Singularity in the quasiparticle interaction function in a 2D Fermi gas

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The interaction function of Landau quasiparticles, $f(\mathbf{p}, \mathbf{p}')$, of a 2D Fermi gas is calculated in second-order perturbation theory. This function has a square-root singularity near the Fermi surface when the momenta $\mathbf{p}$ and $\mathbf{p}'$ are nearly parallel. Although this singularity does not disrupt the Fermi-liquid picture in the 2D case, it does lead to some nontrivial corrections to thermodynamic properties and to the zero sound velocity.

Some recent papers by Anderson[1] have provoked a lively debate on whether a 2D Fermi gas exists even in the case of a weak interaction. Anderson formulated three fundamental points which lead him to doubt the applicability of the standard Galitskiĭ–Bloom approach[2] in the 2D case. (a) There is the question of whether the scattering phase shift for particles with nearly parallel momenta and opposite spins is nonzero. Such a value would lead to the vanishing of the Z factor on the Fermi surface (a Luttinger Fermi liquid). (b) There is the related question of whether the upper Hubbard band plays an important role in lattice models even in the case of a low electron density. (c)The final question concerns a singularity which arises in the Landau f -function in a 2D Fermi gas even in the absence of a lattice.

In the ensuing debate,[1,3–6] it has become clear that both the authors who agree with Anderson and those who disagree (who adhere to Fermi-gas ideas) have some fairly strong arguments. The debate actually centers on the choice of the correct state to serve as the basis for constructing a regular procedure of successive approximations in the interaction (more precisely, in only that part of the interaction which is ignored in the choice of the ground state). For example, it was shown by perturbation theory in Refs. 3–6 that a singularity in the t-matrix which is associated with the exclusion principle in momentum space for the scattering of two particles with parallel momenta and opposite spins, and which is also associated with the existence of an upper Hubbard band, leads to only nontrivial corrections to the Z-factor and the quasiparticle lifetime τ; it does not disrupt the self-consistent Fermi-liquid picture. We, too, agree with the Fermi-liquid ideas. Using those ideas as a framework, we attempt in this letter to analyze in more detail the third point of the debate, concerning the existence of a singularity in the Landau quasiparticle interaction function $f(\mathbf{p}, \mathbf{p}')$. This third point has previously received little attention. We show that singularities which arise in the f-function even in second-order perturbation theory lead to nontrivial corrections to the Fermi-liquid parameters, but they do not disrupt the overall Fermi-liquid picture.

According to the classical results of Landau's Fermi-liquid theory,[7] the f-function is given by the following expression in second-order perturbation theory:

$$f_{+-}(\mathbf{p},\mathbf{p}')\equiv f_s-f_a=\frac{\delta^2 E}{\delta n_+(\mathbf{p})\delta n_-(\mathbf{p}')}$$

$$=g-\frac{4mg^2}{(2\pi)^2}\int\left[\frac{\delta(\mathbf{p}+\mathbf{p}'-\mathbf{p}_1-\mathbf{p}_2)}{p^2+p'^2-p_1^2-p_2^2}+\frac{1}{4}\frac{\delta(\mathbf{p}+\mathbf{p}_1-\mathbf{p}'-\mathbf{p}_2)}{p^2+p_1^2-p'^2-p_2^2}\right.$$

$$\left.+\frac{1}{4}\frac{\delta(\mathbf{p}'+\mathbf{p}_1-\mathbf{p}-\mathbf{p}_2)}{p'^2+p_1^2-p^2-p_2^2}\right]$$

$$\cdot\,\theta(p_F^2-p_1^2)d^2p_1d^2p_2, \qquad (1)$$

$$f_{++}(\mathbf{p},\mathbf{p}')\equiv f_s+f_a=\frac{\delta^2 E}{\delta n_+(\mathbf{p})\delta n_+(\mathbf{p}')}=-\frac{2mg^2}{(2\pi)^2}\cdot\frac{1}{4}$$

$$\times\int\left[\frac{\delta(\mathbf{p}+\mathbf{p}_1-\mathbf{p}'-\mathbf{p}_2)}{p^2+p_1^2-p'^2-p_2^2}+\frac{\delta(\mathbf{p}'+\mathbf{p}_1-\mathbf{p}-\mathbf{p}_2)}{p'^2+p_1^2-p^2-p_2^2}\right]$$

$$\times\theta(p_F^2-p_1^2)d^2p_1d^2p_2, \qquad (2)$$

where the coupling constant g is given by

$$g=\frac{mU_0/4\pi}{1+mU_0/4\pi\cdot\ln(1/p_F^2r_0^2)},$$

which is the standard formula for the 2D case. Here p_F is the Fermi momentum, and U_0 and r_0 are the amplitude and range of the potential. For a non-Born repulsive potential we would have

$$g=\frac{1}{2\ln(1/p_Fr_0)}.$$

The notation f_{++} and f_{+-} refers to the spin. Ordinarily, $f(\mathbf{p},\mathbf{p}')$ is calculated on the Fermi surface ($p=p'=p_F$). In the 3D case, it takes the familiar form found by Abrikosov and Khalatnikov.[7,8] Taking a careful look at the expression for f_{+-}, we see that it is of the following form:

$$f_{+-}=g+g^2K+g^2\Pi,$$

where

$$K=\int\frac{1-\theta(p_F^2-p_1^2)-\theta(p_F^2-(\mathbf{p}+\mathbf{p}'-\mathbf{p}_1)^2)}{p^2+p'^2-p_1^2-(\mathbf{p}+\mathbf{p}'-\mathbf{p}_1)^2}d^2p_1$$

is the expression for a Cooper loop. In complete analogy with the 3D case, this expression becomes ($p=p'=p_F$)

$$K=\frac{m}{4\pi}\ln\frac{4p_F^2-k^2}{k^2},$$

where $\mathbf{k}=\mathbf{p}+\mathbf{p}'$ is the momentum of the center of mass.

This expression contains a standard logarithmic singularity when the angle between $\mathbf{p}$ and $\mathbf{p}'$ is equal to π. This situation leads to a Cooper pairing in the case of an attractive potential. That case is of no further interest for our purposes here.

The third and fourth terms in f_{+-} correspond to a diagram of an exchange nature. For a short-range potential, this diagram is equivalent to a polarization operator at a nonzero frequency:

$$\Pi(\pm\Omega,\mathbf{q})=\int\frac{\theta(\varepsilon_F-\varepsilon_p)-\theta(\varepsilon_F-\varepsilon_{p+q})}{\varepsilon_p-\varepsilon_{p+q}\pm\Omega}\,d^2p,$$

where the role of the frequency Ω is played by the energy difference between the scattered particles, $(p^2-p'^2)/2m$, and $\mathbf{q}=\mathbf{p}-\mathbf{p}'$ is exactly equal to the momentum transfer. If, as usual, we restrict the discussion to an evaluation of the f-function on the Fermi surface, then we have $p=p'=p_F$ and $\Omega=0$. According to Kagan and Afanas'ev,[9] the polarization operator in this case is

$$\Pi(0,\mathbf{q})=\frac{m}{4\pi}\left(1-\mathrm{Re}\sqrt{1-\frac{4p_F^2}{q^2}}\right).$$

In other words, it contains no singularities.

The situation changes radically if we allow a departure from the Fermi surface in terms of the momenta $\mathbf{p}$ and $\mathbf{p}'$. In this case, according to Stern and a later detailed analysis by Fukuyama,[4] we have

$$\Pi(\Omega,\mathbf{q})\sim\frac{m}{4\pi}\left[1+\mathrm{sign}\left(\frac{m\Omega+q^2}{2p_Fq}\right)\frac{p_F}{q}\mathrm{Re}\sqrt{\left(\frac{m\Omega+q^2}{2p_Fq}\right)^2-1}\right.$$
$$\left.-\mathrm{sign}\left(\frac{m\Omega-q^2}{2p_Fq}\right)\frac{p_F}{q}\mathrm{Re}\sqrt{\left(\frac{m\Omega-q^2}{2p_Fq}\right)^2-1}\right].$$

We introduce some small deviations from the Fermi surface, $\varepsilon=(p-p_F)/p_F$ and $\varepsilon'=(p'-p_F)/p_F$, for the magnitudes of the vectors $\mathbf{p}$ and $\mathbf{p}'$. If we also introduce the angle between these vectors, $\varphi=\mathbf{p}\cdot\hat{\mathbf{p}}'$, we can reduce the polarization operator for nearly parallel vectors near the Fermi surface (ε, ε', $\varphi\to 0$) to the following form through some straightforward calculations:

$$\frac{2\pi}{m}\Pi(\Omega,\mathbf{q})-1\sim\frac{\mathrm{sign}(\varepsilon-\varepsilon')}{(\varepsilon-\varepsilon')^2+\varphi^2}\mathrm{Re}\,\{\sqrt{2\varepsilon(\varepsilon-\varepsilon')^2-\varphi^2)}-\sqrt{2\varepsilon'(\varepsilon-\varepsilon')^2-\varphi^2}\}. \tag{3}$$

It can be seen from this expression that, as we move upward from the Fermi surface in terms of the magnitude of one of the momenta (e.g., $\varepsilon>0$, $\varepsilon'=0$), there exists a small angular interval $\varphi\sim\varepsilon^{3/2}$ near an almost parallel orientation of $\mathbf{p}$ and $\mathbf{p}'$ in which the polarization operator and thus the Landau f-function acquire a square-root singularity:

$$f(\varepsilon>0,\quad\varepsilon'=0)\sim\frac{g^2}{\sqrt{\varepsilon}},\quad\varphi<\varepsilon^{3/2}. \tag{4}$$

If we move away from the Fermi surface upward in terms of the magnitudes of both momenta ($\varepsilon>0$, $\varepsilon'>0$), the singularity in the f-function is of the form $g^2/(\sqrt{\varepsilon}+\sqrt{\varepsilon'})$.

The singular part of the f-function in a 2D Fermi gas was originally calculated by Prokofiev (see the review by Stamp[6]). It was later found independently by two of the present authors (M.A.B. and M.Yu.K.). In Anderson's papers[1] it is assumed, by analogy with the situation in 1D Fermi systems, that the f-function has a more singular form:

$$f_{+-}\sim[(\varepsilon-\varepsilon')^2+\varphi^2]^{-\frac{1}{2}}$$

(in our notation).

If the perturbation-theory results did indeed confirm Anderson's hypothesis, the meaning would be that Landau's Fermi-liquid theory fails completely in the 2D case, since the Landau harmonics $f_0, f_1, \ldots$ would become logarithmically divergent. With $\varepsilon=\varepsilon'=0$, we find $f\sim\varphi^{-1}$ for the f-function and, for example,

$$f_0=\int f(\varphi)d\varphi\sim\int d\varphi/\varphi\sim\ln\varphi \text{ as } \varphi\to 0.$$

Actually, the singularity found in second-order perturbation theory, (3), (4), is far weaker: $f_{+-}\sim 1/\sqrt{\varepsilon}$, not $1/\varepsilon$. In addition, it exists in a very narrow angular interval $\varphi\sim\varepsilon^{3/2}$. (Here we are also seeing a distinction between the 2D and 1D cases, since the presence of one more integration variable, φ, renders the divergences weak.)

To find the temperature corrections to some thermodynamic properties—the compressibility, susceptibility, and effective mass—we use equations which generalize the standard Fermi-liquid expressions for these properties to the case in which quasiparticles deviate from the Fermi surface. In this case we find

$$\frac{m^*}{m}=1+\frac{m}{4\pi}\int f^s(\varepsilon,\varepsilon',\varphi)\cos\varphi\frac{\partial n}{\partial\varepsilon}\frac{\partial n'}{\partial\varepsilon'}d\varepsilon d\varepsilon'\frac{d\varphi}{2\pi}+O\left(\frac{T^2}{\varepsilon_F^2}\right), \tag{5}$$

$$\chi=\chi_0\frac{m^*}{m}\left(1+\frac{m}{4\pi}\int f^a(\varepsilon,\varepsilon',\varphi)\frac{\partial n}{\partial\varepsilon}\frac{\partial n'}{\partial\varepsilon'}d\varepsilon d\varepsilon'\frac{d\varphi}{2\pi}\right)^{-1}+O\left(\frac{T^2}{\varepsilon_F^2}\right), \tag{6}$$

$$u^2=\frac{N}{m}\left(\frac{\partial\mu}{\partial N}\right)_T=\frac{v_F^2}{2}\left(1+\frac{m}{4\pi}\int f^s(\varepsilon,\varepsilon',\varphi)\frac{\partial n}{\partial\varepsilon}\frac{\partial n'}{\partial\varepsilon'}d\varepsilon d\varepsilon'\frac{d\varphi}{2\pi}\right)+O\left(\frac{T^2}{\varepsilon_F^2}\right). \tag{7}$$

After substituting the singular part of the f-function into these expressions, we find the following expression [the singular part contributes to only f^s, as can be seen from Eqs. (1) and (2)]:

$$\frac{m^*(T)-m^*(0)}{m^*(0)}\sim\frac{\chi(T)-\chi(0)}{\chi(0)}\sim\frac{u^2(T)-u^2(0)}{u^2(0)}\sim g^2\frac{T}{\varepsilon_F}. \tag{8}$$

Correspondingly, the corrections to the specific heat and the entropy are proportional to g^2T^2/ε_F. The correction terms in the thermodynamic potential are evidently of the form g^2T^3/ε_F^2 in the 2D case. (The summation of ladder diagrams does not strengthen the singularity.) Corrections of the same type to the specific heat were recently found in Ref. 13 through an analysis of corrections to the eigenenergy part. The temperature dependence of the latter corrections is the same as the standard paramagnon corrections, $\sim -T^2\ln T$, in the 2D case. On the other hand, the corrections to the susceptibility in (8) are much larger than the paramagnon corrections at low temperatures. (We should stress, however, that there are no paramagnons at all in our case of a low-density 2D Fermi gas.)

Calculations of the susceptibility are motivated by some recent experiments by Hallock's group[10] and Sanders's group[11] and also by some experiments planned by Godfrin. The latter experiments will involve using cw NMR to measure the temperature dependence of χ in submonolayers of ^3He on graphite and on the free surface of a thin film of superfluid ^4He. At low temperatures, the corrections which we have found will determine the temperature dependence of the susceptibility. So far, experiments have been carried out primarily at intermediate and high temperatures (at which there is a transition to a Curie law).

As we have already mentioned, $f(\mathbf{p}, \mathbf{p}')$ acquires a singular part because of a singularity in the 2D polarization operator $\Pi(\Omega,q)$ as $q=|\mathbf{p}-\mathbf{p}'| \to 0$, $\Omega = (p^2-p'^2)/2m \to 0$, and $\Omega/v_F q \to 1$. In a sense, this singularity is an exchange analog of a zero-sound pole (a collective mode) in a vertex function. The solution of the Bethe–Salpeter equation in the zero-sound channel (which is equivalent to the solution of a collisionless kinetic equation) leads to the following corrections to the velocity of 2D zero sound:

$$\frac{c(T)-c(0)}{c(0)} \sim g^2 \frac{T}{\varepsilon_F}$$

at

$$T/\varepsilon_F < g^2 .$$

[At $T=0$, the 2D zero sound velocity is $c(0) \approx v_F(1+g^2)$.]

Attempts at a parquet intensification of the zero-sound singularity involving mutual insertions of polarization loops singular in terms of the variables $q=|\mathbf{p}-\mathbf{p}'| \to 0$ and $t=|\mathbf{p}+\mathbf{p}'|-2p_F \to 0$ do not result in an intensification of the singularity.

In summary, it can be concluded that the singularity we have found does not lead to a disruption of the Fermi-liquid picture. It leads to only some nontrivial temperature corrections to Landau harmonics and thus to thermodynamic properties.

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