

Crystallization waves in ^3He

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The growth and melting of ^3He crystals spin-polarized by a magnetic field at temperatures below the antiferromagnetic transition (and below the superfluid transition in the liquid) are accompanied by superfluid spin currents in addition to mass flows. In fields weak in comparison with exchange fields, the crystallization waves change in nature, because the spin currents begin to play a predominant role. The wave spectrum acquires an acoustic nature, with a velocity inversely proportional to the magnetic field.

Crystallization waves arise at the interface between solid and superfluid ^4He because of the quantum, dissipationless nature of the processes by which the crystal grows and melts.^{1,2} The potential energy and kinetic energy of the oscillations are, respectively, the surface energy of the crooked interfacial boundary and the kinetic energy of the superfluid mass flow which arises because of the difference between the densities of the solid and liquid phases.³ We would naturally expect a similar phenomenon in ^3He at temperatures below the temperature of the transition of the liquid to a superfluid state and below the temperature of the antiferromagnetic transition in the solid phase. The magnetism of the liquid and solid phases gives the crystallization waves in ^3He some distinctive features; these features are the subject of the present letter.

The most important of these features is the change in the nature of the crystallization and melting in a magnetic field. In this case, the solid and liquid phases are characterized by different values of not only the mass density but also another (approximately) conserved quantity: the spin density. In the high-temperature region, because of the extremely large value of the mean free time of Fermi quasiparticles, the liquid which appears upon the melting of the spin-polarized crystals has a high nonequilibrium spin density.^{4–6} In the low-temperature case under consideration here, each phase is completely ordered. Accordingly, instead of nonequilibrium spin densities, superfluid spin currents arise during the growth and melting of spin-polarized crystals, along with the mass flows in the liquid. This is true of both the liquid and the crystal.

The kinetic energy of the oscillations of the interface in the crystallization wave is thus the sum of the kinetic energies of the mass flow and the spin current. In contrast with the relative difference between the mass densities of the solid and liquid phases, the relative difference between the spin densities is by no means small in a magnetic field. Accordingly, the spin current begins to play a decisive role in comparatively weak fields, as we will see below. As a result, crystallization waves take on an extremely unusual nature—oscillations in which the potential energy is related to

orbital degrees of freedom, while the kinetic energy is related to spin degrees of freedom.

We denote by $z = \zeta(x, t) = \zeta_0(t)e^{ikx}$ the equation of the boundary between the solid phase ($z < \zeta$) and the liquid phase ($z > \zeta$). This interface is slightly displaced from its $z=0$ position. We assume that the temperature satisfies the condition $T \ll T_N$ (T_N is the Néel temperature of the solid phase), and we assume the magnetic field \mathbf{H} , directed along the normal, \hat{z} , satisfies the condition $H \ll H_0$, where $\mu H_0 \sim T_N$, and μ is the magnetic moment of the ^3He nucleus. Under these conditions the solid phase has an antiferromagnetic $u2d2$ structure, while the liquid phase is $^3\text{He-B}$.

Let us assume, however, that the magnetic field is strong in comparison with the characteristic fields at which the anisotropy vector \mathbf{n} of $^3\text{He-B}$ aligns parallel to the field, i.e., along the z axis, and at which the antiferromagnetic vector \mathbf{l} of the solid phase runs perpendicular to the z axis.

We also assume that the length of the crystallization wave, $1/k$, is large in comparison with the dipole length $l_D \sim 10^{-3}$ cm, over which the nonconservation of the spin in the liquid phase due to the dipole-dipole interaction of the spins is influential. Since the corresponding dipole length in the solid phase, l_s , satisfies the condition $l_s \ll l_D$, the spin currents in each phase are nonzero only in narrow regions with thicknesses l_D and l_s near the interface. The problem of calculating these currents is actually a 1D problem. It can be assumed that the angles through which the spins rotate in the equations of spin hydrodynamics⁷⁻⁹ depend on the coordinate z only. Only the angle θ , of the spin rotation around the z axis, is nonzero in both phases.

The hydrodynamic equations for the liquid ($z > 0$) and for the crystal ($z < 0$) are

$$\tilde{\theta} - c_l^2 \frac{\partial^2 \theta}{\partial z^2} + \Omega_l^2 \theta = 0, \quad \tilde{\theta} - c_s^2 \frac{\partial^2 \theta}{\partial z^2} + \Omega_s^2 \theta = 0, \quad (1)$$

where c_l and c_s are the velocities of the corresponding spin waves in the liquid and solid phases, Ω_l is the frequency of the longitudinal NMR resonance in liquid $^3\text{He-B}$, and Ω_s is the frequency of the uniform oscillations of the vector \mathbf{l} in the solid phase in the plane perpendicular to the magnetic field. This frequency depends on the angle φ , between the normal \hat{z} and the direction of one side of the cubic cell of the ^3He crystal, along which there is a $uudd$ alternation of spins: $\Omega_s^2 = \Omega_{s0}^2 \sin^2 \varphi$, where Ω_{s0} is a constant.⁸

The flux densities of the z component of the spin along the \hat{z} direction in the liquid phase, j_l , and in the solid phase, j_s , are

$$j_l = -\frac{\chi_l c_l^2}{\gamma^2} \frac{\partial \theta}{\partial z}, \quad j_s = -\frac{\chi_s c_s^2}{\gamma^2} \frac{\partial \theta}{\partial z}, \quad (2)$$

where χ_l is the magnetic susceptibility of $^3\text{He-B}$, χ_s is the magnetic susceptibility of the solid phase in the direction perpendicular to \mathbf{l} , and $\gamma = 2\mu / \hbar$ is the gyromagnetic ratio.

Under our conditions, the frequency of the crystallization wave is low in comparison with Ω_l and Ω_s . We can thus ignore the first terms $\tilde{\theta}$ in both equations in (1).

The boundary conditions on these equations are that $\theta(z)$ is finite as $z \rightarrow \pm \infty$, $\theta(z)$ is continuous at $z=0$, and the z component of the spin is conserved at $z=0$:

$$j_l - j_s = -(S_s - S_l)\dot{\xi} = -\frac{\chi_l}{\gamma} H \dot{\xi}. \quad (3)$$

Here $S_s = (\chi_l / \gamma)H$ and $S_l = (\chi_l / \gamma)H$ are the equilibrium spin densities in the crystal and the liquid, and $\dot{\xi}$ is the velocity of the interface. We have used the condition $\chi_l \gg \chi_l$. The energy density of the spin waves in the solid phase is given by

$$E_s = \frac{\chi_l}{2\gamma^2} \left\{ c_s^2 \left(\frac{\partial \theta}{\partial z} \right)^2 + \Omega_s^2 \theta^2 \right\}. \quad (4)$$

The energy density in the liquid, E_l , is found from (4) by replacing χ_l by χ_l , c_s by c_l , and Ω_s by Ω_l .

Solving Eqs. (1) under the boundary conditions formulated above, substituting the solution into (4), and integrating the energy density over z , we find the following expression for the spin part of the kinetic energy of the crystallization wave:

$$\mathcal{E}_{\text{kin}} = \frac{1}{4} M_{\text{sp}} |\dot{\xi}|^2,$$

where the quantity

$$M_{\text{sp}} = \frac{\chi_l^2 H^2}{c_s \Omega_s \chi_l + c_l \Omega_l \chi_l} \quad (5)$$

plays the role of the spin part of the "mass" per unit surface area.

The potential energy of the crystallization wave,

$$\mathcal{E}_{\text{pot}} = \frac{1}{4} \kappa(k) |\xi|^2,$$

is determined by the surface stiffness $\tilde{\alpha}$, as in ${}^4\text{He}$.^{1,3} This stiffness evidently varies only slightly upon the ordering of the phases at low temperatures. Here we have³ $\kappa(k) = \tilde{\alpha} k^2$.

The frequency of the crystallization wave is given by

$$\omega^2(k) = \frac{\tilde{\alpha} k^2}{M(k)}, \quad (6)$$

where $M(k) = M_{\text{sp}} + M_m$, and M_m is determined by the kinetic energy of the superfluid mass flows, as in ${}^4\text{He}$:³ $M_m = (\Delta\rho)^2 / \rho |k|$, where $\Delta\rho = \rho_c - \rho_l$, ρ_l is the density of the crystal, and ρ_c is that of the liquid. Since the condition $\Delta\rho \ll \rho$ holds, we will use the letter ρ , without a subscript, to denote the common value of the density: $\rho \approx \rho_l \approx \rho_c$.

The general expression for the mass $M(k)$ can be written in the form

$$M(k) = \rho l \left[\left(\frac{H}{H_0} \right)^2 + \left(\frac{\Delta\rho}{\rho} \right)^2 \frac{1}{|k|l} \right], \quad (7)$$

where we have introduced a characteristic length

$$l = \chi c_l^2 / (c_l \Omega_l \chi_l + c_s \Omega_s \chi_s)$$

and a characteristic field

$$H_0 = (c_l / \chi_l) (\rho \chi_l)^{1/2}.$$

The characteristic length l is smaller than the dipole length in the liquid, $l_D \sim c_l / \Omega_l$, by a factor of only several units. The characteristic field H_0 is on the order of the exchange field: $\mu H_0 \sim T_N$.

We assume that the magnetic field satisfies the condition $H \gg H_0 \Delta\rho / \rho$, i.e., that the parameter $\epsilon = H_0 \Delta\rho / (\rho H)$ is small in comparison with one. Spin currents then dominate the mass in (7) over a broad range of wavelengths satisfying the inequality $kl \gg \epsilon^2$. The spectrum of the crystallization waves, $\omega(k)$, is linear, $\omega = uk$, with a wave velocity

$$u = \left(\frac{\tilde{\alpha}}{\rho l} \right)^{1/2} \frac{H_0}{H}. \quad (8)$$

Only under the condition $kl \ll \epsilon^2$ can we ignore the contribution of spin currents, and only under this condition does the usual law¹ $\omega \propto k^{3/2}$ hold.

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