Scattering by an isolated impurity in a quantum channel in a magnetic field

V. A. Geĭler, V. A. Margulis, and I. I. Chuchaev
Mordovian State University, 430000 Saransk, Russia

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The conductance of a ballistic quantum channel with a constriction in a quantizing magnetic field is analyzed in the case of scattering by an isolated impurity in the channel. The transmission coefficient and the conductance are derived for this microstructure. The parameters of a Breit–Wigner resonance and oscillations in the transmission coefficient are analyzed.

The transport of charge carriers in ballistic quantum microstructures is attracting progressively increasing interest because of progress in the technology of fabricating these structures.

Experimental and theoretical research on the conductance in ballistic channels with a constriction has shown that the scattering of carriers by an isolated impurity in the channel has an extremely strong effect on the conductance.[1–7] Our purpose in this letter is to take a theoretical look at this effect in the case in which there is a quantizing magnetic field. Following Refs. 2–4, 8, and 9, we use a saddle-shaped potential to describe the microconstriction and a short-range potential to describe the electron–impurity interaction.

The Hamiltonian of a 2D electron in a saddle potential $V(x,y) = m(-\alpha_1 x^2 + \alpha_2 y^2)$ and in a uniform magnetic field $\mathbf{B}$ oriented perpendicular to the confinement plane is

$$H_0=\frac{1}{2m}\left(\mathbf{p}-\frac{e}{c}A\right)^2+V(\mathbf{r}). \tag{1}$$

Here $\mathbf{A}=\mathbf{B}\times\mathbf{r}/2$ is the vector potential of the magnetic field $\mathbf{B}$ in a symmetric gauge. The parameters of the potential V are related to the geometry of the microstructure by[2] $\alpha_1=\hbar/mLd$, $\alpha_2=\hbar/md^2$, where d and L are the width and length of the constriction. The complete Hamiltonian of the system is $H=H_0+U$, where $U(\mathbf{r})$ is the impurity potential. Following Refs. 2–4, we choose U to be a zero-radius potential. To pursue the analysis of the Hamiltonian H we use the theory of operator expansions.[10,11] Using the Krein formula of that theory, we find the following expression for the Green's function G of the operator H:

$$G(\mathbf{r},\mathbf{r}';E)=G_0(\mathbf{r},\mathbf{r}';E)-[Q(E)+\lambda^{-1}]^{-1}G_0(\mathbf{r},0;E)G_0(0,\mathbf{r}';E). \tag{2}$$

Here G_0 is the Green's function of the operator H_0, λ is the coupling constant in the electron–impurity scattering potential, and $Q(E)$ is the so-called Krein function for H (Refs. 10, 12, 13). Denoting by ψ_0 the scattering state of a particle for Hamiltonian H_0 with energy E, we have

$$\psi(\mathbf{r})=\psi_0(\mathbf{r})=[Q(E)+\lambda^{-1}]^{-1}\psi_0(0)G_0(\mathbf{r},0;E) \tag{3}$$

as the scattering state for Hamiltonian H with the same energy. Equations (2) and (3) reduce the problem of finding the state of a particle scattered by a short-range potential to the determination of a Krein function. Within an additive constant C we find the function $Q(E)$ from

$$Q(E)=[G_0(\mathbf{r},0;E)-G_0(\mathbf{r},0;E_0)]_{\mathbf{r}=0}, \tag{4}$$

where E_0 is some fixed energy. We will not reproduce here the complicated expression for the exact value of C, since it is unimportant to the discussion below. We introduce the notation

$$\begin{gathered} D^2=[(-\alpha_1^2+\alpha_2^2+\omega^2)^2+4\alpha_1^2\alpha_2^2]^{1/2}, \quad \omega=|eB/mc|, \\ \omega_{1,2}=[(D^2\pm(\alpha_1^2-\alpha_2^2-\omega^2))/2]^{1/2}, \quad W_1=(\alpha_1\omega_2-\alpha_2\omega_1)^{1/2,} \\ W_2=(\alpha_1\omega_1-\alpha_2\omega_2)^{1/2}, \quad W^2=W_1^2+iW_2^2, \quad \Omega=\omega_2+i\omega_1, \\ F(t)=2(\alpha_1\alpha_2)^{-1}\mathrm{Im}[W^2\sin(\Omega t/2)]^2. \end{gathered} \tag{5}$$

The integral representation for $Q(E)$ is then

$$Q(E)=(m/2\pi\hbar^2)\oint_\gamma [F(t)]^{-1/2}[\exp(i(E+i0)t/\hbar)-1]dt, \tag{6}$$

where the contour γ lies in the lower complex half-plane and does not contain zeros of $F(t)$.

We know that there exists a simplicial transformation $(p_x,p_y,x,y)\to(p_1,p_2,q_1,q_2)$ of the phase space for the quadratic Hamiltonian H_0 such that this Hamiltonian acquires a canonical form:

$$H_0=\frac{1}{2m}p^2+m(-\omega_1^2q_1^2+\omega_2^2q_2^2)/2. \tag{7}$$

This phase space is transformed by the generators of a unitary transformation from a state in the $\mathbf{r}$ representation to a state in the $\mathbf{q}$ representation. A direct calculation leads to

$$\langle\mathbf{r}|\mathbf{q}\rangle=K\exp[-iM(\beta_1xy+\beta_2q_1q_2+\beta_3yq_1+\beta_4xq_2)], \tag{8}$$

where

$$\begin{gathered} K=mD(\alpha_1\alpha_2)^{1/2}/2\pi\hbar W_1, \quad M=m/2\hbar W_1^2, \\ \beta_1=\omega(\alpha_1\omega_2+\alpha_2\omega_1), \quad \beta_2=2\omega\alpha_1\alpha_2, \quad \beta_3=-2\alpha_2D(\alpha_1^2-\omega_1^2)^{1/2}, \\ \beta_4=-2\alpha_1D(\omega_2^2-\alpha_2^2)^{1/2}. \end{gathered}$$

When the electron transition occurs between two equilibrium states, the conductance of the microstructure is determined by the expression[8] $\sigma = Te^2/h$. Here $T = \Sigma T_{nm}$ is the total transition probability, and T_{nm} is the probability for a transition from scattering channel n into another channel m.

There are two independent solutions of the Schrödinger equation with Hamiltonian H_0 in the $\mathbf{q}$ representation. For state n these solutions are

$$\psi_n^{\pm}(\mathbf{q}) = \Phi_n(\eta) E(-\epsilon_n, \pm\xi), \tag{9}$$

where Φ_n is the Hermite function, $E(a,x)$ is the Weber function, $\epsilon_n = [E - (n+1/2)\hbar\omega_2]/\hbar\omega_1$, $\xi = (2m\omega_1/\hbar)^{1/2} q_1$, and $\eta = (m\omega_2/\hbar)^{1/2} q_2$ (Refs. 2 and 9). We introduce

$$\kappa = \omega(\alpha_1\alpha_2)^{1/2}/W_1^2, \quad \tau_n^{\pm} = \int_{-\infty}^{\infty} \Phi_n(\kappa\xi) E(-\epsilon_n, \pm\xi) d\xi,$$

$$t(\epsilon) = [1 + \exp(-2\pi\epsilon)]^{-1/2},$$

$$A_{nm} = (-1)^m i^{n+m} D^2 t(\epsilon_n) \tau_m^+ \tau_n^- / 8\pi^2 W_1^2 \, [Q(E) + \lambda^{-1}].$$

Using (3), (8), and (9), we find

$$T_{nm} = t^2(\epsilon_m) |\delta_{nm} + A_{nm}|^2. \tag{10}$$

Expression (10) is too complicated for an analytic study. The problem simplifies for a narrow ballistic channel, i.e., in the limit $d/L \to 0$ ($\alpha_1 \ll \alpha_2$). In this case we can ignore the q_z dependence of H. Writing $\delta(\mathbf{r})$ in the $\mathbf{q}$ representation, we find a 1D scattering problem for the Hamiltonian

$$H_1 = -\frac{\hbar^2}{2m}\frac{d^2}{dq^2} - \frac{m\omega_1^2 q^2}{2} + \Lambda\delta(q), \tag{11}$$

where $\Lambda = \lambda_0 D W_1/\omega(\alpha_1\alpha_2)^{1/2}$, and λ_0 depends on only the coupling constant λ. The 1D problem with Hamiltonian (11) is also of interest in its own right, since it corresponds to the conditions for a pinch in the microstructure.[4] In this connection, we consider the more general case in which the impurity is at an arbitrary position in the conducting channel. In this case the Hamiltonian, which depends on the impurity position q_0, is

$$H_1(q_0) = -\frac{\hbar^2}{2m}\frac{d^2}{dq^2} - \frac{m\omega_1^2 q^2}{2} + \Lambda\delta(q - q_0), \tag{12}$$

for which a Krein function can be calculated easily. We introduce

$$\xi = (2m\omega_1/\hbar)^{1/2} q, \quad \epsilon = E/\hbar\omega_1, \quad \mu = (2m/\hbar^3\omega_1)^{1/2}\Lambda.$$

It is convenient to write the asymptotic values of Λ and μ: in the limit $\omega \to 0$,

$$\mu \sim \lambda_0 (2m/\hbar^3\alpha_1)^{1/2} (1 + \omega^2/\alpha_2^2) \tag{13}$$

[here $\Lambda/\lambda_0 = 1 + O(\omega^2)$]; in the limit $\omega \to \infty$,

$$\mu \sim \lambda_0 (2m/\hbar^3\alpha_1)^{1/2} \omega \, [\alpha_2^{-1} + O(\omega^{-2})]. \tag{14}$$

The Krein function $Q_1(\epsilon, \xi_0)$ is

$$Q_1(\epsilon,\xi_0)=2^{-1}t(\epsilon)E(-\epsilon,\xi_0)E(-\epsilon,-\xi_0). \tag{15}$$

For the transition coefficient we find

$$T=T_0/|\mu Q_1(\epsilon,\xi_0)+1|^2, \tag{16}$$

where $T_0=T|_{\mu=0}$. The result $T_0=t^2(\epsilon)$ has been established elsewhere.[9] If the impurity is at the constriction in the channel ($\xi_0\simeq 0$), we find

$$T=T_0/(1+A\mu+B\mu^2), \tag{17}$$

where

$$A=\left|\Gamma\left(\frac{1}{4}-\frac{i\epsilon}{2}\right)\right|^2\exp(-\pi\epsilon/2)/2^{3/2}\pi, \quad B=\left|\Gamma\left(\frac{1}{4}-\frac{i\epsilon}{2}\right)\right|^4\cosh(\pi\epsilon)/8\pi^2.$$

If $\mu<0$ (an attractive impurity), then T has a sharp peak (a Breit–Wigner resonance). The position of the resonant energy ϵ_r is found from the equation

$$\mathrm{Re}[\mu Q_1(\epsilon,0)+1]=0, \tag{18}$$

and the height of the resonance, T_r, is found from

$$T_r=[1+\exp(2\pi\epsilon_r)]^{-1}. \tag{19}$$

The solution of Eq. (18) is $\epsilon_r\simeq -\mu^2/4$ ($\mu \ll -1$); ϵ_r corresponds to the ground state of H_1 in the case $\omega_1=0$. We thus have $T_r\to 1$ if $\omega\to\infty$ ($\lambda_0<0$) or $\lambda_0\to-\infty$. On the other hand, near the average value of μ [$\mu\leqslant-(2/\sqrt{2}\pi)|\Gamma(1/4)|^{-2}\approx-0.5$] the transition coefficient T has a maximum $T_{\max}$ at negative values of $\epsilon=\epsilon(\mu)$ with an asymptotic behavior $T_{\max}\sim|\mu|^{-1}$. Accordingly, the peak in T shrinks at intermediate values of μ if ω increases (this result agrees with a result from Ref. 7).

If $|\xi_0|\gg 1$, then

$$Q_1(\epsilon,\xi_0)=\frac{1}{q}\left\{t(-\epsilon)\exp\left[i\left(2\epsilon\ln q+\frac{q^2}{2}-\Phi_2(\epsilon)\right)\right]+i\right\}+O(q^{-5/2}). \tag{20}$$

Expression (20) shows that $Q_1(\epsilon,\xi_0)$ and thus T oscillate as functions of ξ_0. Near energies $\epsilon\ll-1$, the coefficient T has a maximum of approximately one if μ and ξ_0 obey the condition $|\mu/\xi_0|=\exp(-\pi\epsilon)$. The minimum $T_{\min}$ closest to this maximum tends toward the value $[1+8\exp(-3\pi\epsilon)\cosh(\pi\epsilon)]^{-1}$.

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