# Effect of $Z^3$ correction in ionizational energy losses on the ranges of heavy ions

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(Submitted 7 July 1993; resubmitted 20 September 1993)
Pis'ma Zh. Eksp. Teor. Fiz. **58**, No. 9, 711–714 (10 November 1993)

A first multifaceted study has been made of how a $Z^3$ correction in the ionizational energy losses affects the ranges of Ne, Ar, Fe, Au, and U nuclei with energies of 0.3–1.2 GeV/nucleon in a nuclear emulsion. The results of this study indicate a systematic discrepancy between the experimental ranges of heavy relativistic nuclei and the ranges calculated from the standard Bethe–Bloch stopping theory. The $Z^3$ correction improves the agreement between theory and experiment.

Bethe's theoretical derivation of the ionizational energy losses, with $dE/dx \propto Z^2$, is based on the first Born approximation for describing the collision of a heavy particle with atomic electrons. Calculations carried out in the next approximation generate another term in the formula for $dE/dx$, which is correspondingly proportional to $Z^3$, where $Z$ is the charge of the incident particle. Numerical estimates of the $Z^3$ correction were found by Jackson and McCartly.[1] They also showed that the estimates found for the $Z^3$ effect in $dE/dx$ can explain the experimental differences between the ranges of $\pi^+$ and $\pi^-$ mesons. Nevertheless, the model developed in Refs. 2 and 3, which is successful in calculations on the passage of nonrelativistic heavy ions with an energy of 0.01–10 MeV/nucleon through homogeneous media, ignores the $Z^3$ correction in the ionizational energy losses of multiply charged particles. In addition, Ziegler and Iafrate,[2] concluded that the $Z^3$ correction according to Ref. 1 degrades the agreement between theory and experiment. In this letter we are reporting the first study of how the $Z^3$ correction in the ionizational losses affects the ranges of Ne, Ar, Fe, Au, and U nuclei with energies in the interval 0.3–1.2 GeV/nucleon in a BR-2 nuclear emulsion. The initial energies and masses of the particles studied here are higher than the mass and energy discussed in Refs. 1–3 by a factor of more than 100.

Chambers consisting of 30–35 layers of BR-2 emulsion of the standard composition[4] were exposed at the Bevalac accelerator (Berkeley, California). (The numbers of nuclei per 1 cm$^3$, specifically, $n_i \times 10^{-22}$, are as follows: 3.148 H, 1.412 C, 0.396 N, 0.956 O, 0.004 S, 0.002 I, 1.031 Br, 1.036 Ag.) The chambers were wrapped in two layers of black paper with a total thickness of 0.024 g/cm$^2$ and in one layer of polyethylene with a thickness of 0.017 g/cm$^2$. Before the chambers were developed, at the High Energy Laboratory, Joint Institute for Nuclear Research (Dubna, Russia), the thickness of each emulsion layer was measured at four symmetric points. It was found that the thickness of the undeveloped layer fluctuates with a standard deviation of close to 4 $\mu$m. The tracks were traced by scanning along the track on an MBI-9 microscope at a magnification of 40×15 (the scale division on the

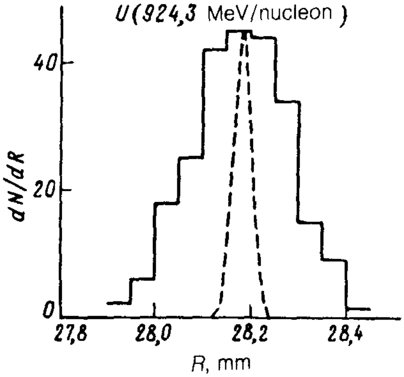

FIG. 1. Distribution of the ranges of $^{238}U$ nuclei in a BR-2 emulsion ($dN/dR$ is the number of nuclei whose ranges fall in the given interval of track lengths $R$). Histogram: Experimental results of the present study ($R_t = 28.185 \pm 0.02$ mm; $\Delta R_t = 130 \pm 20$ $\mu$m). The total number of tracks is $N_{\text{tot}} = 241$. Dashed line: Gaussian distribution with the parameters $R_t = R_{\text{expt}} = 28.185$ mm and $\Delta R_t = \Delta R_{\text{calc}} = 17.5$ $\mu$m, normalized at the peak.

eyepiece was 2.7 $\mu$m). The total statistical base was 1327 tracks. For each case, on the order of 200 tracks were traced (the actual number ranged from 94 for Au to 307 for Ne). The experimental errors in the measurements of the lengths of individual tracks were determined by repeating the measurements on the actual tracks. These errors turned out to be 30 $\mu$m for U nuclei and 20 $\mu$m for the others. The reason for the larger error in the case of U was the need to trace each track in two or three plates. In all cases the tracks of the accelerated nuclei terminated in the same plate in which they began.

Figure 1 illustrates the results with the experimental distribution of the lengths of the tracks of U nuclei with an energy of 924.3 MeV/nucleon stopped in the BR-2 emulsion. Also shown here is a Gaussian distribution with a variance corresponding to the calculated straggling. This straggling was calculated from data on the straggling of protons of the same velocity in accordance with Ref. 5, with allowance for an inverse proportionality between the relative straggling $\Delta R_t/R_t$ and $M^{1/2}$, where $M$ is the ion mass. We see from Fig. 1 that the actual scatter in the ranges, $(\Delta R_t/R_t)_{\text{exp}}$, is larger than the corresponding theoretical estimate. The explanation may be any of the following: Coulomb scattering of the heavy nucleus by atoms of the medium, a beam energy spread $\Delta E/E$ during the bombardment, and measurement errors. The observed relative scatter $(\Delta R_t/R_t)_{\text{exp}}$, which has an average value of 0.8% (the figures range from 0.46% for U to 0.98% for Ar), can be used as an upper estimate of $\Delta\ E/E$. In determining the errors in the mean range $R_t$, we allowed for the experimental variance of the distribution of track lengths.

The ranges were calculated by the PRAL program[3] and the RANGE program,[6] whose validity has been demonstrated previously for energies up to 1 MeV/nucleon. The validity test[3,6] for particles with energies up to 1 GeV/nucleon was based on experimental and theoretical data of Barkas[7] on the ranges of protons in an Ilford G5 emulsion. A comparative analysis of the data showed that the calculations by each program (PRAL and RANGE) systematically underestimate the particle ranges by 2% in comparison with the experimental and theoretical results of Barkas.[7,8] When this systematic underestimate of the ranges is taken into account, we find that the greatest difference between the theoretical and experimental results in the calculations

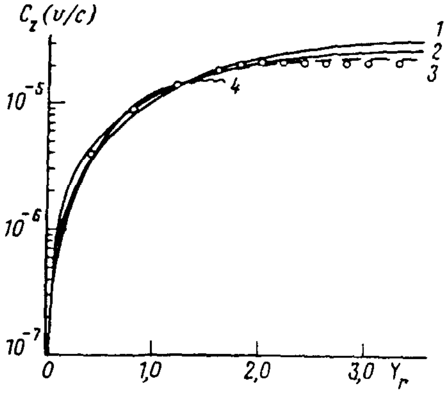

FIG. 2. Correction to the range of an ion in an Ilford G5 emulsion ($C_z$) versus the velocity of the ion, expressed in units of the velocity of a $K$-shell electron [$Y_r = 137(v/c)/Z$; see the text proper]. Points— Universal curve[9] constructed from experimental data[9] for nuclei with $6 \leqslant Z \leqslant 20$; curves—calculated by the RANGE program for several nuclei: $^{20}$Ne (1), $^{40}$Ar (2), $^{56}$Fe (3), and $^{197}$Au (4).

by Barkas and the present study is no more than 1.5–2%. The difference between the ranges calculated by the PRAL and RANGE programs is no greater than 0.1%. Testing of the programs was pursued through calculations on the passage of heavy ions with energies up to 1 GeV/nucleon in the BR-2 and G5 emulsions. Heckman *et al.*[9] have used a semiempirical formula in which the range $R(\beta)$ of a heavy ion of mass $M$ and charge $Z$ is related to the range of a proton of the same velocity $v$, i.e., $\lambda(\beta)$ ($\beta = v/c$, where $c$ is the velocity of light), by

$$R(\beta) = \frac{M}{Z^2}\lambda(\beta) + C_z(\beta/Z)MZ^{2/3}, \qquad (1)$$

where the correction $C_z(\beta/Z)$ is for the capture of electrons at low ion velocities. This correction is usually treated as a universal function of $\beta/Z$ for a given emulsion.[9] Calculations by the RANGE program and a comparison of these data with the results of Ref. 9 show (Fig. 2) that $C_z(\beta/Z)$ can be accepted as "universal" only within an error about 10%, since for heavy ions such as Au and U the correction $C_z(\beta/Z)$ reaches the plateau with increasing ion velocity considerably sooner than for Ne, Ar, and Fe nuclei, and in addition the plateau is smaller. However, the relative contribution of the correction associated with $C_z$ in Eq. (1) does not exceed 2% in the energy range studied, for either the Ilford G5 or the BR-2 emulsion.

Table I shows experimental and theoretical ranges of heavy ions in the BR-2 emulsion of standard composition.[4] {Shown in parentheses is the relative deviation of the theoretical ranges from the experimental ones, $\delta = [(R_{\rm exp} - R_{\rm calc})/R_{\rm exp}] \cdot 100\%$.} We see in this table that the ranges calculated by the two programs are essentially the same, and in satisfactory agreement with the experimental data: The relative discrepancy $\delta$ varies systematically with increasing $Z$ of the ion, from $+5\%$ for Ne to $-8.98\%$ for U.

To evaluate the effect of the $Z^3$ correction, we carried out some calculations which incorporate this correction in the model of Ref. 1. The stopping power for a heavy ion of velocity $v$, i.e., $S_e(v)$, is written as

$$S_e(v) = Z_{\rm eff}^2(v)S_p(v)[1 + Z_{\rm eff}(v)(J/I)], \qquad (2)$$

TABLE I. Ranges of heavy ions in BR-2 nuclear emulsion of standard composition.

| Ion | Energy MeV/nucleon | Experiments of present study $R_p$, mm | Calculations of present study (RANGE) without $Z^3$ correction $R_p$, mm | Calculations of present study (RANGE) with $Z^3$ correction $R_p$, mm | Calculations by PRAL program $R_p$, mm |
|---|---|---|---|---|---|
| Ne | 389.4 | 61.75±0.06 | 58.67<br>(+4.99%) | 58.06<br>(+5.21%) | 58.68 |
| Ar | 504.2 | 53.11±0.05 | 54.23<br>(−2.11%) | 53.33<br>(−0.40%) | 54.25 |
| Fe | 498.7 | 34.84±0.01 | 35.71<br>(−2.50%) | 34.91<br>(−0.20%) | 35.72 |
| Fe | 980.84 | 95.16±0.04 | 96.05<br>(−0.94%) | 94.05<br>(+1.17%) | 96.10 |
| Au | 1147.2 | 42.83±0.05 | 45.76<br>(−6.89%) | 43.44<br>(−1.42%) | 45.81 |
| U | 924.3 | 28.185±0.02 | 30.66<br>(−8.78%) | 28.86<br>(−2.40%) | 30.68 |

where $S_p(v)$ is the stopping power for a proton of the same velocity $v$, $Z_{\text{eff}}$ is the effective charge of the ion calculated as in Ref. 3, and $J/I$ is calculated as in Ref. 1. The ionizational energy losses are calculated by the Bragg rule. The relative increment in the "standard" $dE/dx$ is 5.7% and 5.1%, respectively, for U ions (with an energy of 924.3 MeV/nucleon) and Au ions (1147.2 MeV/nucleon). It is less than 2% for the other ions. The results calculated for the total ranges of the ions with the $Z^3$ correction are shown in the next-to-last column of Table I. The agreement with experiment for the heavy nuclei (Au, U) is seen to be improved significantly, and the systematic overestimate of the theoretical range with increasing $Z$ is essentially eliminated.

In summary, this study of the total ranges of heavy nuclei in a nuclear emulsion has demonstrated unambiguously that calculations which use the proportionality $dE/dx \propto Z^2$, which follows from the first Born approximation, result in a significant systematic error for nuclei such as Au and U. The incorporation of higher powers of $Z$ of the ion results in a satisfactory description of the experimental data. We wish to place particular stress on the point that this phenomenon is general in nature. The approximations for $J/I$ used in Ref. 1 to reconcile the ranges of $\pi^+$ and $\pi^-$ mesons turn out to be valid for the passage of nuclei with a charge higher by a factor of nearly 100 and with a mass higher by a factor of nearly 1500.

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Translated by D. Parsons