $(\xi/d)^{16/21}$ it is wider than the region over which the jump is smeared (to the point of discontinuity) on the current–voltage characteristic.

A corresponding picture arises in the second case, i.e., that of a droplet of a normal metal on the surface of an insulator. In the voltage region defined by conditions (14) the current is given by (15) with

$$SeR_N j_{(1)} = \mathrm{Re}\sum_i \sqrt{\pi}\sqrt{\Delta_2}R_i^3 \frac{\nu_N^{(1)}\Delta_1^2}{3\nu_S^{(1)}D_S^{(1)}}\int_{\circlearrowleft} d\epsilon \frac{1}{\epsilon\sqrt{eV-\Delta_1-\Delta_2-\epsilon}}$$

$$\times\frac{1}{(i\cdot 2\sqrt{2\Delta_1\epsilon})^{1/2}}\int_0^\infty dr\,\sqrt{r}K_{1/2}(r) = \sum_i \frac{\pi\sqrt{\Delta_2}}{(eV-\Delta_1-\Delta_2)^{3/4}}$$

$$\times\frac{\nu_N^{(1)}}{3\nu_S^{(1)}\sqrt{D_S^{(1)}}}\frac{\Delta_1^{7/4}}{2^{3/4}}R_i^3 B(-1/4,1/2). \qquad (18)$$

The integration contour in (18) is the same as that in (16). The sum over i in (18) means a sum over all the droplets on the insulator. The current $j_{(2)}$ is given by (18) with interchanged indices ($1\rightleftharpoons 2$).

We introduce an effective layer thickness:

$$d_{\mathrm{eff}} = \sum_i V_i/S, \qquad (19)$$

where $V_i = 2\pi R_i^3/3$ is the volume of the hemispherical droplet. Equations (16) and (18), expressed in terms of d_{eff}, then become completely the same.

In summary, in the BCS model, any layer of a normal metal or superconductor with a lower transition temperature, deposited on a layer of an insulator, gives rise to a region with a negative value of the differential resistance above the threshold.

In real superconductors, the picture is considerably more complex. As was mentioned earlier, various depairing mechanisms smear the jump on the current–voltage characteristic.

Paramagnetic impurities, for example, lead to a monotonic I–V characteristics at all voltages. As a result, a region of negative differential resistance may not arise. Experimentally, both types of current–voltage characteristics are observed.[1]

[1] A. Barone and G. Paternó, *Physics and Applications of the Josephson Effect* (Wiley, New York, 1982).
[2] A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **51**, 1535 (1966) [Sov. Phys. JETP **24**, 1035 (1966)].
[3] A. I. Larkin and Yu. N. Ovchinnikov, *Quantum Mechanics of Josephson Junctions* (Elsevier Science Publishers B. V., New York, 1992); *Quantum Tunnelling in Condensed Media* (ed. Yu. Kagan and A. J. Leggett)
[4] A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **73**, 299 (1977) [Sov. Phys. JETP **46**, 155 (1977)].
[5] Z. G. Ivanov, M. Yu. Kupriyanov, and K. K. Liharev *et al.*, J. Phys. (Paris) **39**, C6-556 (1978).

Translated by D. Parsons