

Long-range intensity correlations in the backscattering of light from disordered media

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(Submitted 23 June 1993; resubmitted 1 September 1993)

Pis'ma Zh. Eksp. Teor. Fiz. **58**, No. 8, 608–613 (25 October 1993)

Long-range intensity correlations in the speckle patterns formed by coherent light reflected from a multiple-scattering medium are analyzed. In the case of purely elastic scattering, one should see a triangular dip in the fluctuation spectrum.

The interference of elastically scattered waves is recognized as the cause of numerous mesoscopic effects which arise in the transport of electrons and classical waves through disordered media.¹ Among these effects are the universal conductance fluctuations of small metal samples and the formation of speckle patterns—intensity distributions which fluctuate rapidly in space—in the multiple scattering of coherent electromagnetic radiation.

One of the mesoscopic effects observed in the scattering of light and microwaves is the appearance of long-range spatial correlations in the local values of the intensity transmitted through and reflected from a medium.^{1–6} Correlations in the transmission coefficients of a slab for coherent radiation have been studied in detail, both theoretically^{1–4,6} and experimentally.^{5,6} In the case of backscattering, on the other hand, there has been essentially no study of the corresponding effects. The few results which are available² for this case cannot be accepted, since they contradict conservation of the total flux in the case of purely elastic scattering and therefore draw a distorted picture of the long-range correlations. The same comments apply to some results^{7,8} calculated on the variance of fluctuations in the reflection coefficient.

In this letter we discuss intensity correlations in the reflection of coherent electromagnetic radiation from a disordered medium. We derive some general relations describing the fluctuation spectrum for an arbitrary multiple-scattering law. The results show that the behavior of the spectrum of fluctuations in the reflected intensity is closely related to conservation of the total radiation flux at low spatial frequencies $q \ll l_{tr}^{-1}$, where l_{tr} is the transport elastic scattering length. If a large fraction of the incident radiation is reflected ($1 - R \ll 1$, where R is the backscattering coefficient of the medium), there will always be a minimum in the fluctuation spectrum at $q = 0$. In the case of purely elastic backscattering from a semi-infinite medium ($R = 1$), the dip near $q = 0$ will be triangular, like the peak in the angular distribution spectrum of coherent backscattering.^{9,10}

Let us consider the intensity correlations for the scattering of a plane wave by a slab of disordered centers. To calculate the correlation function $C(\rho) = \langle I(\rho)I(0) \rangle - \langle I \rangle^2$ and its Fourier transform, i.e., the fluctuation spectrum $M(q)$, we use an expansion in the multiplicity of the interference of “ladders.”^{2,3}

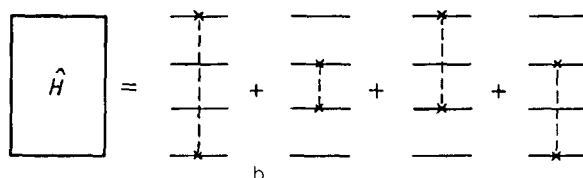
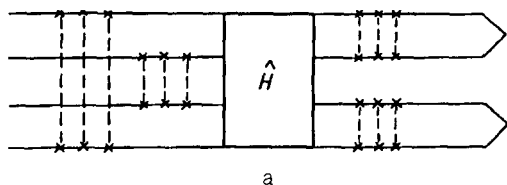


FIG. 1.

When a plane wave is incident on a medium, the leading diagram in terms of the parameter $\lambda/l_{tr} \ll 1$ (λ is the wavelength) is that in Fig. 1. We will use a definition of the Hikami vertex \hat{H} slightly different from that of Refs. 2 and 3. This definition follows from the equation for the fourth moment of the wave field in the ladder approximation.¹¹ It is valid for an arbitrary scattering law. In this definition of \hat{H} , in contrast with that of Refs. 2 and 3, all four diagrams in Fig. 1a correspond to the same number of collisions. It is thus a very simple matter to express the fluctuation spectrum in terms of ladder propagators, i.e., the solutions of the linear transport equation,¹² and to trace the cancellations in the diagrams of Fig. 1 in general form, through the use of nothing more than consequences of conservation laws. This circumstance turns out to be particularly important in calculations of diagrams of higher order in λ/l_{tr} , for which the problem of the cancellation of contributions which diverge at values of the momentum transfer greater than l_{tr}^{-1} in the case of the ordinary definition of \hat{H} requires a special analysis.^{3,13}

A direct calculation of the diagrams in Fig. 1 leads to the following expression for the spectrum of fluctuations in the reflected intensity:

$$M_R(\mathbf{q}) = A \langle (\delta R)^2 \rangle \mathbf{q} = \frac{(2\pi)^2}{k_0^2} \int_0^L dz \int d\Omega \int d\Omega' I_i(z, \Omega) I_i(z, \Omega') \times n \frac{d\sigma}{d\Omega}(\Omega\Omega') |E_f(z, \Omega|0, \mathbf{q}) - E_f(z, \Omega'|0, \mathbf{q})|^2, \quad (1)$$

where $I_i(z, \Omega)$ is the angular distribution of the radiation in the medium at a distance z from the surface,

$$E_f(z, \Omega|0, \mathbf{q}) = \int d^2\rho \exp(-i\mathbf{q}\rho) F_f(z, \Omega|0, \rho),$$

$E_f(z, \mathbf{\Omega} | 0, \mathbf{p})$ is the distribution over the surface $z=0$ of the radiation emerging from the medium after being emitted in the direction $\mathbf{\Omega}$ by a point source at a depth z , $(d\sigma/d\Omega)$ is the differential cross section for elastic scattering, n is the number of scatterers per unit volume, $k_0=2\pi/\lambda$, L is the thickness of the scattering slab, and A is the surface area of this slab. In the 2D case, the common factor in (1) is $2\pi/k_0$.

An expression analogous to (1) for the spectrum of the intensity transmitted through the slab, $M_T(\mathbf{q})$, is found by substituting into (1) the functions $E_f(z, \mathbf{\Omega} | L, \mathbf{q})$ corresponding to the spatial distribution of the flux emerging at the opposite boundary of the slab. Expressions like (1) are also valid for the spectrum of intensity fluctuations on the different sides of the slab and for the total spectrum:

$$M_{\text{tot}}(\mathbf{q}) = A \langle (\delta T + \delta R)^2 \rangle_{\mathbf{q}} \\ = \frac{(2\pi)^2}{k_0^2} \int_0^L dz \int d\mathbf{\Omega} \int d\mathbf{\Omega}' I_i(z, \mathbf{\Omega}) I_i(z, \mathbf{\Omega}') n \frac{d\sigma}{d\Omega}(\mathbf{\Omega}) | [E_f(z, \mathbf{\Omega} | 0, \mathbf{q}) \\ + E_f(z, \mathbf{\Omega} | L, \mathbf{q})] - [E_f(z, \mathbf{\Omega}' | 0, \mathbf{q}) + E_f(z, \mathbf{\Omega}' | L, \mathbf{q})] |^2. \quad (2)$$

If the incident waves differ in frequency by an amount $\Delta\omega$ (Refs. 2-6, 11), the only change in Eqs. (1) and (2) is in the product of intensities $I_i(z, \Delta\omega, \mathbf{\Omega}) I_i(z, \Delta\omega, \mathbf{\Omega}')^*$, where $I_i(z, \Delta\omega, \mathbf{\Omega})$ satisfies the same transport equation as before, but now with a complex absorption coefficient $[l_a^{-1} + i(\Delta\omega/c)]$, where l_a is the absorption length, and c the velocity of light.

Working from general relations (1) and (2), we can immediately draw several conclusions regarding the properties of the fluctuation spectrum.

In the case of purely elastic backscattering of waves from a semi-infinite medium, we have $R=1$ and $\langle (\delta R)^2 \rangle = 0$; at $\mathbf{q}=0$, expression (1) should vanish. The reason is that at $\mathbf{q}=0$ the quantity $E_f(z, \mathbf{\Omega} | 0, \mathbf{q})$ does not depend on the direction¹² of $\mathbf{\Omega}$, and we have $M_R(\mathbf{q}=0)=0$. For $|\mathbf{q}|>0$, E_f becomes dependent on $\mathbf{\Omega}$. As a result, $M_R(\mathbf{q})$ increases. A minimum thus arises in the $M_R(\mathbf{q})$ spectrum near $\mathbf{q}=0$ because of conservation of the total flux. This structural feature is not disturbed even in the next orders of the expansion of $M_R(\mathbf{q})$ in terms of the parameter λ/l_{tr} , since factors of the type

$$[E_f(z, \mathbf{\Omega} | 0, \mathbf{q}) - E_f(z, \mathbf{\Omega}' | 0, \mathbf{q})] [E_f(z', \mathbf{\Omega}_1 | 0, \mathbf{q}) - E_f(z', \mathbf{\Omega}'_1 | 0, \mathbf{q})]^*$$

are explicitly present in all terms of the series expansion of the spectrum in the multiplicity of the interference of intensities.

The calculation method used in Ref. 2 does not lead to a cancellation in the limit $\mathbf{q} \rightarrow 0$ at small values of z ($z < l_{tr}$). This circumstance is the reason for the incorrect conclusions drawn in Ref. 2 regarding the behavior of the spectrum and of the correlation function. For the same reason, the flux conservation is disrupted in the results of Ref. 8.

In the case of elastic scattering of waves by a finite slab, flux conservation ($R + T = 1$) has exactly the same effect on the behavior of fluctuation spectrum (2). In the case $\mathbf{q}=0$, the sum $E_f(x, \mathbf{\Omega} | 0, \mathbf{q}) + E_f(z, \mathbf{\Omega} | L, \mathbf{q})$ in (2) determines the total radi-

ation flux across the two boundaries of the slab. In the absence of absorption, it does not depend on Ω . We thus have $M_{\text{tot}}(\mathbf{q}=0) = A\langle(\delta T + \delta R)^2\rangle = 0$, and the spectrum has a minimum at $\mathbf{q}=0$.

Relations (1) and (2) were derived without any assumptions regarding the form of the functions I_i and E_f . These relations are the most general results which have been proposed for describing correlations in the transmission and reflection coefficients of a disordered medium for coherent radiation. The spectra of fluctuations in the transmitted intensity for the case of radiation diffusion^{2-4,6} and for the case of small-angle multiple scattering¹¹ follow as limiting cases from an equation like (1) for $M_T(\mathbf{q})$ when the corresponding approximate expressions for I_i and E_f are substituted in it.

Let us analyze the long-range intensity correlations which arise in the speckle pattern in the case of purely elastic reflection of coherent light from a semi-infinite medium. For isotropic scattering ($l_{\text{tr}}=l$, where l is the mean free path) the quantities in (1) can be found through an exact solution.¹⁰ In this case we find the following expressions for the fluctuation spectrum ($\mathbf{q} \ll l^{-1}$),

$$M_R(\mathbf{q}) = \frac{Z^2(\mu_0, 1)(2\pi)^{d-1}}{\pi(d-1)k_0^{d-1}} |\mathbf{q}|l, \quad (3)$$

and for the asymptotic behavior of the intensity correlation function ($\rho \gg l$),

$$C_R(\rho) = -\frac{Z^2(\mu_0, 1)(2\pi)^{d-1}l}{\pi^2(d-1)^2k_0^{d-1}\rho^d}. \quad (4)$$

Here d is the dimensionality of the space, μ_0 is the cosine of the angle of incidence of the radiation on the surface of the medium, $Z(\mu_0, 1)$ is the Chandrasekhar function or its 2D analog,¹⁰ and the values of (3) and (4) have been normalized to a unit z component of the incident flux. According to (3), the spectrum of fluctuations in the backscattered intensity has a structural feature near $q=0$: a triangular dip (Fig. 2). The shape of this feature in $M_R(\mathbf{q})$ is reminiscent of the peak in the angular distribution of coherent backscattering.^{9,10} As follows from (4), this feature corresponds to the same type of attenuation of spatial correlations. With decreasing μ_0 , the dip in spectrum (3) becomes progressively more rounded. This behavior, also characteristic of the backscattering peak,¹⁰ stems from a shortening of the trajectories traced out by the waves in the medium in the case of oblique incidence. The fact that the triangular structural feature corresponds to a minimum of $M_R(\mathbf{q})$, while the long-range asymptotic behavior of $C_R(\rho)$ corresponds to an anticorrelation of intensities, is a consequence of flux conservation in the case of purely elastic scattering. The sign of $C_R(\rho)$ changes at $\rho \sim l$. At $q > l^{-1}$ the spectrum is essentially independent of q : $M_R(\mathbf{q}) \propto 1/k_0^{d-1}$.

When the waves incident on the medium differ in frequency by a small amount $\Delta\omega$, there is additional attenuation of long-range correlations. Although the total flux is conserved in this case, and we have $M_R(\mathbf{q}=0)=0$, the triangular dip in $M_R(\mathbf{q})$ is replaced by a parabolic one, and the linear behavior in (3) sets in at

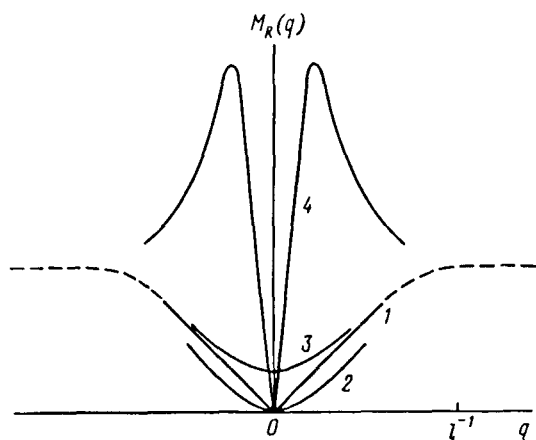


FIG. 2. Fluctuation spectrum near $q=0$ for backscattering from a semi-infinite medium. 1—No absorption, $\Delta\omega=0$; 2—no absorption, $\Delta\omega\neq 0$; 3—weak absorption; 4—strong Fresnel reflection at the boundary of the medium. The dashed curve shows the part of the spectrum corresponding to short-range correlations.

$|\mathbf{q}| > (\Delta\omega/cl)^{1/2}$ (Fig. 2). Expression (4) is valid only under the condition $l < \rho < (cl/\Delta\omega)^{1/2}$; at $\rho > (cl/\Delta\omega)^{1/2}$ the intensity correlation function falls off more rapidly: $C_R(\rho) \propto -(cl^3/\Delta\omega)^{1/2}/(k_0^{d-1}\rho^{d+2})$.

It is a straightforward matter to generalize the results found above, with the help of Ref. 10, to the case of a backscattering of waves from a thick slab ($L \gg l$) and to the case of a weakly absorbing medium ($l_a \gg l$). The flux losses due to the absorption and due to the escape of waves at the opposite boundary of the slab lead to similar qualitative effects in the fluctuation spectrum. For example, for the reflection of waves from a weakly absorbing medium, the $M_R(\mathbf{q})$ spectrum near the minimum is

$$M_R(\mathbf{q}) = A \langle (\delta R)^2 \rangle \left(1 + 2q^2 \frac{ll_a}{d} + \dots \right), \quad (5)$$

where

$$\langle (\delta R)^2 \rangle = \frac{Z^2(\mu_0, 1)(2\pi)^{d-1}}{4\pi(d-1)k_0^{d-1}A} \left(d \frac{l}{l_a} \right)^{1/2}. \quad (6)$$

The linear behavior in (3) sets in at $|\mathbf{q}| > l_D^{-1} = (ll_a/d)^{-1/2}$ (Fig. 2). The intensity correlation function $C_R(\rho)$ in the interval $l < \rho < l_D$ is again described by (4), while at $\rho > l_D$ it falls off in accordance with

$$C_R(\rho) \propto -\frac{l}{k_0^{d-1}\rho^d} \left(\frac{\rho}{l_D} \right)^{(d-1)/2} \exp\left(-\frac{\rho}{l_D}\right).$$

For purely elastic scattering from a finite slab, the thickness L serves as the cutoff in place of l_D . The variance of the backscattering coefficient for a plane wave in this case is

$$\langle (\delta R)^2 \rangle = \langle (\delta T)^2 \rangle = -\langle \delta T \delta R \rangle = \frac{Z^2(\mu_0, 1)(2\pi)^{d-1}}{3\pi(d-1)k_0^{d-1}A} \left(\frac{l}{L} \right). \quad (7)$$

Flux conservation thus leads to a value of $\langle (\delta R)^2 \rangle$ which is much smaller than that which follows from Refs. 2, 7, and 8 [$\langle (\delta R)^2 \rangle \sim 1/(k_0^{d-1}A)$].

The experiments of Refs. 14 and 15 showed that Fresnel reflection from the boundary of a medium has a strong influence on multiple scattering of light and microwaves. This effect leads to a "blocking" of the waves in the medium; this blocking should influence the amplification of long-wave intensity correlations. The functions in (1) can be calculated in this case by using the approximate boundary conditions proposed in Ref. 14. For strong internal reflection, the problem acquires a new length scale $l_F \sim l/(1-R_F) \gg l$ (R_F is the Fresnel reflection coefficient averaged over angle), and the shape of the spectrum changes qualitatively. The dip in the fluctuation spectrum becomes narrower, and a maximum appears at $q \sim l_F^{-1}$: $M_R^{\max} \propto [1/(k_0^{d-1})](l_F/l)$ (the spectrum is normalized to a unit incident flux in the medium). Beyond the maximum ($l_F^{-1} < q < l^{-1}$) the spectrum falls off as $M_R(q) \propto M_R^{\max}(ql_F)^{-1}$ to a value $M_R \sim 1/k_0^{d-1}$ (Fig. 2), which is determined by short-range correlations. Corresponding to this behavior of the spectrum are positive correlations at $l < \rho < l_F$,

$$C_R(\rho) \sim 1/k_0^2 l \rho \quad (d=3), \quad C_R(\rho) \sim \frac{1}{k_0 l} \ln \frac{l_F}{\rho} \quad (d=2),$$

and correlations at $\rho > l_F$ which are similar to (4) but have a larger amplitude,

$$C_R(\rho) \sim -\frac{l}{k_0^{d-1} \rho^d} \left(\frac{l_F}{l} \right)^2.$$

A picture of the long-range correlations like that drawn above is also valid for the total spectrum, (2), although the functional relations which follow from (2) differ from those found for the spectrum of the reflected intensity. With regard to correlations in the transmission coefficients of the slab, these relations are insensitive to flux-conservation effects at $L \gg l$, so the results of Refs. 1-4 and 6 draw a picture which is qualitatively correct. A calculation of $M_T(q)$ from an equation like (1) leads to values of the common factors which are slightly different from those of Refs. 1-4 and 6 and to a dependence on μ_0 [see (7)].

We are indebted to E. E. Gorodnichev and S. L. Dudarev for interest in this study and for valuable advice. One of us (D.R.) wishes to thank S. Feng for a useful discussion of problems touched on in this paper.

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Translated by D. Parsons