Critical behavior of 3D electrodynamics

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Three-dimensional QED with N four-component fermions is analyzed in the leading and succeeding orders of a $1/N$ expansion, in the approach of Appelquist *et al.* As the gauge parameter increases, the interval of N values in which a dynamic mass is generated shrinks markedly. In the Landau gauge, for exmple, a dynamic breaking of chiral symmetry occurs at $N < 3.31$, and in the Feynman gauge a dynamic mass is not generated at all.

The model discussed in this letter is 3D QED with N four-component fermions. In the massless case, to which this letter is restricted, the model contains only infrared divergences, which vanish when a $1/N$ expansion is used.[1,2] As a result, the only scale is the dimensional interaction constant $a=Ne^2/8$, which we will hold fixed as $N\to\infty$.

The Lagrangian of the model is

$$\mathscr{L}=\bar{\Psi}(i\hat{\partial}-e\hat{A})\Psi-\frac{1}{4}F_{\mu\nu}^2, \tag{1}$$

where Ψ is a four-component complex fermion. In the four-component case we can introduce matrices γ_3 and γ_5 which anticommute with γ_0, γ_1, and γ_2. Lagrangian (1) is invariant under transformations $\Psi\to\exp(i\alpha_3\gamma_3)\Psi$ and $\Psi\to\exp(i\alpha_5\gamma_5)\Psi$. In addition to the unit matrix and $[\gamma_3,\gamma_5]$, we have $U(2)$ symmetry for each spinor, so the complete global "chiral" symmetry is $U(2N)$. The introduction of a mass term in Lagrangian (1) breaks this symmetry to $U(N)*U(N)$. The dynamic generation of such a mass is analyzed in this letter. There is the further possibility of incorporating a mass which does not conserve parity in the analysis (Ref. 3, for example), but we will not take up that effect here.

Following Ref. 4, we study the solution of the Schwinger–Dyson equation. We write the inverse fermion propagator in standard form:

$$S(p)=(1+A(p))[-\hat{p}+\Sigma(p)],$$

where $A(p)$ is a wave-function renormalization coefficient, and $\Sigma(p)$ is a dynamic, parity-conserving mass, which is assumed to be the same for all fermions.

The Schwinger–Dyson equation is

$$\Sigma(p)=\frac{2a}{N}\mathrm{Tr}\int\frac{d^3k}{(2\pi)^3}\frac{\gamma^\mu D_{\mu\nu}(p-k)\{\hat{k}+\Sigma(k)\}\Gamma^\nu(p,k)}{(1+A(k))[k^2+\Sigma^2(k)]}, \tag{2}$$

where[1)]

$$D_{\mu\nu}(p)=\frac{g_{\mu\nu}-(1-\xi)p_\mu p_\nu/p^2}{p^2[1+\Pi(p)]}$$

is the photon propagator, and $\Gamma^\nu(p,k)$ is a vertex function.

1. In the leading order in $1/N$ we have

$$A(p)=0,\quad \Pi(p)=a/|p|,\quad \text{and}\quad \Gamma^\nu(p,k)=\gamma^\nu,$$

where we are ignoring a contribution of the fermion mass to the polarization operator. The Schwinger–Dyson equation becomes

$$\Sigma(p)=\frac{16a}{N}\int\frac{d^3k}{(2\pi)^3}\frac{\Sigma(k)}{k^2[(p-k)^2+a|p-k|]},\tag{2a}$$

where, as in Ref. 4, we have ignored a term $\Sigma^2(k)$ in the denominator.

As in Ref. 4, we use the following ansatz for the dynamic mass:

$$\Sigma(k)\sim(k^2)^\alpha.\tag{3}$$

It is clear from Eq. (2a) that in the limit of an infinitely large coupling constant a the right side of Eq. (2a), under condition (3), can easily be found by means of the standard rules for calculating massless perturbation-theory diagrams (Ref. 7, for example). In the limit $a\to\infty$ we find, solving (2a),

$$1=\frac{(2+\xi)}{L\beta}\tag{3a}$$

with $\beta=(-\alpha)(\alpha+\frac{1}{2})$ and $L=\pi^2N$. Alternatively, we have

$$\alpha_\pm=\frac{1}{4}\left[-1\pm\left(1-\frac{16(2+\xi)}{L}\right)^{1/2}\right].\tag{3b}$$

We have thus reproduced a solution offered by Appelquist *et al.*[4] Their analysis leads to the critical value $N_c=16(2+\xi)/\pi^2\approx1.62(2+\xi)$ for the number of fermions such that at $N>N_c$ we have $\Sigma(p)=0$, while for $N<N_c$ we have

$$\Sigma(0)\simeq\exp\left[-\frac{2\pi}{(N_c/N-1)^{1/2}}\right].$$

A breaking of chiral symmetry occurs when the index α become complex.

2. The next order in terms of an expansion in $1/N$ has already been analyzed by Nash.[5] We would point out, however, that the result in Ref. 5 was found in the form of the ansatz

$$\beta=\frac{d(\xi)}{L}\left(1-\frac{b(\xi)}{L}\right)\tag{4}$$

for the behavior of $\beta(L)$. Furthermore, a complete result was found only in the Feynman gauge: $d(1)=\frac{8}{3}\approx2.67$, $b(1)\approx7.81$.

We have exactly evaluated the Feynman integrals corresponding to the $1/N$ corrections (see Fig. 1 of Ref. 5), using the rules for evaluating massless diagrams of standard perturbation theory. The result contains terms in the form of double and triple series; i.e., the result is rather complicated, and it will be studied in a separate publication. We restrict the present letter to an analysis of only a simplified dependence corresponding to the selection of terms $\sim 1/(-\alpha)^k$ and $\sim 1/(\alpha+1/2)^k$ from the complex series. These terms are most important for a study of the vicinity of the critical point, $\alpha_c = -\frac{1}{4}$, $N = N_c$. We have the equation

$$1=\frac{(2+\xi)}{L\beta}+\frac{1}{(L\beta)^2}[f(\xi)+\varphi(\xi)\beta], \tag{5}$$

where

$$f(\xi)=\frac{4}{3}(1-\xi)-\xi^2, \quad \varphi(\xi)=\frac{176}{9}-4\pi^2-\frac{16}{3}\xi+4\xi^2.$$

In using ansatz (4) as the solution of Eq. (5), we find a rather weak gauge dependence for the parameters d and b and also N_c, found from (4) with $\beta_c=1/16$:

$$d(\xi)=\{2.53;2.62;2.67;2.62\}, \quad b(\xi)=\{6.52;7.19;8.24;9.51\},$$

$$N(\xi)=\{3.27;3.32;3.19;2.77\} \quad \text{for } \xi=\{0.0;0.3;0.7;1.0\},$$

respectively. For the values $\xi=2/3$ and $\xi=-2$, the solution of Eq. (5) in the form in (4) is exact.

To find the exact critical value N_c from (5), we set $\alpha_c = -1/4$. We find that the quantity

$$N_{c,\pm}=\frac{8}{\pi^2}[(2+\xi)\pm\{(2+\xi)^2+4f(\xi)+\varphi(\xi)/4\}^{1/2}] \tag{6}$$

has the values

$$N_{c,+}=\{3.31;3.35;3.09;2.81\}, \quad N_{c-}=\{-0.07;0.38;1.29;1.88\}$$

for

$$\xi=\{0.0;0.3;0.7;0.9\}.$$

A curious fact follows from Eq. (6): Incorporating the $1/N$ correction leads to the appearance of a second critical point (for $0.05 < \xi < 0.95$), below which the chiral symmetry is not broken. A dynamic mass is generated only in the interval between these critical points. At $\xi \gtrsim 0.95$, this interval collapses, and the chiral symmetry remains unbroken. For small values of the gauge parameter ($\xi \lesssim 0.05$), the new critical point does not appear.

Incorporating the coefficients of the exact solution of the Schwinger–Dyson equation in the first two orders of a $1/N$ expansion may slightly alter the numerical values for the critical points $N_{c,+}$ and $N_{c,-}$, but there is no change in the qualitative picture of the decrease in the interval of N values in which a dynamic mass is generated.

The solution of Eq. (5),

$$\beta_{\pm}=\frac{1}{2L}\left[2+\xi+\frac{\varphi(\xi)}{L}\pm\left\{\left(2+\xi+\frac{\varphi(\xi)}{L}\right)^2+4f(\xi)\right\}^{1/2}\right], \tag{7}$$

has a simple expression in the Landau gauge for the + component at $L\sim L_c$:

$$\beta_+(\xi=0)\approx\left(1+\sqrt{\frac{7}{3}}\right)\frac{1}{L}+\left(1+\sqrt{\frac{3}{7}}\right)\frac{\varphi(0)}{2L^2}\approx\frac{2.52}{L}\left(1-\frac{6.52}{L}\right)$$

with coefficients approximately the same as those found in Ref. 5.

3. The breaking of chiral symmetry in 3D QED is currently being studied actively because QED is a simple but important model in research on the critical behavior of such complex theories as QCD. However, different groups[4,8–11] have derived very contradictory results. Examining the leading order of a $1/N$ expansion of the Schwinger–Dyson equation, Appelquist *et al.*[4] (see also Sec. 1 of the present letter) showed that the mass of fermions is generated for $N < N_c \approx 3.24$. On the other hand, Pennington *et al.*[8] and also Pisarski,[9] using different methods to solve the Schwinger–Dyson equation, found that a dynamic mass arises for all N, vanishing only in the limit $N \to \infty$. There is also a nonperturbative solution of the Schwinger–Dyson equation, found by Atkinson *et al.*[10] That solution preserves the chiral symmetry at fairly large values of N. Monte Carlo calculations[11] confirm that there is a critical value N_c.

Since N_c is not large, the contributions of orders other than the leading order may turn out to be extremely important. Incorporating terms $O(1/N^2)$ in the analysis, Nash[5] studied the solution of the Schwinger–Dyson equation in the form of ansatz (4) and demonstrated a pronounced weakening of the gauge dependence which prevailed in the result of the leading order. This property stems from the form of ansatz (4), in which the nonleading approximation is actually taken as a $1/N$ correction to the result of the leading order, and it is analogous to the absence of a dynamic mass (i.e., to the absence of gauge-invariant terms also) when a simple perturbation theory in $1/N$ is used without the Schwinger–Dyson equation.[1]

In the present letter we have incorporated a $1/N$ correction exactly. We have found a strong gauge dependence for not only quantitative but also the qualitative characteristics of the result. The result cannot, in general, be represented as ansatz (4). Accordingly, the hopes for improving the situation in the study of the critical behavior of 3D QED by incorporating terms $O(1/N^2)$ were not justified. We would point out, however, that in the Landau gauge, in which Ward's identities are approximately the same as their realization, even when a free vertex is used in the Schwinger–Dyson equation, the addition of a $1/N$ correction does not alter the qualitative predictions of the leading order, and it causes only exceedingly slight changes in the quantitative predictions. Consequently, the solution offered by Appelquist *et al.* seems to have found solid support in this case.

In conclusion, we would like to point out, following Pisarski,[9] that it would be difficult at this point to claim that we have a good understanding of the breaking of chiral symmetry in such a complex theory as QCD in four dimensions if we still lack a complete understanding of this process in the simple model of 3D QED.

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[1)]Following Nash,[5] we introduce a nonlocal gauge-fixing term. This possibility is analyzed in detail in Ref. 6.

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