Calculation of three-loop thermodynamic potential in non-Abelian theories

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A simple formula is derived for the exact thermodynamic potential of a non-Abelian theory. A Feyman-diagram expression is found for the three-loop approximation of this potential.

Abelian theories contain the simple and familiar formula (Ref. 1, for example)

$$\Omega(g)=\Omega(g=0)+\int_0^g \frac{dg'}{g'}\,\Pi D, \tag{1}$$

which gives the thermodynamic potential of the theory in terms of the polarization operator (or mass operator) of one-particle Green's functions in closed form. This extremely useful formula makes it possible to carry out various perturbative and nonperturbative calculations in a self-consistent manner. It arises from the simple structure of the Abelian Lagrangian and does not hold in a non-Abelian theory:[2]

$$\Omega(g)=\Omega(g=0)+\int_0^g dg'\frac{\partial\Omega}{\partial g'},$$

$$\frac{\partial\Omega}{\partial g}=\frac{1}{2g}\left\{\Pi D-\Sigma_{C\bar{C}}G_{C\bar{C}}-\Sigma_{\psi\bar{\psi}}G_{\psi\bar{\psi}}+\frac{1}{6}\,\Gamma_3^{(0)}DDD\Gamma_3\right\}, \tag{2}$$

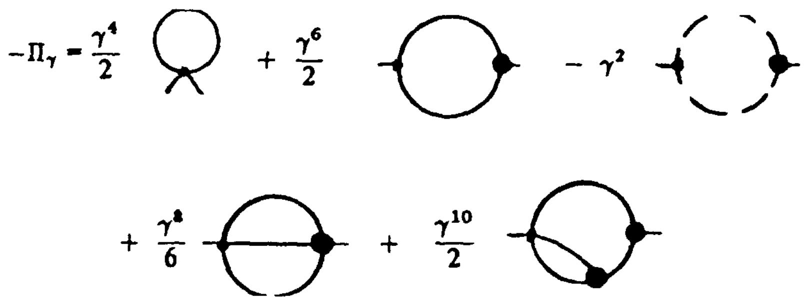
where the structure of the Lagrangian is complicated by terms with a self-effect.

The uncanceled (last) term in (2) and the complicated form of this expression cause several inconveniences in efforts to calculate multiloop corrections. In the construction of nonperturbative schemes, expression (2) simply cannot be used. In this letter we propose a recipe for reconstructing (1). As an example we do this on the basis of a Feynman-diagram series which represents the three-loop potential of the $SU(N)$ model.

In order to derive a formula analogous to (1) in non-Abelian theories, it is first necessary to expand (modify) the Lagrangian of the original theory. After carrying out several operations, we then go back into that theory by means of a simple (auxiliary) integration. The theory is modified by introducing a new effective charge (here, the parameter γ) in front of each interaction term. The power of this charge is the same as the power of the gauge fields which appear in the given interaction term. After several simple transformations, it is a simple matter to prove the necessary formula,

$$\Omega=\Omega(\gamma=0)+\int_0^1 \frac{d\gamma}{\gamma}\,\Pi_\gamma D^\gamma, \tag{3}$$

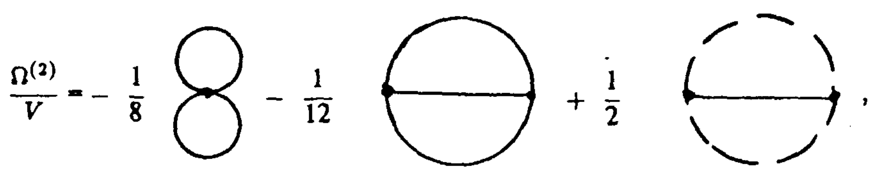
but the operator Π_γ is given by a new expression, which is slightly more complicated than that of the original theory (Ref. 2, for example):



(4)

The dashed lines and vertices here are also functions of γ . Formula (3) is valid for both perturbative and nonperturbative calculations, and it is more convenient than expression (2).

It is easy to verify that formula (3), along with (4), correctly reproduces the two-loop thermodynamic potential

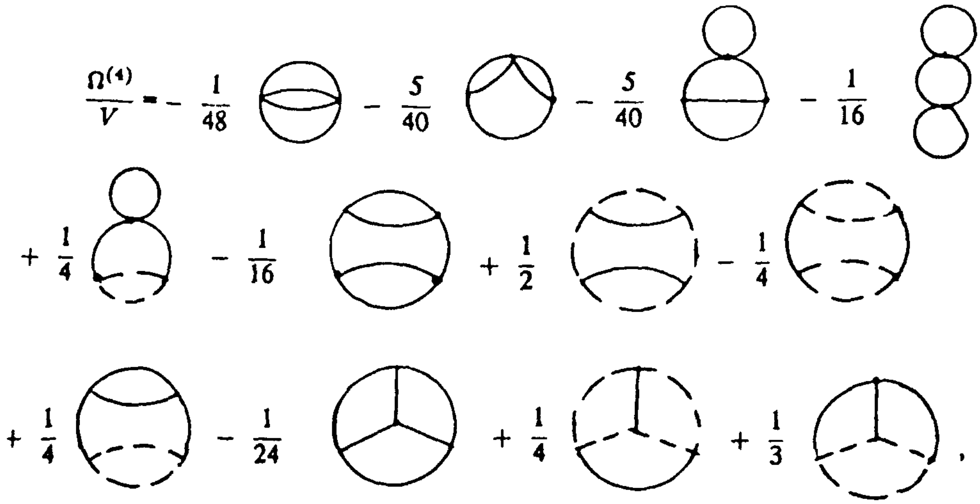


(5)

and also makes it possible to derive (through an integration over γ) the correct coefficients in the three-loop diagram expansion of Ω :

$$\frac{\Omega^{(4)}}{V}=\int_0^1 \frac{d\gamma}{\gamma}\{\Pi_\gamma^{(4)}D_0-\Pi^{(2)}D_0D_0\Pi^{(2)}\}. \tag{6}$$

The expression for $\Pi^{(4)}$ here can be taken from Ref. 3. There are no additional difficulties in the integration over γ in (6), or in any other order of perturbation theory. The resulting diagram series is



(7)

where all the Green's functions and vertices are of a seed nature. The solid lines here are the propagation functions of gauge fields in the α gauge; the dashed lines are the Green's functions of some fictitious fields, a familiar attribute of this gauge.

Equation (7) is a new and important result, which determines an analytic expression for the three-loop thermodynamic potential and raises the hope that the complex and laborious calculations will have a successful outcome. The basic difficulties stem from the last three diagrams, which themselves stem from an iteration of the three-gluon vertex function. Their status deserves a separate discussion. Without regard to the basic calculations, within the framework of Eq. (7) it is necessary to first verify that the infrared singularities, which are not present in the g^4 corrections to Ω, cancel out. It is also important to recall that the introduction of a $\gamma \neq 1$ disrupts the gauge invariance of the theory in all intermediate stages of the calculations, so many of the ordinary properties of this theory are lost. With $\gamma \neq 1$, for example, not only the longitudinal part but also the transverse part of the two-loop polarization tensor has a finite infrared limit:

$$\Pi_\gamma^{\|}(|\bar{k}| \to 0,\ k_4=0) = \frac{g^2NT^2}{12}\left(\frac{3}{2}\gamma^2 + 3\gamma^4 - \frac{1}{2}\gamma^2\right),$$

$$\Pi_\gamma^{\perp}(|\bar{k}| \to 0,\ k_4=0) = \frac{g^2NT^2}{12}\left(-\frac{7}{2}\gamma^6 + 3\gamma^4 - \frac{1}{2}\gamma^2\right). \qquad (8)$$

This point must not be overlooked in explicit calculations.

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[2]O. K. Kalashnikov and Klimov, Yad. Fiz. **33**, 1572 (1981) [Sov. J. Nucl. Phys. **33**, 847 (1981)]; O. K. Kalashnikov, Fortschr. Phys. **32**, 525 (1984).
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