Spectral properties of a Josephson junction in a strong magnetic field

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Spectral properties are derived for the mixed state of vortices in a long Josephson junction in a strong magnetic field for the case in which the length scale of the vortex harmonics is smaller than the London depths, but large in comparison with the correlation lengths.

In this letter we present some theoretical results on the frequency spectrum of a long Josephson junction in a strong magnetic field oriented in the plane of the junction. As we know,[1] there is a mixed state in this situation, characterized by a periodic structure with a period

$$L=\frac{\phi_0}{2\pi(\lambda_++\lambda_-+2d)\bar{H}}\,. \tag{1}$$

Here $\phi_0=\pi\hbar c/|e|$ is the quantum of magnetic flux, and λ_+ and λ_- are the London penetration depths in the superconductors on the two sides of the tunnel junction, $2d$ is the width of the junction, and $\bar{H}$ is the magnetic field averaged along the junction. If this field is sufficiently strong, e.g., if it approaches the lower critical field in magnitude but remains lower,[2]

$$H_{c1}^{+(-)}=\frac{\phi_0}{4\pi\lambda_{+(-)}^2}\ln\frac{\lambda_{+(-)}}{\xi_{+(-)}},$$

where $\xi_{+(-)}$ is the correlation length, and $\lambda_{+(-)}\gg\xi_{+(-)}$, then the period in (1) can evidently be smaller than the London depth. Under such conditions, according to Refs. 3–5, Josephson vortex structures can be described by the following basic equation of nonlocal electrodynamics for the phase difference (φ) between Cooper pairs on the two sides of an infinitely long junction:

$$\sin\varphi+\frac{\beta}{\omega_y^2}\frac{\partial\varphi}{\partial t}+\frac{1}{\omega_y^2}\frac{\partial^2\varphi}{\partial t^2}=\lambda_0^3\frac{\partial}{\partial z}\int_{-\infty}^{+\infty}dz'Q(z-z')\frac{\partial\varphi(z',t)}{\partial z'}\,. \tag{2}$$

Here ω_y is the Josephson frequency, β characterizes the resistive properties of the junction, $\lambda_0^3=\lambda_y^2(\lambda_+ + \lambda_- +2d)$, and λ_y is the Josephson length. Finally,

$$Q(z)=\int_{-\infty}^{+\infty}\frac{dk}{2\pi}e^{ikz}K(k) \quad\text{and}\quad K(k)=[\lambda_+\sqrt{k^2\lambda_+^2+1}+\lambda_-\sqrt{k^2\lambda_-^2+1}+2d]^{-1}\,. \tag{3}$$

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The same kernel relates the magnetic field inside the Josephson junction to the phase difference:[5]

$$H_y(z,t) = -\frac{\phi_0}{2\pi}\int_{-\infty}^{+\infty} dz'\frac{\partial\varphi(z',t)}{\partial z'} Q(z-z'). \tag{4}$$

In accordance with Ref. 3, linearized equation (2) describes modified Swihart waves, whose frequency ω is related to their wave vector k by

$$\omega^2 = \omega_y^2\left[1+\frac{\lambda_y^2(\lambda_+ + \lambda_- + 2d)k^2}{\lambda_+\sqrt{k^2\lambda_+^2+1}+\lambda_-\sqrt{k^2\lambda_-^2+1}+2d}\right]. \tag{5}$$

For our purposes, we are interested in this formula in the short-wave limit, with $k\lambda_{+(-)} \gg 1$, and $k(\lambda_+^2 + \lambda_-^2) \gg 2d$. Finally, we ignore the one in square brackets on the right side of (5). We then find

$$\omega = c_0\sqrt{\frac{\lambda_+ + \lambda_- + 2d}{\lambda_+^2+\lambda_-^2}}\, k, \tag{6}$$

where $c_0 = \omega_y\lambda_y$ is the velocity of an ordinary Swihart wave.[2]

In a strong magnetic field $\bar{H}$ (this is again the average along the junction), a solution of Eq. (2) can be written in the approximate form

$$\varphi(z,t) = -(z/L) + \phi(z,t), \tag{7}$$

where L is given by (1), and the small quantity $\phi(z, t)$ can be written as the expansion $\phi = \phi_1 + \phi_2 + \ldots$. In this case we find

$$\phi_1(z,t) = \sin\frac{z}{L}\left\{\frac{1}{\eta(1/L)} + C_1\sin\left(\sqrt{\omega_1^2 - \frac{\beta^2}{4}}t + \psi_1\right)e^{-\beta t/2}\right\}, \tag{8}$$

$$\phi_2(z,t) = \sin\frac{2z}{L}\left\{-\frac{1}{2\eta(L/L)\eta(2/L)} + C_2\sin\left(\sqrt{\omega_2^2 - \frac{\beta^2}{4}}t + \psi_2\right)e^{-\beta t/2} + \frac{C_1\sin\left(\sqrt{\omega_1^2 - \frac{\beta^2}{4}}t + \psi_1\right)e^{-\beta t/2}}{2[\eta(1/L) - \eta(2/L)]}\right\}, \tag{9}$$

where

$$\eta(n/L) = \lambda_0^3(n/L)^2K(n/L), \tag{10}$$

$$\omega_n = c_0\frac{n}{L}\sqrt{\frac{\lambda_+ + \lambda_- + 2d}{\lambda_+\sqrt{(n\lambda_+/L)^2+1}+\lambda_-\sqrt{(n\lambda_-/L)^2+1}+2d}}, \tag{11}$$

and C_1, C_2, ψ_1, and ψ_2 are arbitrary constants, which can be found from the initial-value problem. Before we discuss the spectral properties of the Josephson junction, which are becoming apparent, let us examine the conditions under which our approximate solution is applicable. If this solution is to be valid, the quantity η (n/L) must be large, and the integration constants C_n must of course be small. These constants determine the size of small time-varying perturbations. The latter condition on C_n is not related to spectral properties. We should thus take a careful look at the condition

$$\eta(n/L) \gg 1. \tag{12}$$

In the case of strong fields, in which we are interested here, and in which the conditions $\lambda_{+(-)} > (L/n)$ hold, the average magnetic field strength satisfies the inequality

$$n\bar{H} > \frac{\phi_0}{2\pi(\lambda_+ + \lambda_- + 2d)\lambda_{+(-)}}. \tag{13}$$

We assume for simplicity that the inequalities $(\lambda_{+(-)}^2/2d) > (L/n)$ also hold. This assumption does not impose any limitations if the width of the tunnel junction is small in comparison with the London depths. Condition (13) then reduces to

$$n\bar{H} > \frac{\phi_0(\lambda_+^2 + \lambda_-^2)}{2\pi\lambda_y^2(\lambda_+ + \lambda_- + 2d)^2}. \tag{14}$$

Under the usual conditions $\lambda_y \gg \lambda_{+(-)}$, condition (14) is weaker than condition (13). It can be stronger only for tunnel junctions with an anomalously large Josephson critical current. In this case we would have $\lambda_y < \lambda_{+(-)}$. Since conditions (13) and (14) can be satisfied at $\bar{H} < H_{c1}$, our analysis allows us to use expression (11) under these conditions. For a thin junction $(2d < \lambda_{+(-)})$, expression (11) becomes

$$\omega_n = c_0 \frac{\lambda_+ + \lambda_-}{\sqrt{\lambda_+^2 + \lambda_-^2}} \sqrt{\frac{2\pi\bar{H}}{\phi_0}} \sqrt{n} \operatorname{sgn} n. \tag{15}$$

This expression differs in a qualitative way, in both the n dependence and the $\bar{H}$ dependence, from the corresponding limit of a weak field, in which we have $\omega_n = \text{const} \cdot n\bar{H}$.

In discussing the spectral properties of the Josephson junction in a strong magnetic field, we ignore the resistivity. Assuming $\beta = 0$, we can then assert that the frequencies in (15) and the combinational frequencies $\omega_N = \Sigma_{l,n} l\omega_n$ are characteristic frequencies of the tunnel junction. The onset of various values of n corresponds to the effect of spatial harmonics (nz/L) established in Eqs. (8) and (9). The appearance of various values of l corresponds formally to the effect of the nonlinearity of $\sin\varphi$, which is not brought out in Eqs. (8) and (9) but which is nevertheless obvious. Clearly, a dependence $\sim \sqrt{\bar{H}}$ is a comparatively simple general dependence of ω_N in a strong field. In conclusion we can say that expression (15) of this paper corresponds to the dependence of the frequency of generalized Swihart waves, (6), on the wave vector, if we use $k = 2\pi\, n/L$, with L from (1).

In summary, this analysis establishes the frequency spectrum of a long Josephson junction in a strong magnetic field.

[1] I. O. Kulik and I. K. Yanson, *Josephson Effect in Superconducting Tunneling Structures* (Halsted, New York, 1972).

[2] A. A. Abrikosov, *Fundamentals of the Theory of Metals* (Elsevier, New York, 1988).

[3] Yu. M. Aliev, V. P. Silin, and S. A. Uryupin, Sverkhprovodimost (KIAE) **5**, 228 (1992) [Superconductivity **5**, 230 (1992)].

[4] Yu. M. Aliev and V. P. Silin, Phys. Lett. A **117**, 259 (1993).

[5] Yu. M. Aliev and V. P. Silin, Zh. Eksp. Teor. Fiz. **104**, 2526 (1993) [Sov. Phys. JETP **77**, 142 (1993)].

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